Exchange Rates and Fundamentals: Evidence for Canada

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Abstract

We use the methods of Engel and West (2005) to evaluate forward looking models of the exchange rate for Canada. The first prediction we examine is whether the CAD-USD exchange rate Granger causes future fundamentals. The second prediction we examine is whether solutions for the exchange rate in terms of expected present discounted value measures of fundamentals, constructed from estimated dynamic models, better match the data when the exchange rate is included in the forecasting model. We find evidence of Granger causality, using the methods of Toda and Yamamoto (1995) for integrated series, for a number of fundamentals with the strongest results for commodity price indices. We further find evidence of strong correlations between actual and predicted exchange rates based on forward looking models of the exchange rate, particularly when using commodity prices as a fundamental. In summary, we have evidence that the Canada-US exchange rate is consistent with forward looking models of the exchange rate and that its variation in recent years is consistent with anticipated movements in commodity prices as predicted by these models.

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1 Introduction

The Canadian dollar is widely viewed as a commodity currency, a currency for which there is a significant relationship between commodity prices and currency value. The seminal study for commodity currencies is Chen and Rogoff (2003), which examines the Canadian, Australian, and New Zealand currencies. Although in the sample period Chen and Rogoff consider there is little relationship between the Canadian US dollar exchange rate and commodity prices, in recent years the relationship has strengthened; see for example Berg, Guerin, and Imura (2016).

Our analysis uses the methods in Engel and West (2005), who adapt the methods of Campbell and Shiller (1987) to assess the forward looking nature of the exchange rate. As Engel and West show, most asset-based exchange rate models give rise to a solution for the exchange rate in terms of the expected present discounted value of future fundamentals. This has two implications. The first is that the exchange rate should Granger cause future fundamentals. This is a relatively weak prediction of these models that has been investigated by a number of studies, most notably Chen, Rogoff, and Rossi (2010). The second implication more explicitly relies on the forward looking solutions for the exchange rate. These solutions depend upon the expected present discounted value of a fundamental (or fundamentals), which can be constructed from estimated dynamic models. Engel and West argue that if the forward looking model is correct then solutions based upon dynamic models that include the exchange rate will better match the data than solutions based upon models that do not include the exchange rate.

We employ these methods for the Canada-US dollar exchange rate over the last fifteen years, where there is an evident strong relationship between commodity prices and the exchange rate. We first establish that the exchange rate Granger causes a number of fundamentals, including the Bank of Canada's commodity price index for which we get the strongest results. We also show that this is a new phenomena, not evident in earlier samples. We are not the first to consider Granger causality of commodity prices for the Canadian dollar; Chen *et al* (2010) examine the Canadian dollar as well as other currencies. However, we are the first to our knowledge to use the Granger causality methods of Toda and Yamamoto (1995), which are designed to handle causality tests for possibly integrated series. This allows us to focus on Granger causality for the levels of the exchange rate and associated fundamentals, in contrast to using the differences. Our results show much stronger evidence of Granger causality than previous studies.

We next estimate dynamic bivariate models for the exchange rate and various fundamentals, accounting where necessary for non-stationarity and cointegration, and use these to construct solutions for the exchange rate based upon forward looking exchange rate models. This adds to the analysis in Engel and West in two ways. First, we include commodity prices, which they do not. Second, we have evidence of cointegration in a number of instances, which they do not. What we find in a number of instances is strong correlation between the actual and predicted exchange rate; this is particularly true when we use commodity prices as the fundamental but also holds true when we use broader relative price measures as the model fundamental. This contrasts with the generally weak results in Engel and West and their very weak results specific to Canada. In summary, we have evidence that the Canada-US exchange rate is consistent with forward looking models of the exchange rate and that its variation in recent years is consistent with anticipated movements in commodity prices as predicted by these models.

2 Empirical Framework

2.1 Theoretical Framework

The framework for the empirical work is Engel and West (2005), hereafter EW, which emphasizes the means by which it is possible to investigate and, to a limited extent, test the forward looking nature of the nominal exchange rate with respect to macroeconomics fundamentals. This framework is extended by Chen, Rogoff, and Rossi (2010) to include commodity prices as a possible fundamental for currencies such as the Canadian dollar.

EW (see also Chen *et al*, 2010) demonstrates that many standard open economy models provide forward looking solutions for the nominal exchange rate, solutions that depend upon macroeconomic fundamentals. Based upon these models, EW provide a general framework for the exchange rate as a function of observable and unobservable fundamentals:[1]

$$s_t = (1-b)(f_{1t} + z_{1t}) + b(f_{2t} + z_{2t}) + bE_t s_{t+1} \tag{1}$$

where s_t is the nominal exchange rate (logarithm), defined as the domestic currency price of foreign currency, $f_{it},\ i = 1, 2$ denotes observable fundamentals, $z_{it},\ i = 1, 2$ unobservable fundamentals, and b is referred to as the discount factor. The exact nature of the fundamentals and the discount factor depend upon the specific model one has in mind. Both f_{it} and z_{it} can be single variables or linear combination of variables, and both may depend upon structural parameters.

Equation (1) is the basis for the empirical analysis; however, in order to proceed it is helpful to be more specific about the underlying exchange rate models as these determine which fundamentals one might consider, how these fundamentals and unobservables enter equation (1), and what structural parameters underlie the discount factor b. For this purpose, we next present two macroeconomic exchange rate models EW consider with an amendment for commodity prices as a determinant of the real and hence nominal exchange rate. The purpose is not to present these models as the basis for estimation and testing but rather to motivate the set of fundamentals we focus on and how we need treat these fundamentals in the empirical analysis.

2.1.1 Money-income Model

The traditional model-income exchange rate model, as presented in EW, is:

$$\begin{aligned}
m_t &= p_t + \gamma y_t - \alpha i_t + \upsilon_{mt} \\
m_t^* &= p_t^* + \gamma y_t^* - \alpha i_t^* + \upsilon_{mt}^* \\
s_t &= p_t - p_t^* + q_t \\
i_t &= i_t^* + E_t s_{t+1} - s_t + \rho_t
\end{aligned}$$

The first two equations are money market equilibrium conditions for the domestic and foreign economy. The domestic variables are: m_t, money supply; y_t, output; p_t, price level, all in logarithms, the nominal interest rate i_t, and a money demand shock υ_{mt}. The foreign variables are the same but denoted with an asterisk. The home and foreign money demand elasticities are assumed to be

[1] EW discuss exchange rate models that provide forward looking solutions for the exchange rate based upon financial markets, specifically through uncovered interest rate parity. Chen *et al* (2010) in addition discuss how intertemporal dependent economy models can generate forward looking solutions for the (real) exchange rate.

the same. The third equation defines the real exchange rate q_t and the final equation is uncovered interest rate parity with ρ_t a time varying risk premium. E_t is the conditional expectations operator. Combining we obtain a representation for the nominal exchange rate comparable to equation (1),

$$s_t = \frac{1}{1+\alpha}\left(m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (\upsilon_{mt} - \upsilon_{mt}^*) - \alpha\rho_t\right) + \frac{\alpha}{1+\alpha}E_t s_{t+1}$$

From this expression, we see that $b = \alpha/(1+\alpha)$. The first two rows of the table below present the two fundamentals that are immediate from the equation above. We could also consider $m_t - m_t^* - \gamma(y_t - y_t^*)$ as a single fundamental, as EW do; however, we choose to focus on the components rather than theoretical combinations. Row (3) extends the EW presentation to include domestic and foreign velocity equations: $m_t + v_t = p_t + y_t$ and $m_t^* + v_t^* = p_t^* + y_t^*$.

Money-Income Model				
	Fundamental 1 f_{1t}	Fundamental 2 f_{2t}	Unobservable 1 z_{1t}	Unobservable 2 z_{2t}
(1)	$m_t - m_t^*$	-	$q_t - \gamma(y_t - y_t^*) - (\upsilon_{mt} - \upsilon_{mt}^*)$	$-\rho_t$
(2)	$-\gamma(y_t - y_t^*)$	-	$m_t - m_t^* + q_t - (\upsilon_{mt} - \upsilon_{mt}^*)$	$-\rho_t$
(3)	$p_t - p_t^*$	-	$q_t - (v_t - v_t^*) - (\upsilon_{mt} - \upsilon_{mt}^*)$	$-\rho_t$
(4)	$\mu_0 - \mu_1 cx_t$	-	$m_t - m_t^* - \gamma(y_t - y_t^*) + \epsilon_t - (\upsilon_{mt} - \upsilon_{mt}^*)$	$-\rho_t$

We can add to the money-income model the assumption that the real exchange rate q_t depends concurrently upon commodity prices, cx_t, as well as other unspecified determinants, ϵ_t:

$$q_t = \mu_0 - \mu_1 cx_t + \epsilon_t \tag{2}$$

The explanation for this relationship, which is the focus of Chen and Rogoff's initial examination of commodity currencies, is that commodity prices represent an exogenous and significant component of the terms of trade, a theoretical determinant of the real exchange rate in many open economy models.[2] Here we follow Chen and Rogoff (2003) and use a real commodity price index measure, deflating a US dollar commodity price index by the US price level. We expect $\mu_1 > 0$, indicating that an increase in commodity prices (an improvement in the terms of trade) will appreciate the real exchange rate. A key motivation for focusing only on the commodity component of the terms of trade is that it can reasonably be treated as exogenous for a small open economy such as Canada. This is in contrast to the other fundamentals; this issue is discussed in the empirical work below.

Since the parameters μ_0 and, more importantly, μ_1 are not known, this in principle simply replaces the (assumed) unobservable real exchange with an unobservable function of commodity prices. However, under certain conditions it is possible to proceed with estimates of these parameters; this is discussed further below.[3]

Two points are worth making concerning the decompositions above. First, there is no requirement that the unobservables be stationary, though this does have some implications for estimation strategies. Second, whether a variable is viewed as an observable fundamental or otherwise is

[2] See Chen *et al* (2010) for a discussion. In the context of the Canadian dollar, see also the earlier work Amano and van Norden (1995).

