201801 Math 122 A01 Quiz #6

This is a **take-home quiz**. It is due at the start of class: 8:30AM on Thursday, April 5. Late quizzes will not be accepted except in documented cases of illness, emergency, accident, or affliction.

This quiz is **to be done individually**. You may consult any pre-existing resources on the course page or elsewhere. Any form of collaboration or communication between persons is not permitted.

There are 4 questions with marks as shown, and a total of 15 marks available. For each question, it is necessary to show clearly organized work in order to receive full or partial credit. **Answers must be written in your own words in a way that reflects your own understanding**.

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1. [4] Answer each question **TRUE** or **FALSE**. In each case, give a brief explanation for your answer.
   (a) The last digit of 122^{122} is 4.
   (b) If $k \equiv -5 \pmod 7$ the the remainder when $4k^3 + 6k$ is divided by 7 is 2.
   (c) If $x^2 \equiv y^2 \pmod 4$, then $x \equiv y \pmod 2$.
   (d) $4 \times 25 + 6 \times 15^5 - 8 \equiv 8 \pmod 5$.

2. Let $f : \mathbb{Q} \to \mathbb{Q}$ be the function defined by $f(x) = \frac{3}{2}x - \frac{7}{3}$.
   (a) [3] Determine whether f is 1-1, and whether it is onto. In each case give a proof or counterexample, as appropriate.
   (b) [1] Is f invertible? If so, find a formula for f^{-1}. If not, explain why not.

3. Let $\sim$ be the relation on $A = \{10, 11, \ldots, 122\}$ defined by $x \sim y \Leftrightarrow$ the second digit in the decimal representation of x equals the second digit in the decimal representation of y.
   (a) [2] Prove that $\sim$ is an equivalence relation.
   (b) [1] How many distinct equivalence classes are there? Explain.

4. [4] Let $\mathcal{R}$ be a relation on $A = \{1, 2, 3, 4\}$. Answer each question **TRUE** or **FALSE**. In each case, give a brief explanation for your answer.
   (a) If $\mathcal{R}$ is symmetric and transitive, and $(1, 2) \in \mathcal{R}$, then $(1, 1) \in \mathcal{R}$.
   (b) If $\mathcal{R}$ is antisymmetric and transitive, and $(1, 2), (2, 3) \in \mathcal{R}$, then $(3, 1) \notin \mathcal{R}$.
   (c) If $(4, 4) \in \mathcal{R}$, then $\mathcal{R}$ is reflexive.
   (d) It is possible for $\mathcal{R}$ to be reflexive, symmetric, and antisymmetric (i.e. to have all 3 properties).