201909 M122 A01 Q6 Sol'ns.

1. No. If $y = \frac{1}{x} + 2$ then we must have $x = 1/(y-2)$.

$\therefore$ There is no x such that $f(x) = 2$.

2. a) By inspection, f is 1-1 and onto. $\therefore$ f is invertible

b)

x	1	2	3	4	5	6
$f(f(x))$ $=$ $f \circ f(x)$	6	4	1	5	2	3

We have $f(x) = y \iff f \circ f(y) = x$

$\therefore$ $f^{-1} = f \circ f$.

3. Since R is reflexive $(1,1), (2,2), (3,3) \in R$
Since $(2,1), (1,3) \in R$ we have $(2,3) \in R$ by transitivity
$\therefore$ $(1,2), (3,1), (3,2) \notin R$ by anti-symmetry.

$\therefore$ $R = \{(1,1), (2,2), (3,3), (2,1), (1,3), (2,3)\}$.

4. a) reflexive : the product of the digits of x is the same as the product of the digits of x
$\therefore$ R is reflexive

symmetric : Suppose the product of the digits of x is the same as the product of the digits of y.
Then the product of the digits of y is the same as the product of the digits of x
$\therefore$ R is symmetric.

transitive: Suppose the product of the digits of x equals the product of the digits of y, and the product of the digits of y equals the product of the digits of z. Then the product of the digits of x equals the product of the digits of z.
$\therefore$ R is transitive.

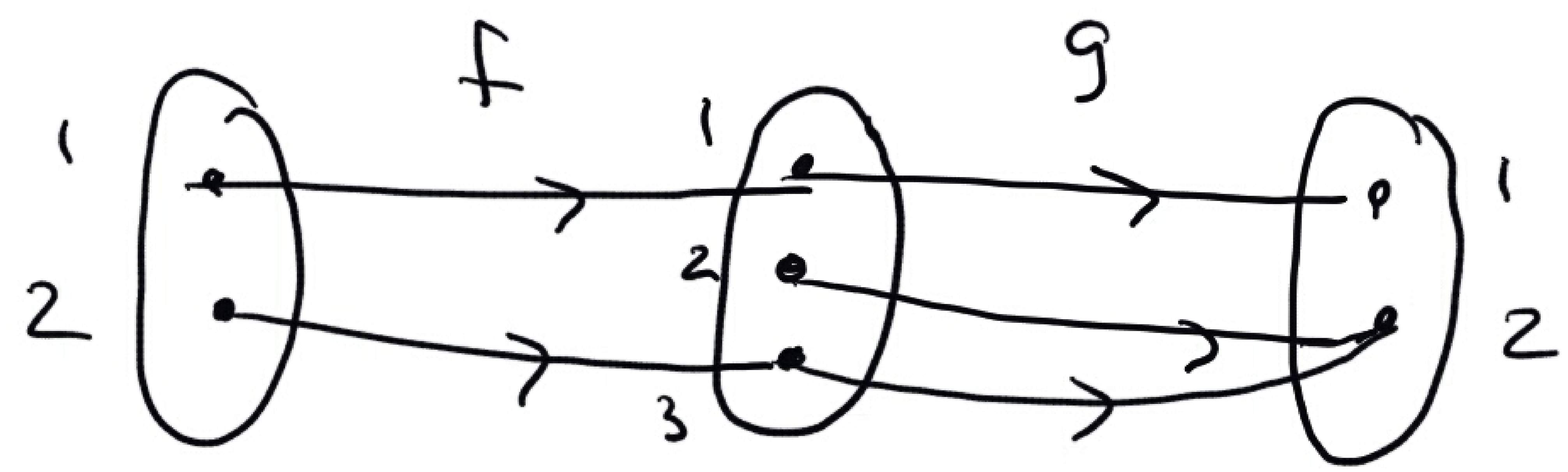
$\therefore$ R is an equivalence relation

b) # equivalence classes = # different products
~~[20], [21], ..., [29], [34], [35], ... [39], [44], [45], [47],~~
~~[48], [49], [55], [56], [57] cover the possible products~~

$\therefore$ 24 different equivalence classes

[20], [21], ..., [29], [31], [33], [35], [38], [39], [45], [47],
[48], [49], [51], [55], [56], [57]

5. a. True



$g \circ f$ is the identity function, $\therefore$ it is 1-1.
but g is not 1-1 because $g(2) = g(3)$ and $2 \neq 3$.

b. True. $A = \mathbb{Z}$, which is countable

c. True. $(0,1)$ is uncountable, so any set that contains $(0,1)$ as a subset is uncountable.

d. True. For example ϕ is always a subset, and it is countable.