[3] Briefly, we can recover the parameter estimates when we can use cointegration estimation methods as discussed below. The same applies to the γ parameter on $y_t - y_t^*$. Note that if we relax the symmetry conditions then the situation is more complicated for most of these fundamentals.

something of a matter of taste. For example, one might argue that while the real exchange rate is measurable it is perhaps not very well measured. Similarly, available money measures may not adequately capture payment methods. In what follows, we follow Engel and West and focus on bivariate relationships — that is, one fundamental at a time — and sweep the other fundamentals into the unobservables. One reason to provide a table such as the one above is so that we are clear what is at least theoretically being set aside in the various estimation setups considered below.

2.1.2 Taylor Rule Model

The home country central bank (Canada) has the following monetary policy (Taylor) rule:

$$i_t = \beta_0(s_t - \bar{s}_t) + \beta_1 y_t^g + \beta_2 \pi_t + \nu_t \tag{3}$$

Interest rates and exchange rates are as defined previously. In addition, $\bar{s}_t$ is the equilibrium or target level for the exchange rate; inflation is $\pi_t = p_t - p_{t-1}$, with p_t the log domestic price level; and the output gap is y_t^g. All of the coefficients are assumed to be positive and further by assumption $\beta_2 > 1$ (the Taylor principle). The error term, possibly capturing other non-systematic concerns for monetary policy, is denoted ν_t.[4]

The foreign country also follows a Taylor rule but with no concern for the exchange rate:

$$i_t^* = \beta_1^* y_t^{*g} + \beta_2^* \pi_t^* + \nu_t^* \tag{4}$$

EW specify the target exchange rate to be based on purchasing power parity; we extend this to allow for an equilibrium real exchange rate, $\bar{q}_t$:

$$\bar{s}_t = p_t - p_t^* + \bar{q}_t \tag{5}$$

As above, the equilibrium real exchange rate is assumed to be a linear function of commodity prices and other unspecified factors, ϵ_t: $\bar{q}_t = \mu_0 - \mu_1 cx_t + \epsilon_t$. Nominal and domestic interest rates are related by uncovered interest rate parity:

$$i_t = i_t^* + E_t s_{t+1} - s_t + \rho_t \tag{6}$$

where E_t is the expectations operator conditional on information at time t and ρ_t is a risk premium or some other stationary deviation from UIP.

Combining equations (3)-(6), we get:

$$\begin{aligned} s_t = & \frac{\beta_0}{1+\beta_0}\left(p_t - p_t^* + \mu_0 - \mu_1 cx_t + \epsilon_t\right) + \\ & \frac{1}{1+\beta_0}\left(\beta_1(y_t^{*g} - y_t^g) + \beta_2(\pi_t^* - \pi_t) + \nu_t^* - \nu_t - \rho_t\right) + \frac{1}{1+\beta_0} E_t s_{t+1} \end{aligned} \tag{7}$$

where for simplicity of presentation we have assumed (as do EW) that the monetary authorities in each country have similar responses to inflation and the output gap: $\beta_j = \beta_j^*,\ j = 1, 2$. The decomposition into observables and unobservables is in the table below. For this model, the discount

[4] Note that this presentation is reversed in terms of home and foreign country relative to that in EW. Note also that for the moment, no distinction is made between different possible price indices, i.e. GDP deflators versus consumer price indices. The empirical work examines both of these indices.

factor b is equal to $1/(1+\beta_0)$. As in the previous table we focus on single components (of country relatives) rather than combinations of fundamentals.

Taylor Rule Model				
	Fundamental 1 f_{1t}	Fundamental 2 f_{2t}	Unobservable 1 z_{1t}	Unobservable 2 z_{2t}
(1)	$p_t - p_t^*$	n/a	$\bar{q}_t$	$\beta_1(y_t^{*g} - y_t^g) + \beta_2(\pi_t^* - \pi_t) + \nu_t^* - \nu_t - \rho_t$
(2)	$\mu_0 - \mu_1 cx_t$	n/a	$p_t - p_t^* + \epsilon_t$	$\beta_1(y_t^{*g} - y_t^g) + \beta_2(\pi_t^* - \pi_t) + \nu_t^* - \nu_t - \rho_t$
(3)	n/a	$-\beta_2(\pi_t - \pi_t^*)$	$p_t - p_t^* + \bar{q}_t$	$\beta_1(y_t^{*g} - y_t^g) + \nu_t^* - \nu_t - \rho_t$
(4)	$i_t - i_t^*$	n/a	$p_t - p_t^* + \bar{q}_t$	$\rho_t - (1/(1-\beta_0))\boldsymbol{\beta}'\mathbf{z}_t$

EW note that the model can be re-expressed to identify the difference in nominal interest rates as a fundamental. In the current context, this can be done as follows. We have from above,

$$\begin{aligned} i_t - i_t^* &= \beta_0(s_t - \bar{s}_t) + \beta_1 y_t^g + \beta_2 \pi_t + \nu_t - (\beta_1^* y_t^{*g} + \beta_2^* \pi_t^* + \nu_t^*) \\ &= \beta_0(s_t - \bar{s}_t) + \boldsymbol{\beta}'\mathbf{z}_t \end{aligned}$$

where the notation should be obvious. Using the real exchange rate definition and uncovered interest rate parity,

$$s_t = \beta_0(i_t - i_t^*) + \beta_0(p_t - p_t^* + \bar{q}_t) + (1-\beta_0)\rho_t - \boldsymbol{\beta}'\mathbf{z}_t + (1-\beta_0)E_t s_{t+1}$$

In this case, $b = 1 - \beta_0$. The decomposition is presented in the final row of the table above. (Note that for consistency, the unobservable term $\boldsymbol{\beta}'\mathbf{z}_t$ is arbitrarily scaled by $1/(1-\beta_0)$.)

Together, the two models present a set of observable fundamentals suitable for consideration. We choose, however, to make one final adjustment, replacing $m_t - m_t^*$ with $p_t + y_t - p_t^* - y_t^*$, the difference in nominal GDP; this introduces a velocity term into the unobservable but otherwise leaves the decompositions unchanged. This has the advantage of not requiring a particular choice of monetary aggregates. Nor do we face any difficulties of determining comparable monetary aggregates across the two countries. To keep notation compact, let $n_t \equiv p_t + y_t$ and similarly for n_t^*. Our empirical analysis thus considers the following set of fundamentals to be used in the empirical analysis of equation (1):

$$-cx_t,\ -(y_t - y_t^*),\ (n_t - n_t^*),\ (p_t - p_t^*),\ (i_t - i_t^*),\ -(\pi_t - \pi_t^*)$$

where for future reference we have signed the fundamentals so that the theoretical models predict a positive correlation.

Even within the very stylized theoretical frameworks considered here, some of these variables enter into the exchange rate solution with unknown parameter(s) beyond the discount term b. For example, the inflation differential is weighted by the unknown parameter β_2. Moreover, if the policy rules or the money demand functions were not symmetric the unknown parameters are even greater in number and the relationships more complex. For some of the empirical analysis, notably the Granger causality analysis, this is not of significant concern since the unknown parameters do not pose any limitations in this regard. For estimation that models the forward-looking solution, however, the unknown parameters may play a role. This is discussed further below.

2.2 Empirical Representations of the Theoretical Models

The process in equation (1), under the condition $\lim_{j\to\infty} b^j E_t s_{t+j} = 0$ that rules out rational bubbles, has the solution:

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t(f_{1t+j} + z_{1t+j}) + b\sum_{j=0}^{\infty} b^j E_t(f_{2t+j} + z_{2t+j}) \tag{8}$$

Equation (8) is the basis for the empirical work.

2.2.1 Granger Causality

The first implication is due to EW's extension of a basic result for forward looking models due to Campbell and Shiller (1987). The basic Campbell and Shiller result is as follows. Assume that there is one fundamental and no unobservables. Then equation (8) reduces to,

$$s_t = (1-b)E_t \sum_{j=0}^{\infty} b^j f_{t+j} = f_t + E_t \sum_{j=1}^{\infty} b^j \Delta f_{t+j}$$

$$s_t - f_t = E_t \sum_{j=1}^{\infty} b^j \Delta f_{t+j} \tag{9}$$

where the second equality in the first line is a standard re-representation for this solution and the second line is written to define the variable $s_t - f_t$. If the fundamental f_t is I(1), s_t and f_t are cointegrated with vector $(1, -1)'$. Campbell and Shiller then demonstrate (footnote 7) that this solution implies that $s_t - f_t$ Granger causes Δf_t. By the same logic, if f_t, and hence s_t, is I(0) then s_t must Granger cause (GC) f_t.

As EW note, equation (8) above is somewhat more complicated since the information set is much larger, including multiple observable and unobservable variables. Consequently, the strong result of Campbell and Shiller does not go through and the interpretation of Granger Causality tests is weaker in the current environment. Specifically — and setting aside for the moment qualifications about the stationarity of s_t and the fundamentals — evidence that s_t Granger Causes an observable fundamental f_{1t} or f_{2t} is consistent with the solution in equation (8). However, failure to find such Granger Causality could arise because the model is incorrect and or the fundamental being tested is an exact distributed lag of unobservables or observables that are not under consideration in a bivariate Granger Causality framework. Unlike the case considered by Campbell and Shiller, this second possibility cannot be reasonably ruled out.

In the empirical analysis below, we focus on bivariate Granger Causality tests as a first examination of the forward-looking model. Unlike the earlier work of EW and Chen *et al* (2010), we specifically address the order of integration and possible cointegration in the bivariate models. We first use Granger Causality methods that are robust to non-stationarity; we then examine Granger Causality in the presence of cointegration as predicted by the Campbell and Shiller type argument.

