MATH 122
Quiz #6 - Solutions

1. a) $k \equiv -15 \pmod{11}$

$$\begin{aligned} 5k^2 - 122k &\equiv 5(-15)^2 - 122(-15) \pmod{11} \\ &\equiv 5(225) - 122(-15) \pmod{11} \\ &\equiv 5(5) - 1(-4) \pmod{11} \\ &\equiv 25 + 4 \pmod{11} \\ &\equiv 29 \pmod{11} \\ &\equiv 7 \pmod{11} \end{aligned}$$

The remainder is 7.

b) False. Say $x = 1$ and $y = 3$.
Then $x^2 \equiv y^2 \pmod 8$ since $1^2 \equiv 3^2 \equiv 1 \pmod 8$
but $x \not\equiv y \pmod 4$ since $1 \not\equiv 3 \pmod 4$.

2. a) $f(x)$ is 1-1.

Suppose
$$\begin{aligned} f(x_1) &= f(x_2) \\ 4x_1 - 7 &= 4x_2 - 7 \\ 4x_1 &= 4x_2 \\ x_1 &= x_2 \quad \blacksquare \end{aligned}$$

b) $f(x)$ is not onto.
Look at a fixed $b \in \mathbb{Z}$.
If f was onto there would be an $x \in \mathbb{Z}$ such that $f(x) = b$.
Then $4x + 12 = b$
$x = \frac{b-12}{4}$. Notice that for $b = 1$, $x = \frac{1-12}{4} = \frac{-11}{4} \notin \mathbb{Z}$
So no $x \in \mathbb{Z}$ maps to $b = 1$. $\therefore f$ is not onto. $\blacksquare$

3. a) reflexive?

$\forall x \in A$, the sum of the digits of x = sum of digits of x, so $(x,x) \in R$ $\forall x \in A$. $\therefore R$ is reflexive.

symmetric?

Suppose $(x,y) \in R$, Then sum of digits of x = sum of digits of y. This says sum of digits of y = sum of digits of x so also $(y,x) \in R$ $\therefore R$ is symmetric.

transitive?

Suppose $(x,y) \in R$ and $(y,z) \in R$. Then

sum of digits of x = sum of digits of y

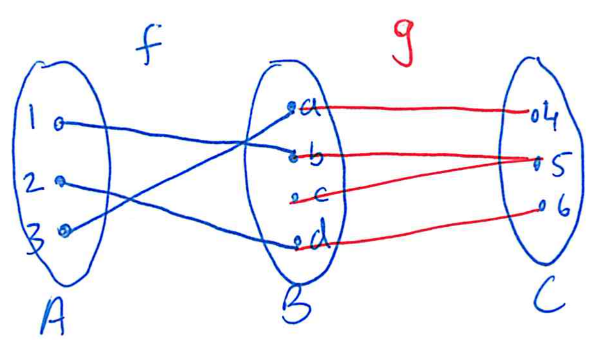
and sum of digits of y = sum of digits of z.

But then sum of digits of x = sum of digits of z so $(x,z) \in R$ also. $\therefore R$ is transitive.

Since R is reflexive, symmetric, and transitive it is an equivalence relation.

b) There are 18 distinct equivalence classes, one for each possible sum of the digits.

4. a)



One possible $g: B \to C$ so that $g \circ f$ is onto is

$g = \{(a,4), (b,5), (c,5), (d,6)\}$.

When we decide the mapping $g: B \to C$ there are

3 options for a
2 options for b
1 option for d
3 options for c

So there are $3 \cdot 2 \cdot 1 \cdot 3 = 18$ possible functions.

(Note that c must be mapped somewhere for g to be a function.)

5. a) In order to be reflexive we have
$(1,1), (2,2), (3,3) \in R$.

We are told that $(2,1), (1,3) \in R$.

Since R is transitive this implies $(2,3) \in R$.

Since R is antisymmetric this implies
$(1,2), (3,1), (3,2) \notin R$.

$\therefore R = \{(1,1), (2,2), (3,3), (2,1), (1,3), (2,3)\}$

b) In order to be reflexive we have
$(1,1), (2,2), (3,3) \in R$.

If $(1,2) \in R$ then $(2,1) \in R$ for R to be symmetric.

But if $(1,2), (2,1) \in R$ then R is not antisymmetric.

Hence $(1,2), (2,1) \notin R$.

Likewise $(1,3), (3,1) \notin R$
and $(2,3), (3,2) \notin R$.

$\therefore R = \{(1,1), (2,2), (3,3)\}$.