201609 Math 122 [A03] Quiz #6 Solution Ideas

1. A bit of trial and error leads to observing that 11 can not be written as a sum of threes and sevens, but each of 12, 13 and 14 can. This turns out to be enough to make the proof work. The argument is almost identical to the example at the top of page 8 in the Induction notes

2. This is just like the example at the start of Section 7.8.3 on page 26 of the Induction notes.

   (a) $a_5 = 431$

   (b) Using the method demonstrated on page 27 of the Induction notes leads to $a_n = 2 \cdot 6^k - 1$ when $n = 2^k$. To eliminate the k, notice that $k = \log_2(n)$. Thus, $a_n = 2 \cdot 6^{\log_2(n)} - 1$

3. In each part below, classify the given set as countable or uncountable, and supply a brief justification for your answer.

   (a) $\mathbb{Q} \cap (0, 1)$ is a subset of $\mathbb{Q}$. Any subset of a countable set of countable.

   (b) The closed interval of real numbers, $[0, 2]$ contains $(0, 1)$. Any set that contains an uncountable subset is uncountable.

   (c) The set of all integers with at most 2^{100} digits in their base 16 representation is finite. Any finite set is countable.

   (d) If $\mathcal{P}(\mathbb{N})$ were countable, it would have the same cardinality as $\mathbb{N}$. Since $\mathcal{P}(\mathbb{N})$ is an infinite set and no set has the same cardinality as its power set, it follows that $\mathcal{P}(\mathbb{N})$ is uncountable.

4. In a group of 35 ex-athletes, 17 play golf, 20 go cycling, and 12 do yoga. Exactly 8 play golf and go cycling, 8 play golf and do yoga, 7 go cycling and do yoga, and 4 do all three activities. How many of the ex-athletes do none of these activities?

   Make and fill in a Venn Diagram as in the example on page 3 of the notes on the Principle of Inclusion and Exclusion that accompanied this quiz. The answer is five.