2.2.2 Further Implications of the Forward Looking Model

While evidence of Granger causality is consistent with the forward looking model of the exchange rate, by itself it does not provide much information about how well the model fits the data, either statistically or in economic terms. To pursue this objective, it is necessary to examine the restrictions implied by the present value model.

To keep the analysis manageable, we focus on one observable fundamental at a time relegating all other possible influence as unobservable in each case. Because we are interested in examining the extent of the role for the forward looking information in the exchange rate, we will be relatively careful about decomposing the solution, keeping track of the fundamental types. Rewrite (8) as either

$$s_t = (1-b)\sum_{j=0}^{\infty} b^j E_t f_{1t+j} + U_{1t} \tag{10}$$

or

$$s_t = b\sum_{j=0}^{\infty} b^j E_t f_{2t+j} + U_{2t} \tag{11}$$

where U_{1t}, U_{2t} are the unobservable, or more correctly the unmodeled, components in each case.

How we proceed next depends importantly upon the time series properties of the exchange rate and fundamental of interest. Consistent with empirical results below, we assume that the exchange rate itself is first difference stationary, I(1); however, the fundamental under consideration may or may not be stationary. If it is stationary (strictly, if it does not have a unit root), I(0), then this implies U_{1t} or U_{2t} is non-stationary. If the fundamental is I(1), then the next question is whether or not the exchange rate and fundamental are cointegrated. We consider the testable implications of each of these cases.

Case 1: Stationary fundamental

If f_t is stationary, we can proceed directly with equations (10) and (11), which following EW, we re-write as,

$$s_t = (1-b)F_{1t} + U_{1t} \quad \text{or} \quad s_t = bF_{2t} + U_{2t}$$

where $F_{it} \equiv f_{it} + E_t \sum_{j=1}^{\infty} b^j f_{it+j}$, $i = 1, 2$. By assumption, s_t is I(1), f_{it} and hence F_{it} is I(0), and U_{it} is by construction I(1). Because of the non-stationarity, we specify the relationship in first differences:

$$\Delta s_t = (1-b)\Delta F_{1t} + \Delta U_t \quad \text{or} \quad \Delta s_t = b\Delta F_{2t} + \Delta U_t$$

Now, to examine the importance of the fundamental within the context of the forward looking exchange rate solution, EW propose examining the simple correlation between Δs_t and an estimate of ΔF_{it}, where the latter is constructed using two different dynamic models. The first, Model 1, is an AR(p) model for f_{it}, from which we can calculate an estimate of F_{it}, denoted $\widehat{F}_{it}^{(1)}$. The second, Model 2, is a VAR(p) model for $(f_{it},\ \Delta s_t)'$, from which we again calculate an estimate of F_{it}, denoted $\widehat{F}_{it}^{(2)}$.[5] Both estimates are conditional on the discount parameter b, the choice of which is discussed in the empirical section.

If the fundamental is *economically* important *and* the present value model is correct then the correlation between Δs_t and ΔF_{it} should be high, though the the presence of the unobservable variables qualifies this conclusion and does so in both possible directions. Depending upon how ΔU_{it} and ΔF_{it} covary, we may observe a high correlation when the model is false and a low correlation when the model is true. This qualification can be addressed to a limited extent by comparing the correlations from Model 1 and Model 2. Model 1 in effect provides a benchmark for the correlation, using limited information to forecast the fundamental. Model 2, which includes the exchange rate, should out perform Model 1 if the exchange rate incorporates a significant amount of information about the future path of the fundamental. EW liken this to an extension of the Granger causality result in the context of these forward looking models.

Finally, as discussed previously, the fundamentals are likely to enter into the exchange rate equation dependent upon unknown parameters, which can be represented as $f_{it} = \gamma_0 + \gamma_1 x_t$, where x_t is one of the measurable fundamentals. If our only interest is the magnitude of the correlation as evidence for the forward looking model, then we can treat $f_{it} = x_t$ as the value of γ_1 has no bearing. Relatedly, Δs_t and $(1-b)\Delta\widehat{F}_{1t}$ or $b\Delta\widehat{F}_{2t}$ are not directly comparable in magnitude, even if we condition on a value for b. However, if we believe our theoretical models then the sign of γ_1, which we cannot recover from estimation, will have a bearing on the sign of the correlation. For this reason, when we estimate the AR or VAR model, we sign x_t so that theory suggests a positive correlation.

Case 2: Non-stationary fundamental with no cointegration

This case is the focus of EW, which finds no evidence of cointegration between the nominal exchange rates and fundamentals they consider. As before, we have

$$s_t = (1-b)F_{1t} + U_{1t} \quad \text{or} \quad s_t = bF_{2t} + U_{2t}$$

where we can write $(1-b)F_{it} \equiv f_{it} + E_t \sum_{j=1}^{\infty} b^j \Delta f_{it+j}, i = 1, 2$. In contrast to the previous case, the expected present discounted value term is in terms of the change in the fundamental, which ensures the terms is stationary as the fundamental is now assumed to be I(1). Since both s_t and f_{it} are assumed to be I(1) but not cointegrated, U_{it} is again I(1), and it is sensible to consider the relationships in first differences:

$$\Delta s_t = (1-b)\Delta F_{1t} + \Delta U_t \quad \text{or} \quad \Delta s_t = b\Delta F_{2t} + \Delta U_t$$

As before, we estimate F_{it}, conditional on a choice of b, using two models: Model 1, an AR(p) model for Δf_{it}; Model 2, a VAR(p) model for $(\Delta f_{it},\ \Delta s_t)'$. The two estimates are denoted $\widehat{F}_{it}^{(1)}$ and $\widehat{F}_{it}^{(2)}$. As with Case 1, we can substitute x_t for the fundamental and the unknown parameters will not influence the magnitude of the correlation. Again, though, we sign x_t so that theory suggests a positive correlation.

Case 3: Non-stationary fundamental with cointegration

As before, we have

$$s_t = (1-b)F_{1t} + U_{1t} \quad \text{or} \quad s_t = bF_{2t} + U_{2t}$$

[5] An appendix provides the detail of this construction.

with $(1-b)F_{it} \equiv f_{it}+E_t \sum_{j=1}^{\infty} b^j \Delta f_{it+j} \equiv f_{it}+H_{it}, i=1,2$, where the second equivalence introduces the term H_{it} for the present discounted value term. The above becomes

$$s_t = f_{1t} + H_{1t} + U_{1t} \quad \text{or} \quad s_t = \left(\frac{b}{1-b}\right) f_{2t} + \left(\frac{b}{1-b}\right) H_{2t} + U_{2t}$$

By assumption, s_t and f_{it} are I(1) and s_t and f_{it} are cointegrated. Since f_{it} is I(1), H_{it} is I(0). These together imply that U_{it} is stationary. Since s_t and f_{it} are cointegrated, we could examine the correlation between the error correction residual and H_{it} (as is examined in Campbell and Shiller, 1987). However, this has the disadvantage of not being immediately comparable to the previous two cases, which focus on the change in the exchange rate. So, to be consistent, we first difference the above

$$\Delta s_t = \Delta(f_{1t} + H_{1t}) + \Delta U_{1t} \quad \text{or} \quad s_t = \Delta(\left(\frac{b}{1-b}\right) f_{2t} + \left(\frac{b}{1-b}\right) H_{2t}) + \Delta U_{2t}$$

and consider the correlation between Δs_t and and the first differences of the constructions involving f_{it} and H_{it}.

To calculate these constructions and the associated correlations, we can take advantage of the cointegration between s_t and the associated fundamental. Because cointegration vector estimates converge more quickly than linear regression estimators for stationary variables, we can estimate these first and then condition the subsequent estimation on them. That is, we first estimate $s_t = \gamma_0 + \gamma_1 x_t$ using standard methods for cointegrated variables to obtain

$$\hat{f}_{it} = \hat{\gamma}_0 + \hat{\gamma}_1 x_t$$

Once we have this estimate, we can then estimate the two models from which we construct H_{it}. The first, Model 1, is an AR(p) model for Δx_t. The second is a VAR(p) model for $(\Delta x_t,\ s_t - \hat{f}_{it})'$. A key difference between this case and the previous cases is that the VAR model does not use Δs_t but rather $s_t - \hat{f}_{it}$, the residual from the cointegrating relationship. This is necessary since a VAR in Δx_t and Δs_t would be mis-specified. More importantly, this allows us to make use of the relationship between the levels of the series.

Constructing H_{it} from the AR(p) and VAR(p) estimates, which depend upon Δx_t rather than Δf_{it}, is straightforward, again relying on the cointegration estimates:

$$\hat{H}_{it} = E_t \sum_{j=1}^{\infty} b^j \Delta f_{it+j} = \hat{\gamma}_1 E_t \sum_{j=1}^{\infty} b^j \Delta x_{t+j}$$

For the first and second fundamental type, the observed part of the forward looking exchange rate solution is

$$\hat{f}_{it} + \hat{H}_{it} = \hat{\gamma}_0 + \hat{\gamma}_1 x_t + \hat{\gamma}_1 E_t \sum_{j=1}^{\infty} b^j \Delta x_{t+j}$$

For the first fundamental, this is immediate. For the second fundamental type, which involves a $b/(1-b)$ term, this may be less obvious. Because the cointegration estimates subsume the $b/(1-b)$ term, when x_t is used in place of f_{it}, these weights are implicit and the construction is exactly as above.

Finally, one significant advantage of the cointegration framework is that the estimates of $\hat{f}_{it}+\hat{H}_{it}$ can be directly compared to the exchange rate s_t both in sign and magnitude. Consequently, x_t need not be signed as in the previous cases and, more importantly, we can compare the construction to the actual exchange rate to see how well the forward looking model matches the behaviour of the exchange rate within sample.

