201709 Math 122 A01 Quiz #6

#V00:________________ Name: Key

This is a take home quiz, and due by 12:20PM on Friday, December 1. Between 11:30 and 12:20, quizzes can be submitted in Maclaurin D288. Before that, they can be dropped off at your instructor's office or at the Math and Stats Office, DTB A425, if for some reason your instructor is not in.

This quiz has 2 pages and 5 questions. There are 15 marks available. Except where explicitly noted, is necessary to show clearly organized work in order to receive full or partial credit. You can write your answers on this paper, or on your own paper.

1. [2] Fill in the blanks. No reasons are necessary.

   (a) If $1000k = 2^6 5^7 7^2 11^4 13^1$, then the prime factorization of k is $2^3 5^4 7^2 11^4 13^1$.

   (b) If p is a prime number and $a \not\equiv 0 \pmod{p}$, then $gcd(a, p) =$ 1.

   (c) Let $A = \{1, 2, \ldots, 5\}$ and $\mathcal{R} = \{(1,1), (2,2), (3,3), (4,4)\}$. Which of the properties reflexive, symmetric, transitive, antisymmetric does the relation $\mathcal{R}$ have?
   symmetric, transitive, antisymmetric

   (d) Let $A = \{1, 2, 3, 4, 5\}$. Take it as given that the relation $R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,4), (4,5), (5,4), (5,5)\}$ is an equivalence relation on A. The partition of A given by the different equivalence classes of $\mathcal{R}$ is $\{\{1,2,3\}, \{4,5\}\}$.

2. Let a, b, and c be any integers such that a and b are relatively prime, $a \mid c$ and $b \mid c$.

   (a) [1] Show that there are integers x and y so that $acx + bcy = c$.

   Since $gcd(a,b) = 1$, there exist integers x, y such that $ax + by = 1$

   $\therefore acx + bcy = c$

   (b) [2] Use part (a) to prove that $ab \mid c$.

   We have $x, y \in \mathbb{Z}$ s.t. $acx + bcy = c$.
   Now $ab \mid ac$ b/c $a \mid a$ & $b \mid c$; and
   $ab \mid bc$ b/c $a \mid c$ & $b \mid b$.

   $\therefore ab \mid acx + bcy = c$.

3. [3] Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(x) = 5x - 8$. Is f an onto function? Give a proof or counterexample, as appropriate.

No. Suppose $5x-8=y$.
Then $x = \frac{y+8}{5}$.

$\therefore$ For $y=0$ to be a value of f, we must have $x = 8/5$ in the domain.
We don't $\therefore$ f is not onto.

4. [3] Let $n = (346)_{11}$. Find the last digit in the base 10 representation of n^{2007}.

We need d s.t. $0 \le d < 10$ & $n^{2007} \equiv d \pmod{10}$

$n^{2007} = (3\times 11^2 + 4\times 11 + 6)^{2007}$
$\equiv (3\times 1^2 + 4 \times 1 + 6)^{2007}$ b/c $11 \equiv 1 \pmod{10}$
$\equiv 13^{2007} \equiv 3^{2007} \equiv (3^4)^{501}\, 3^3$
$\equiv 1^{501}\, 27 \equiv 7 \pmod{10}$

$\therefore$ The last digit is 7.

5. [4] For each of the following, state whether the given assertion is True or False. If the assertion is false then give an example to show it is false, or an explanation of why its truth would contradict some theorem. And if the assertion is true, prove it. Note: all solutions to this question are short!

(a) If $\lfloor n/2 \rfloor = \lceil n/2 \rceil$, then n is even. $\boxed{\text{TRUE}}$

n odd $\Rightarrow n = 2k+1$ for some $k \in \mathbb{Z}$
$\Rightarrow k = \lfloor n/2 \rfloor < \lceil n/2 \rceil = k+1$

(b) The function $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \sqrt{x^2}$ is 1-1. $\boxed{\text{FALSE}}$

$f(1) = \sqrt{1^2} = 1$ & $f(-1) = \sqrt{(-1)^2} = 1$
but $1 \neq -1$.

(c) If $f : A \to B$ is a function, then $f \circ \iota_A = f$. $\boxed{\text{TRUE}}$

$f \circ \iota_A : A \to B$ so they have the same domain & target. And, for $a \in A$, $f\circ\iota_A(a) = f(\iota_A(a))$
$= f(a)$.

(d) If $2k \equiv 4 \pmod 6$ then $k \equiv 2 \pmod 3$. $\boxed{\text{TRUE}}$

$2k = 4 + 6\ell \Rightarrow k = 2 + 3\ell$
$\Rightarrow k \equiv 2 \pmod 3$