3 Empirical Analysis

3.1 Data

The data for the empirical analysis is taken from the United States Federal Reserve's FRED database and Statistics Canada's CANSIM database. Specific details of source and construction for each series used are provided in Table 1.

The data set for this study includes data from 1981Q1–2014Q4 but the focus of the estimation and analysis is the sample 1999Q1–2014Q4. This later sample is coincides with an announced change in policy by the Bank of Canada. Since September 1998, the Bank has maintained a hands off approach to the Canadian dollar and ceased regular interventions in the foreign exchange market.[6] This later sample is also, as we shall see, the period during which there has been a strong relationship between the Canadian dollar and commodity prices.

For purposes of estimation, the sample is denoted $t = -p+1, \dots, 0, 1, \dots, T$ where p is the lag length in use (and varies as required). Unless otherwise noted, the $t = 1$ observation corresponds to 1999Q1. This fixes the estimation sample to 64 observations. The variables and the notation used are presented in the table below.

Variable	Definition
s_t	CAD-USD nominal exchange rate, CAD per USD (log)
cx_{1t}	All Commodities price index (USD), deflated by US GDP Deflator (log)
cx_{2t}	All Commodities less Energy price index (USD), deflated by US GDP Deflator (log)
cx_{3t}	Energy price index (USD), deflated by US GDP Deflator (log)
$y_t - y_t^*$	Canada Real GDP less US Real GDP (log)
$n_t - n_t^*$	Canada Nominal GDP less US Nominal GDP (log)
$i_t - i_t^*$	Canada 3 month TB interest rate less US 3 month TB interest rate
$p_t - p_t^*$	Canada GDP Deflator less US GDP deflator (log)
$\pi_t - \pi_t^*$	Canada GDP Deflator less US GDP deflator (log first differences)
$p_t^c - {p^c}_t^*$	Canada CPI less US CPI (log)
$\pi_t^c - {\pi^c}_t^*$	Canada CPI less US CPI (log first differences)

The eleven series used are presented in Figure 1–3 for the estimation sample 1999Q1–2014Q4. With the exception of the relative interest rates and the relative inflation rates, the series exhibit significant upward trends over the sample. One feature evident in the figures is the apparent relationship between real commodity prices, particularly the All Commodities index, cx_{1t}, and the relative GDP deflator indices, $p_t - p_t^*$. The former is constructed as $c_{1t} - p_t^*$, where c_{1t} is the (log) All Commodities price index in USD, which suggests that p_t and c_{1t} are themselves closely related.

[6] See the Bank of Canada's backgrounder, "Intervention in the Foreign Exchange Market," 2012, available at bankofcanada.ca.

As indeed they are — the correlation between the quarterly (log) growth rates for these two series is 0.87. As p_t is the GDP deflator for Canada and given the relatively significant contribution of commodity production to Canadian GDP, as commodity prices rise so too does the GDP deflator. Although this seems reasonable, it implies that for purchasing power parity to hold, the rise in $p_t - p_t^*$ should be accompanied by a depreciation of the nominal exchange rate. As we might expect from the figures, purchasing power parity for the GDP deflators is not a feature of our sample, not too surprising given the relative short sample length involved and the well known result that PPP across broad price aggregates holds at best in the very long run.[7] In contrast to the GDP deflators, however, we do observe a clear downward trend in the relative CPI measure over the sample. From a PPP perspective, the falling relative price levels is consistent with the tendency for the Canadian dollar to appreciate over this period.

One final definitional issue is the deflation of the commodity price index, which is in US dollars, by the US GDP deflator. Previous studies have used CPI to deflate the commodity price indices, for example, Chen and Rogoff (2003), while others such as Cashin *et al* (2004) use a world manufacturing price index, getting closer to a measure of the terms of trade. We use the GDP deflator for simplicity and because it is a broad measure of the US price level.

3.2 Estimation

The first step in the analysis is to establish the order of integration of the series used in estimation. Table 1 presents Augmented Dickey-Fuller tests for the null of a unit root. The first set of tests are for the series in levels, the second set for the series in differences. Test regressions include four or eight lags. The spot exchange rate, the commodity prices, the nominal and real GDP differentials, the GDP deflator differential, and the CPI differential all exhibit a trend over the sample so for these variables we focus on the test regressions that include a constant and a trend. The remaining two variables, the interest rate and inflation rate differentials, do not exhibit a trend so for these variables we focus on the specification with just a constant term. Summary conclusions at the five percent significance level are provided at the bottom of the table.

For five of the series, we have clear evidence that the series are first difference stationary, or I(1). These are the commodity price indices and the real and nominal GDP differences. Three other series are I(1) if we focus on one of the lag specifications. For the nominal exchange rate and the relative CPI measure, four lags is sufficient to ensure no serial correlation in the residuals (orders 1–4) and based on this lag length, these series are also I(1). In contrast, four lags is insufficient to remove all serial correlation in the relative interest rate series, so the conclusion is based upon a lag length of eight, which indicates this series is also I(1). The remaining non-stationary series is the GDP deflator differential; however, the evidence points to the series being trend stationary. Because the sample is relatively small, however, the ADF test will have low power to discriminate between I(1) and trend stationary; for prudence and simplicity we treat the series as I(1). The remaining two series, the two different inflation differential measures, are both I(0). For the CPI based measure, this conclusion rests on the four lag specification, which is sufficient to remove any serial correlation.

[7] See Taylor and Taylor (2004) and Rogoff (1996).

3.2.1 Toda–Yamamoto Granger Causality Tests

As discussed, forward looking theories of the exchange rate suggest that the the spot exchange rate should Granger cause exchange rate fundamentals. This prediction has been the focus of considerable attention, most notably Engel and West (2005) and Chen, Rogoff, and Rossi (2010). Both of these authors consider the prediction in terms of first differences of the exchange rate and fundamentals on (for the most part) a bivariate basis. In contrast, we consider Granger causality for the levels of the series using the methods proposed in Toda and Yamamoto (1995), which are designed to handle causality tests for possibly integrated series. This has a distinct advantage over using the differences of the series since differencing loses potentially important information about the relationship between series. Moreover, if the series in question are I(1) and cointegrated, a VAR in differences is misspecified.[8]

The first step is to determine the appropriate lag length for the set of bivariate VAR models using the nominal exchange rate and each of the fundamentals both in levels. To do so, we follow the approach in Sims (1980) and estimate a sequence of VAR models with lag lengths between one and eight and select the lag length based upon a LR test for p lags against $p + 1$ using a nominal size of 5%.[9] We then test the residuals for serial correlation of orders one through four and where necessary increment the lag length until the residuals are free of serial correlation. Table 2 reports the appropriate lag length p for each of the bivariate VAR models (models are identified by the included fundamental). Having determined the appropriate lag length, the Today-Yamamoto Granger Causality tests are based upon bivariate VARs with an additional lag added.[10] The tests are Wald tests on the joint significance of the first p lags for the relevant variable. Wald statistics and P-values are reported in the table. The additional lag in the VAR model ensures that the Wald statistic is asymptotically $\chi^2(p)$.

The test of primary interest is whether or not the nominal exchange rate Granger causes the fundamental. The null is the nominal exchange rate does not Granger cause the fundamental, denoted $H_0 : s \not\to x$, and a rejection of the hypothesis is interpreted as evidence of Granger causality. At the five percent test size, there are six such fundamentals: the three commodity price indices, the GDP deflator differential, the CPI differential, and the CPI inflation rate differential. Two other fundamentals show some promise, with marginal significance levels of roughly fifteen percent; these are the nominal GDP differentials and the interest rate differentials. The two remaining fundamentals, the real GDP differential and the GDP deflator inflation differential, are not Granger caused by the nominal exchange rate. A natural concern in an exercise such as this is how robust these results are to alternative specifications. In particular, over-parameterized and hence inefficient VAR models may have low power and incorrectly fail to reject the null hypothesis, finding against Granger causality when in fact such causality exists. It is certainly the case that different lag lengths give different conclusions for some of the variables; for this reason, we have taken some care to select the appropriate lag length. In any event, our principle objective is to demonstrate that the exchange rate has some forward looking predictive content and the results in the table,

[8] Another point of departure for our study relative to CRR is their focus on parameter instability in the Granger Causality tests. As we focus on a shorter reasonably stable sample period we are less concerned about parameter instability in our test regressions.

[9] Nielsen (2006) demonstrates that the test is still asymptotically valid in the presence of non-stationary variables as we have here. He also demonstrates that various information criteria remain valid; generally, we found the information critiera selected relatively parsimonious VAR models that were prone to serial correlation so we relied on the LR procedure.

[10] More generally, the Toda and Yamamoto procedure is to augment the VAR model with m additional lags where m is the maximum order of integration of the variables in the VAR. For all of our models, $m = 1$.

particularly those with respect to commodity prices, provide such evidence.

Tests for Granger causality in the other direction, denoted $H_0 : x \not\to s$, are roughly similar though typically with larger marginal significance levels. If we focus strictly on a five percent test size, only two variables Granger cause the exchange rate — the two different measures of the price levels. This is somewhat interesting since these two series have very different properties over the sample, trending in different directions. In the analysis below, we further explore the relationship between these price levels and the exchange rate. All together, these results provide some evidence in favour of the forward looking model for exchange rates, specifically the strong results for the Granger causality of fundamentals by the exchange rate.[11]

Although not the primary focus of the study, it is of some interest to see whether the Granger causality results, particularly those relating to commodity prices, are a recent phenomena or are evident over a longer time period. To this end, Table 2 presents the same analysis but for an early sample, 1981Q1–1998Q4, and the full sample 1981Q1–2014Q4.[12]

The differences between the later sample, 1999Q1–2014Q4, and the early sample, 1981Q1–1998Q4, are quite stark. There is no evidence, at conventional significance levels, of Granger causality between commodity prices and the exchange rate in either direction in the early sample. Evidently, the Canadian dollar as a commodity currency is a recent phenomenon. This is in fact consistent with earlier studies, such as Chen and Rogoff (2003), where of the countries they consider (Canada, Australia, and New Zealand), Canada has the least evidence of having a commodity currency. If we are willing to be somewhat relaxed about the marginal significance level, there is some (quite weak) evidence that the exchange rate Granger causes real and nominal GDP differentials as well as interest rate differentials. The only strong results concern Granger causality of the exchange rate, in this case by the GDP deflator differential, the interest rate differential, and the deflator inflation differential. Overall, there is little evidence of forward looking behaviour of the nominal exchange rate in the early sample.

Finally, we can consider the full sample of data, though this has to be done in the context of the quite evident parameter instability over this sample. Nonetheless, the longer sample has its own advantages — notably more available information — and is still of interest. For the most part, the full sample results are very similar to the more recent sample results. Commodity prices, price differentials (both measures), and inflation differentials (both measures) are all Granger caused by the exchange rate; in all cases, the P-values are very small indicating strong support. This result, though, is clearly driven by the more recent behaviour of the exchange rate so the causality methods used by Chen *et al* (2010) for longer samples, which allow for parameter instability, are clearly important.

The results in Table 2 provide pretty clear evidence that the exchange rate in recent years is forward-looking, particularly with regard to commodity prices and price levels or inflation rates. The exact nature and extent of these relationships is explored further below. The other important and somewhat puzzling evidence that comes to light in Table 2 is the instability of the relationship between the exchange rate and (some) of its fundamentals; most notably, the apparent transition to a commodity currency in the last (roughly) two decades. Why this should be so is not entirely obvious and a useful area for future research.

[11] In Table 1, there is the suggestion that the nominal exchange rate series is I(2). In this case, the Granger causality test regressions should include two additional lags. Doing so leaves the results in Table 3 essentially unchanged.

[12] As noted previously, our dataset has observations from 1981 onward. For these Granger causality tests, the sample varies as required by the different lag lengths; in contrast, the 1999-2014 sample fixes the initial observation to 1999Q1. Lag lengths for the early and full sample are determined by the same procedure described above.

3.2.2 Cointegration

The theoretical framework discussed in the previous section takes as its working hypothesis that the nominal exchange rate is I(1), which is confirmed in the tests for non-stationarity in Table 1. From this base, the forward looking solutions for the exchange rate then rely on simple bivariate dynamic models between the nominal exchange rate and fundamentals. These dynamic models depend upon whether the exchange rate and the fundamentals are cointegrated. In this subsection, we first test for cointegration on a bivariate basis. We then use information from these tests to specify and estimate appropriate models.

Table 3 reports tests for cointegration and estimates for the cointegrating vector for bivariate models that include the exchange rate and one of the set of fundamentals that are determined in the preceding section to be I(1). The table reports Johansen's (1995) trace test statistics. The test procedure is a sequence of tests as proposed by Johansen, starting with the null hypothesis of no cointegrating vectors against the alternative of at least one cointegrating vector. If this hypothesis is rejected, the next test is for the null of one or less against two cointegrating vectors. The number of cointegrating vectors is determined when the test first fails to reject. The table reports Osterwald-Lenum (1992) 5% critical values.[13]

For each pair of variables, we consider two possible trend specifications. The first, an unrestricted constant; the second, a trend restricted to the error correction term. That is, we are looking at two possible specifications for the cointegrating relationship, identified in the table as (c) or (t). Because there is uncertainty concerning the correct trend process, we follow the method suggested in Demetrescu, Lütkepohl, and Saikkonen (2008). This involves rejecting a specific cointegrating rank (r in the table) if one or both of the two models (constant or restricted trend) rejects that rank. These authors report Monte Carlo results that such a procedure is preferable, in the sense of identifying the true rank, to pretesting for the appropriate trend specification or allowing for the most general trend specification. We also consider the Schwarz Bayesian Information Criteria (SBIC) for the vector error correction (VEC) system under different cointegrating ranks as an alternative for determining the appropriate cointegrating rank. We get a consistent estimate of the rank by choosing the minimum information criteria.[14]

Conclusions concerning the cointegrating rank are reported for each model for the trace statistic with 5% test size, $CI(5\%)$ and the minimum SB information criteria, $CI(SBIC)$. For the commodity price indices cx_1, cx_2, and cx_3, both the trace statistics and the information criteria provide evidence in favour of cointegration ($r = 1$). Similarly, there is also strong evidence of cointegration between the exchange rate and the relative price deflators $p - p^*$ and the relative CPI indices $p^c - p^{c*}$. In contrast, there is no evidence of cointegration for real and nominal GDP differences nor for the interest rate differential (the hypothesis $r = 0$ cannot be rejected).

We have evidence of cointegration based upon considering both the constant and restricted trend specification following Demetrescu *et al* (2008). With this established, it is useful then to look at the role of the trend in the estimated cointegrating vector, the ρ coefficient in the table. Of the fundamentals cointegrated with the exchange rate, only the GDP deflator differential has a statistically significant trend, a trend stationary error correction term. In subsequent analysis, we will include the trend in this specification only.

[13] The lag length for the models are determined from the VAR model in levels as reported in Table 2.

[14] See Gonzalo and Pitarakis (1998); Aznar and Salvador, (2002).

Table 3 reports the coefficients of the cointegration vector in terms of the equation

$$s_t = \gamma_0 + \gamma_1 t + \gamma_2 x_t + e_t$$

Plots of the associated error correction terms, e_t in the equation above and denoted $e(s, x)$ in the plots, are presented in Figure 4. Noticeably, the term involving the GDP deflator differential is much more volatile than the other series, a reflection of the estimated β coefficient, which we now consider.

As linear relationships between the exchange rate and fundamental, the five cointegrating relationships are summarized as:[15]

$$s_t = 1.4 - 0.8 \cdot cx_{1t} \qquad s_t = 0.7 + 0.01 \cdot t - 33.9 \cdot (p_t - p_t^*)$$
$$s_t = 1.6 - 1.2 \cdot cx_{2t} \qquad s_t = 5.2 + 8.2 \cdot (p_t^c - p_t^{c*})$$
$$s_t = 1.6 - 0.6 \cdot cx_{3t}$$

Recall that the exchange rate is defined as the CAD price of USD, so that $\Delta s_t > 0$ is a depreciation. For the commodity price indices, the slope coefficient is negative consistent with the visual evidence in Figure 1a: rising commodity prices are associated with an increasing value of the Canadian dollar.

For the two price differentials, we get contrasting results. For the GDP deflator differential, we get a negative and very large slope coefficient. In addition, we get a positively signed deterministic trend coefficient. Notice that under PPP, the slope coefficient should be 1 and the trend coefficient 0, so the estimated model is (significantly) at odds with this theory. For the CPI differential, the slope coefficient is at least positive though significantly different from 1. Since PPP is a long run theory of the exchange rate it is not too surprising that we do not find much evidence in its favour over the short sample we use here. However, there are two concerns raised with these estimates, one theoretical, one empirical.

First, if we believe that over longer periods of time PPP is likely to reassert itself, which is not unreasonable given the available evidence, then it is likely that the estimated relationship for the price differentials is not stable.[16] Second, the slope coefficient on the deflator differential may be empirically suspect. Our reasoning on this is as follows.

An advantage of systems estimation of cointegration vectors is that single equation estimates may be biased in small samples (see for example the discussion in Lütkepohl (2006, 300–301). However, the implied bias from our MLE estimates is substantial relative to single equation estimation. Consider for example the estimates we obtain using DOLS estimation due to Stock and Watson (1993), which we report for the variables of interest (those we believe cointegrated) in Table 4. Using eight leads and lags and focusing on the specifications with a trend in the test regression, the slope coefficient estimate is 11.6 with a P-value of zero. The significant difference between the systems and single equation estimators in this case leads us to be somewhat sceptical of the slope coefficient on the deflator differential. In contrast, there is considerable agreement between the DOLS estimates and the systems estimates in Table 3 for the other fundamentals, including those for the CPI differentials. As a consequence, we downplay the results using GDP deflators and focus our discussion on the commodity price indices and the CPI differential.[17]

[15] All reported coefficients are statistically significant; see table for standard errors and P-values.

[16] See Rogoff (1996) for a survey of PPP. Bergin, Glick, and Wu (2014) is an example of more recent empirical work on long run real exchange rates.

[17] One lack of consistency across the two methods is the statistical significance of the trend in the cointegrating

Before considering the dynamic models, we return to the issue of Granger causality though now using the cointegration results. Forward looking models of the exchange rate predict that the error correction term involving the exchange rate and the fundamental will Granger cause changes in the fundamental. Since all terms are now stationary, standard GC tests can be applied. For the fundamentals where we find cointegration (based on Table 3), the Granger Causality tests are presented in Table 5. In this case, there is strong evidence of the error correction term Granger causing changes in the fundamental. In contrast, there is only weak evidence for Granger Causality in the other direction and then only for commodity prices. Indeed, only for the most general measure of commodity prices do we find a P-value below ten percent. These results provide additional support that for this more recent study there is a strong linkage between movements in the nominal exchange rate, joint with movements in commodity prices, and future movement in commodity prices. To be more precise in the current context, departures from the long run relationship between the exchange rate and fundamentals anticipate future movements in commodity prices. If one were confident of the long run relationship such information on a timely basis would be very useful to policy makers in understanding current movements in the exchange rate and commodity prices.

3.2.3 Dynamic models

Based on the results in Tables 1 and 3, we can classify our fundamentals as follows. The two inflation differentials, $\pi_t - \pi_t^*$ and $\pi_t^c - \pi^{c*}$, are Case 1 fundamentals as they are both stationary (see Table 1). The real and nominal GDP differentials and the interest rate differentials, $y_t - y_t^*$, $n_t - n_t^*$, and $i_t - i_t^*$, are Case 2 fundamentals as they are each non-stationary and not cointegrated with s_t. For both Case 1 and Case 2, it is necessary to redefine the fundamentals such that, based on the theoretical models, they are expected to be positively related to the exchange rate. Thus, we use $-(\pi_t - \pi_t^*)$, $-(\pi_t^c - \pi^{c*})$, and $-(y_t - y_t^*)$. For the remaining variables, commodity prices $cx_{1t}, cx_{2t}, cx_{3t}$ and price ratios $p_t - p_t^*$ and $p^c - p_t^{c*}$, each is cointegrated with s_t and so are classified as Case 3 fundamentals.

For each of these fundamentals, we calculate the expected present discounted values in order to construct measures of F_t as defined above. In order to do this, however, we need a value of b. The theoretical models we discussed above, however, do not provide a simple focal point for choosing b since, as was shown, it varies depending upon the model under consideration. Engel and West (2005) choose to focus on a few values that they motivate from simple theoretical values. Our approach is to consider a range of values for b from 0.01 to 0.99.

For Case 1, we estimate a AR(4) using x_t (Model 1) and VAR(4) using $(x_t,\ \Delta s_t)'$ for $x_t = -(\pi_t - \pi_t^*)$ and $x_t = -(\pi_t^c - \pi^{c*})$ (Model 2). For each of these two inflation differentials, we calculate $\widehat{F}_t^{(1)}$ (the AR model) and $\widehat{F}_t^{(2)}$ (the VAR model). For both inflation differentials, a lag length of four is sufficient to ensure no serial correlation in the residuals. Figure 5 shows the correlation between Δs_t and $\widehat{F}_t^{(1)}$ and the correlation between Δs_t and $\widehat{F}_t^{(2)}$ for both inflation differentials across the range of b. For the GDP inflation differentials, the correlation is positive but low. For the model 2 construction, which uses information from the exchange rate, the correlation is 0.2 across all values of b while it drops to zero for the model 1 construction as b gets closer to one. On the whole, this is only very weak evidence in favour of the forward looking exchange rate model for these inflation differentials. The situation is worse for the CPI inflation differentials, where the

vector. We work with the specifications in Table 3.

correlation is uniformly negative and there is almost no difference between the model 1 and model 2 constructions of ΔF_t.

Similarly weak results arise for the three I(1) fundamentals that are not cointegrated with the exchange rate, $y_t - y_t^*$, $n_t - n_t^*$, and $i_t - i_t^*$. The correlations are reported in Figure 6. Both the real and nominal relative output measures give rise to negative correlations for all b and the correlations are, in absolute value, quite weak. There is some discrepancy between Model 1 and Model 2 at higher levels of b but in the case of real GDP the difference is negligible and in the case of nominal GDP the correlation becomes more negative, opposite to what the simple theories above predict.

Figure 7 reports the correlations across the remaining variables, all of which are individually cointegrated with the exchange rate. Here we do get quite strong evidence in favour of the forward looking model of the exchange rate. In all cases, the correlations are positive. As well, the Model 2 correlations are greater than or equal to those of Model 1 for all values of b and substantially so for larger values of b. All five series have correlations for Model 2 that peak out at about 0.9, for values of b ranging between 0.8 and 1.0. So, the first takeaway from these results is evidence in favour of the forward looking model. The second take away is that there is not really any meaningful difference between the results for any of these series, as far as this aspect of the forward looking model is concerned. No clear winner emerges from this perspective. To make this more evident, Figure 8 presents the Model 2 correlations for the three commodity price indices in the same figure.

Finally, an alternative means of assessing just how well the forward looking model works for these cointegrated series is to construct an empirical measure of the exchange rate s_t using the present discounted value formula and compare this with the actual exchange rate. This requires a value for b; Figure 9 presents the comparison using $b = 0.2$ and Figure 10 using $b = 0.85$. Again, we focus only on the commodity price indices. As we might expect from the previous results, the fit is quite close especially in terms of the longer run movements of the two series. And again as we would expect the results are stronger for $b = 0.85$.

4 Summary

Forward looking models of the exchange rate are used to make and test a number of hypotheses about the Canadian-US dollar exchange rate and a number of fundamentals motivated by economic theory, in particular commodity prices.

We first find strong evidence that the Canadian dollar exchange rate Granger causes a number of fundamentals with the strongest relationship evident for commodity price indices. Unlike previous papers, we use Granger causality methods suitable for non-stationary series and for cointegrated series. Using these methods, the evidence in favour of Granger causality is much stronger and provides strong support for interpreting the Canadian dollar as a commodity currency. An important qualification to this, however, is that there is evident instability for these relationships over longer periods of time. Strong evidence of cointegration between the Canadian dollar and commodity price indices is a recent phenomenon (the last fifteen years) and it is quite possible that this relationship will change over time. Further research is required to obtain a better understanding of the long run relationships examined in this paper.

We also use methods suggested by Engel and West (2005) to provide a sense of how well forward looking models describe the Canadian dollar. Again, where we have cointegration between the exchange rate and the fundamental we find some evidence that, at least within sample, supports forward looking models. Notably the evidence is strongest when we use commodity price indices.

Overall, there is considerable evidence in support of an important role for commodity prices to understanding the Canadian-US dollar exchange rate. As emphasized, however, this is contingent upon a stable cointegrating relationship. If we were confident about this relationship then there is potentially much useful information in the quarterly variations in exchange rates and commodity prices that can be used to gauge both future movements in commodity prices and the exchange rate itself. The results in this paper suggest that a more detailed study of the short and long run relationships between the US-Canadian dollar exchange rate and commodity prices, including the stability of this relationship, is warranted.

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Appendix

The text discusses three cases concerning the exchange rate s_t and a fundamental f_{it}. For this discussion, the distinction between the two different types of fundamentals, $i = 1, 2$, is not important and we denote the generic fundamental as f_t. In all cases, s_t is I(1). In case 1, f_t is stationary; in case 2, f_t is I(1) but not cointegrated with s_t; and in case 3, f_t is I(1) and cointegrated with s_t with a $(1,\ -1)'$ cointegrating vector.

Recall that the generic fundamental is possibly related to the underlying measured macroeconomic variable as $f_t = \gamma_0 + \gamma_1 x_t$. For case 1 and 2, the γ parameters are not recoverable, while they are for case 3. To keep the presentation simple, we will use f_t. The text also introduces the notation $F_t \equiv E_t \sum_{j=0}^{\infty} b^j f_{t+j}$ (again, dropping the i subscript), which can be written in two ways:

$$F_t \equiv f_t + E_t \sum_{j=1}^{\infty} b^j f_{t+j}$$

$$(1-b)F_t \equiv f_t + E_t \sum_{j=1}^{\infty} b^j \Delta f_{t+j} \equiv f_t + H_t$$

The first is useful when f_t is stationary; the second when f_t is first difference stationary. In both situations, we require an estimate of an expected present discounted value term. To construct this term, we use dynamic models, either an AR model or a VAR model. The models for each case are as follows:

Case	Model 1	Model 2
Case 1:	AR(p) for $\boldsymbol{w}_t = f_t$	VAR(p) for $\boldsymbol{w}_t = (f_t,\ \Delta s_t)'$
Case 2:	AR(p) for $\boldsymbol{w}_t = \Delta f_t$	VAR(p) for $\boldsymbol{w}_t = (\Delta f_t,\ \Delta s_t)'$
Case 3:	AR(p) for $\boldsymbol{w}_t = \Delta f_t$	VAR(p) for $\boldsymbol{w}_t = (\Delta f_t,\ s_t - f_t)'$

To keep the discussion simple, we focus on the VAR(p) model. The AR(p) model is identical in treatment. We specify a stationary VAR(p) for the vector $\boldsymbol{w}_t$:

$$\boldsymbol{w}_t = \boldsymbol{c} + \boldsymbol{A}_1 \boldsymbol{w}_{t-1} + \ldots + \boldsymbol{A}_p \boldsymbol{w}_{t-p} + \boldsymbol{\varepsilon}_t \tag{12}$$

where $\boldsymbol{\varepsilon}_t$ is a mean zero vector of innovations with

$$E\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_{t-j} = \begin{cases} \boldsymbol{\Omega} & j = 0 \\ \boldsymbol{0} & j \neq 0 \end{cases}$$

The objective is to use the VAR model to estimate the expected discounted sum of the fundamental. In general terms, we wish to calculate the term $E_t \sum_{j=1}^{\infty} b^j w_{1t+j}$ where w_{1t} is the first element of $\boldsymbol{w}_t$. First, rewrite the VAR(p) in companion form,

$$\boldsymbol{W}_t = \boldsymbol{C} + \boldsymbol{A}\boldsymbol{W}_{t-1} + \boldsymbol{\mathcal{E}}_t$$

Then write this forward to obtain

$$\boldsymbol{W}_{t+j} = \sum_{\tau=1}^{j} \boldsymbol{A}^{\tau-1}\boldsymbol{C} + \boldsymbol{A}^j \boldsymbol{W}_t + \sum_{\tau=1}^{j} \boldsymbol{A}^{j-\tau} \boldsymbol{\mathcal{E}}_{t+\tau}$$

Taking conditional expectations, we get:

$$E_t \boldsymbol{W}_{t+j} = \sum_{\tau=1}^{j} \boldsymbol{A}^{\tau-1}\boldsymbol{C} + \boldsymbol{A}^j \boldsymbol{W}_t$$

Or if we define a selection vector $\boldsymbol{e}_1'$ which selects the first variable of the VAR, w_{1t}, then we get

$$E_t w_{1t+j} = \boldsymbol{e}_1' E_t \boldsymbol{W}_{t+j} = \boldsymbol{e}_1' \left(\sum_{\tau=1}^{j} \boldsymbol{A}^{\tau-1}\boldsymbol{C} + \boldsymbol{A}^j \boldsymbol{W}_t \right)$$

Finally, we use this expression to calculate the expected present discounted value:

$$E_t \sum_{j=1}^{\infty} b^j w_{1t+j} = \boldsymbol{e}_1' \left(\frac{b}{1-b} \right) (\boldsymbol{I} - b\boldsymbol{A})^{-1} \boldsymbol{C} + \boldsymbol{e}_1' b\boldsymbol{A}(\boldsymbol{I} - b\boldsymbol{A})^{-1} \boldsymbol{W}_t$$

The details for the last expression are as follows. We use the conditional expectation from the companion VAR to find

$$\begin{aligned} E_t \sum_{j=1}^{\infty} b^j w_{1t+j} &= \sum_{j=1}^{\infty} b^j E_t w_{1t+j} = \sum_{j=1}^{\infty} b^j \boldsymbol{e}_1' \left(\sum_{\tau=1}^{j} \boldsymbol{A}^{\tau-1}\boldsymbol{C} + \boldsymbol{A}^j \boldsymbol{W}_t \right) \\ &= \boldsymbol{e}_1' \sum_{j=1}^{\infty} b^j \left(\sum_{\tau=1}^{j} \boldsymbol{A}^{\tau-1}\boldsymbol{C} \right) + \boldsymbol{e}_1' \sum_{j=1}^{\infty} b^j \boldsymbol{A}^j \boldsymbol{W}_t \end{aligned}$$

Simplifying; first the constant term:

$$\boldsymbol{e}_1' \sum_{j=1}^{\infty} b^j \left(\sum_{\tau=1}^{j} \boldsymbol{A}^{\tau-1}\boldsymbol{C} \right) = \boldsymbol{e}_1' \sum_{j=1}^{\infty} b^j \left(\sum_{\tau=1}^{j} \boldsymbol{A}^{\tau-1} \right) \boldsymbol{C} = \boldsymbol{e}_1' \left(\sum_{j=1}^{\infty} b^j \boldsymbol{S}_j \right) \boldsymbol{C}$$

Find a solution for the summation term:

$$\begin{aligned} \sum_{j=1}^{\infty} b^j \boldsymbol{S}_j &= b\boldsymbol{I} + b^2(\boldsymbol{I} + \boldsymbol{A}) + b^3(\boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2) + \ldots \\ &= b(1 + b + b^2 + \ldots)\boldsymbol{I} + b^2(1 + b + b^2 + \ldots)\boldsymbol{A} + b^3(1 + b + b^2 + \ldots)\boldsymbol{A}^2 + \ldots \\ &= \frac{b}{1-b} \left(\boldsymbol{I} + b\boldsymbol{A} + b^2\boldsymbol{A}^2 + \ldots \right) = \frac{b}{1-b} \left(\boldsymbol{I} - b\boldsymbol{A} \right)^{-1} \end{aligned}$$

The full constant term is then $\boldsymbol{e}_1' \left(\sum_{j=1}^{\infty} b^j \boldsymbol{S}_j \right) \boldsymbol{C} = \boldsymbol{e}_1' \left(\frac{b}{1-b} \right) (\boldsymbol{I} - b\boldsymbol{A})^{-1} \boldsymbol{C}$.

Now, the remaining term, $\boldsymbol{e}_1' \left(\sum_{j=1}^{\infty} b^j \boldsymbol{A}^j \right) \boldsymbol{W}_t$. The summation term in brackets simplifies as:

$$\sum_{j=1}^{\infty} b^j \boldsymbol{A}^j = b\boldsymbol{A} + b^2\boldsymbol{A}^2 + b^3\boldsymbol{A}^3 + \ldots = b\boldsymbol{A}(\boldsymbol{I} + b\boldsymbol{A} + b^2\boldsymbol{A}^2 + \ldots) = b\boldsymbol{A}(\boldsymbol{I} - b\boldsymbol{A})^{-1}$$

So that, $\boldsymbol{e}_1' \left(\sum_{j=1}^{\infty} b^j \boldsymbol{A}^j\right) \boldsymbol{W}_t = \boldsymbol{e}_1' b\boldsymbol{A}(\boldsymbol{I} - b\boldsymbol{A})^{-1}\boldsymbol{W}_t$. Putting this altogether,

$$E_t \sum_{j=1}^{\infty} b^j w_{1t+j} \quad = \quad \boldsymbol{e}_1' \left(\frac{b}{1-b}\right) (\boldsymbol{I} - b\boldsymbol{A})^{-1} \boldsymbol{C} + \boldsymbol{e}_1' b\boldsymbol{A}(\boldsymbol{I} - b\boldsymbol{A})^{-1}\boldsymbol{W}_t$$

Throughout we assume that the summations all converge; this should be true if $\boldsymbol{A}$ satisfies the usual conditions for a stable VAR and if $|b| < 1$.

To construct these terms for a particular variable w_{it}, we use the estimated $\boldsymbol{C}$ and $\boldsymbol{A}$ matrices from the AR and VAR models.

As discussed in the text, the expected present discounted value terms are functions of the unknown discount parameter b. What is evident from the solution here is that for b very close to one, the constant term has a term $b/(1-b)$ that gets arbitrarily large. For this reason, it may be useful to restrict the constant terms in the AR and VAR models to be zero. This is certainly reasonable for the equations that include the error correction term since we replace f_t with estimates of $\gamma_0 + \gamma_1 x_t$ (including as well a trend if there is one empirically), which by construction have mean zero.

The implementation of these calculations are done in *Stata 13*; programmes are available on request.

Table 1: Augmented Dickey-Fuller

1999:Q1–2014:Q4

Levels	s	cx_1	cx_2	cx_3	$y-y^*$	$n-n^*$	$p-p^*$	p^c-p^{c*}	$i-i^*$	$\pi-\pi^*$	$\pi^c-\pi^{c*}$
Constant, no trend											
$Z(t), p=4$	-1.260	-1.898	-1.646	-2.332	-1.040	-1.515	-2.105	-0.827	-2.830	-4.628	-3.825
P-value	0.647	0.333	0.459	0.162	0.738	0.526	0.243	0.811	0.054	0.000	0.003
$Z(t), p=8$	-1.250	-1.590	-1.394	-2.067	-0.974	-0.972	-1.529	-1.043	-2.068	-5.031	-2.227
P-value	0.652	0.488	0.585	0.258	0.763	0.763	0.519	0.737	0.258	0.000	0.197
Constant, trend											
$Z(t), p=4$	-1.010	-1.786	-2.592	-2.052	-1.778	-2.879	-3.556	-1.700	-3.149	-4.651	-3.798
P-value	0.943	0.711	0.284	0.573	0.715	0.170	0.034	0.751	0.095	0.001	0.017
$Z(t), p=8$	-1.091	-1.289	-2.366	-1.064	-1.827	-2.261	-3.645	-2.795	-2.329	-5.010	-2.208
P-value	0.931	0.891	0.398	0.935	0.692	0.456	0.026	0.199	0.418	0.000	0.485
Differences	Δs	Δcx_1	Δcx_2	Δcx_3	$\Delta(y-y^*)$	$\Delta(n-n^*)$	$\Delta(p-p^*)$	Δp^c-p^{c*}	$\Delta i-i^*$	$\Delta\pi-\pi^*$	$\Delta\pi^c-\pi^{c*}$
Constant, no trend											
$Z(t), p=4$	-4.249	-4.366	-3.775	-4.796	-4.457	-4.938	-4.628	-3.825	-4.507	-6.379	-6.535
P-value	0.001	0.000	0.003	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.000
$Z(t), p=8$	-2.364	-3.965	-3.409	-4.376	-3.051	-4.520	-5.031	-2.227	-3.210	-5.045	-3.481
P-value	0.152	0.002	0.011	0.000	0.030	0.000	0.000	0.197	0.019	0.000	0.008
Observations	64	64	64	64	64	64	64	64	64	64	64
Conclusions (5%)											
	I(1)*	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)/TS	I(1)*	I(1)**	I(0)	I(0)*

Test regressions use the sample $t=-p+1,\ldots,1,\ldots,T$, with observations $1,\ldots,T$ corresponding to 1999Q1–2014Q4. The statistic $Z(t)$ is the Augmented Dickey-Fuller test statistic; P-values are from MacKinnon (1994) as reported in *Stata13*.

Conclusions are based upon 5% marginal significance level and refer to the properties of the variable in levels; I(0) indicates stationary, I(1) first difference stationary, I(2) second difference stationary, and TS trend stationary.

*Choice is based upon $p=4$, which is sufficient to ensure no serial correlation in the test regression residuals.

**Choice is based upon $p=8$, which is sufficient to ensure no serial correlation in the test regression residuals; $p=4$ is insufficient.

Table 2: Levels-based Granger Causality Tests

	Fundamental, x									
Test	cx_1	cx_2	cx_3	$y-y^*$	$n-n^*$	$p-p^*$	p^c-p^{c*}	$i-i^*$	$\pi-\pi^*$	$\pi^c-\pi^{c*}$
Sample: 1999:Q1–2014Q4										
$H_0: f \not\rightarrow s$										
$\chi^2(p)$	8.903	8.883	8.401	2.095	5.312	8.371	26.095	2.198	14.541	11.584
P-value	0.064	0.180	0.078	0.351	0.257	0.015	0.001	0.974	0.069	0.171
$H_0: s \not\rightarrow f$										
$\chi^2(p)$	21.134	19.442	11.158	0.365	6.915	6.770	44.377	12.080	9.205	25.065
P-value	0.000	0.003	0.025	0.833	0.140	0.034	0.000	0.148	0.325	0.002
Lag length p	4	6	4	2	4	2	8	8	8	8
Observations	64	64	64	64	64	64	64	64	64	64
Sample: 1981:Q1–1998Q4										
$H_0: f \not\rightarrow s$										
$\chi^2(p)$	2.343	3.679	1.053	1.326	1.764	23.097	–	31.503	9.700	–
P-value	0.673	0.596	0.902	0.857	0.779	0.000		0.000	0.046	
$H_0: s \not\rightarrow f$										
$\chi^2(p)$	2.339	4.542	4.139	6.905	8.113	6.123	–	11.638	5.401	–
P-value	0.674	0.474	0.388	0.141	0.088	0.294		0.168	0.249	
Lag length p	4	5	4	4	4	5		8	4	
Observations	67	66	67	67	67	66		63	66	
Sample: 1981:Q1–2014Q4										
$H_0: f \not\rightarrow s$										
$\chi^2(p)$	8.428	8.572	5.092	3.590	6.772	14.220	13.802	7.210	13.383	10.961
P-value	0.077	0.127	0.278	0.166	0.238	0.014	0.087	0.514	0.010	0.204
$H_0: s \not\rightarrow f$										
$\chi^2(p)$	22.715	21.029	10.211	0.438	9.941	26.620	34.871	5.734	14.847	23.157
P-value	0.000	0.001	0.037	0.803	0.077	0.000	0.000	0.677	0.005	0.003
Lag length p	4	5	4	2	5	5	8	8	4	8
Observations	131	130	131	133	130	130	83	127	130	82

Each column refers to test results from a bivariate VAR model that includes the nominal exchange rate and the indicated fundamental. Lag length chosen by sequential LR tests, 5% nominal size, starting with a maximum lag length of eight. Where necessary, lag lengths are extended to ensure that residuals are not serially correlated, based upon Johansen's (1995) LM test for autocorrelation in VAR residuals.
The test regressions have lag length $p+1$. The hypothesis tests are for the exclusion of the first p lags of the relevant variable. Test statistics use a small sample adjustment to the covariance estimator.
For the samples starting in 1981:Q1, actual starting observations vary depending upon the lag length and, in the case of p^c and π^c, available observations. Since these series start in 1992, early sample estimates are not available.

Table 3: Tests for Cointegration

1999:Q1–2014:Q4

	x_t															
	cx_1		cx_2		cx_3		$(y-y^*)$		$n-n^*$		$p-p^*$		p^c-p^{c*}		$i-i^*$	
CI Relationship $s_t=\gamma_0+\gamma_1 t+\gamma_2 x_t$	c	t	c	t	c	t	c	t	c	t	c	t	c	t	c	t
γ_0	1.409	1.258	1.579	1.407	1.600	1.596	-32.933	41.515	-20.126	31.465	0.605	0.710	5.223	5.345	0.453	0.476
γ_1	.	-0.001	.	-0.001	.	-0.000	.	-0.019	.	-0.026	.	0.012	.	0.000	.	-0.010
s.e.	.	0.001	.	0.001	.	0.003	.	0.005	.	0.006	.	0.004	.	0.002	.	0.001
P-value	.	0.308	.	0.138	.	0.987	.	0.000	.	0.000	.	0.006	.	0.912	.	0.000
γ_2	-0.765	-0.640	-1.193	-1.007	-0.567	-0.564	-14.719	18.240	-9.286	13.777	-15.376	-33.858	8.153	8.365	-0.308	0.151
s.e.	0.066	0.105	0.077	0.117	0.064	0.121	4.477	5.574	2.657	3.911	1.543	5.234	0.469	1.891	0.189	0.024
P-value	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.103	0.000
Trace Test																
$H_0: r=0;\ H_1: r\geq 1$	16.069	17.334	17.078	18.187	18.837	19.429	9.424	13.305	6.522	14.049	29.775	34.738	17.369	17.593	7.525	17.571
$H_0: r\leq 1;\ H_1: r=2$	1.574	2.233	1.726	1.754	2.001	2.593	2.609	5.566	1.720	2.123	1.941	3.471	1.444	1.654	2.083	4.166
SBIC																
$r=0$	-5.907	-5.907	-6.461	-6.461	-4.797	-4.797	-11.315	-11.315	-9.950	-9.950	-10.623	-10.623	-11.338	-11.338	-2.440	-2.440
$r=1$	-5.939	-5.883	-6.506	-6.458	-4.865	-4.800	-11.227	-11.176	-9.830	-9.876	-10.863	-10.852	-11.392	-11.327	-2.330	-2.389
$r=2$	-5.898	-5.788	-6.468	-6.355	-4.831	-4.710	-11.202	-11.133	-9.792	-9.779	-10.828	-10.776	-11.350	-11.223	-2.297	-2.324
CI (5%)	1	0	1	0	1	0	0	0	0	0	1	1	1	0	0	0
CI (SBIC)	1	0	1	0	1	1	0	0	0	0	1	1	1	0	0	0
Lag length	4	4	6	6	4	4	2	2	4	4	2	2	8	8	8	8
Observations	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64

Estimation and testing is based on Johansen's MLE for cointegration; see Johansen (1995). The variables x_t have been signed so that $\gamma_2 > 0$ is consistent with theory. See text for details. Models in columns headed by c include a constant in the VEC but exclude a trend in the error correction term. Models in columns headed by t include a trend in the error correction term. Standard errors are not directly available for the constant term γ_0. The parameter r refers to the number of cointegrating vectors. CI(95%) identifies whether a single cointegrating vector is accepted based upon a five percent test size. Critical values for the trace test are given below. CI(SBIC) identifies the number of cointegrating vectors associated with the minimum of Swartz Bayesian Information Criteria (SBIC).

Trace Test: 5% Critical Values

Hypothesis	c	t
$H_0: r=0;\ H_1: r\geq 1$	15.41	25.32
$H_0: r\leq 1;\ H_1: r=2$	3.76	12.25

Table 4: Additional Cointegration Vector Estimates

1999:Q1–2014:Q4

CI Vector $s_t + \mu + \rho t + \beta f_t$	f_t									
	cx_1		cx_2		cx_3		$p - p^*$		$p^c - p^{c*}$	
	c	t	c	t	c	t	c	t	c	t
μ	1.321	1.079	1.493	1.318	1.311	0.977	0.535	0.543	5.327	5.810
s.e.	0.072	0.080	0.028	0.036	0.131	0.111	0.023	0.020	0.069	0.237
P-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ρ	.	-0.004	.	-0.002	.	-0.006	.	-0.002	.	0.001
s.e.	.	0.001	.	0.000	.	0.001	.	0.002	.	0.000
P-value	.	0.000	.	0.000	.	0.000	.	0.341	.	0.037
β	-0.718	-0.475	-1.124	-0.909	-0.458	-0.230	-14.650	-11.61	8.366	9.223
s.e.	0.045	0.074	0.042	0.042	0.052	0.065	0.663	3.228	0.042	0.418
P-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Estimates are Stock and Watson (1993) dynamic least squares estimates with eight lags and leads of the differences of the dependent variables s. Standard errors are based upon the long run covariance matrix estimator using a Bartlett kernel and bandwidth selected according to Newey and West (1994); see the details in Wang and Wu (2012).

Table 5: Cointegration-based Granger Causality Tests

1999:Q1–2014:Q4

	Fundamental, x				
Test	cx_1	cx_2	cx_3	$p - p^*$	$p^c - p^{c*}$
$H_0 : \Delta x \nrightarrow e(s, x)$					
$\chi^2(p)$	11.979	10.262	7.1206	1.8076	8.4049
P-value	0.018	0.114	0.130	0.405	0.395
$H_0 : e(s, x) \nrightarrow \Delta x$					
$\chi^2(p)$	26.744	29.575	17.686	28.479	16.244
P-value	0.000	0.000	0.001	0.000	0.039
Lag length p	4	6	4	2	4
Observations	64	64	64	64	64

Each column refers to test results from a bivariate VAR model that includes the estimated error correction term for the nominal exchange rate and the indicated fundamental, denoted as $e(s, x)$, and the difference of the fundamental, Δx. The error correction terms are from the estimates in Table 3. Lag length p is chosen based on the results in Table 3.

The hypothesis tests are for the exclusion of p lags of the relevant variable. Test statistics use a small sample adjustment to the covariance estimator.

Table A1: Data Sources

Variable	Source	Series Code	Description
Monthly Data Series			
S	CANSIM	v37426	CAD-USD, noon spot rate, average (CAD)
C_1	CANSIM	v52673496	Bank of Canada, All commodities, price index (USD, 1972=100)
C_2	CANSIM	v52673497	Bank of Canada, All commodities ex. energy, price index (USD, 1972=100)
C_3	CASNIM	v52673498	Bank of Canada, Energy commodities, price index (USD, 1972=100)
i	CASNIM	v122531	Canada, 3 Month treasury bills (percent)
i^*	FRED	TB3MS	US, 3 Month treasury bills (percent)
P^c	CANSIM	v41690914	Canada, All items CPI, seasonally adjusted (2002=100)
P^{c*}	FRED	CPIAUCSL	US, All items CPI, seasonally adjusted (1982–84=100)
Quarterly Data Series			
N	CANSIM	v62305783	Canada Nominal GDP, SAAR (current dollars)
Y	CANSIM	v62305752	Canada Real GDP, chained (2007) SAAR
Y^*	FRED	GDPC1	US Real GDP, chained (2009) SAAR
P	CANSIM	v62307282	Canada GDP Deflator, seasonally adjusted (2007=100)
P^*	FRED	GDPDEF	US GDP Deflator, seasonally adjusted (2009=100)

Notes for data source and variable construction:

1. All US series are taken from FRED. Full citations are as follows:
   (a) Board of Governors of the Federal Reserve System (US), 3-Month Treasury Bill: Secondary Market Rate [TB3MS], FRED, Federal Reserve Bank of St Louis.
   (b) US. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items [CPIAUCSL], FRED, Federal Reserve Bank of St Louis.
   (c) US. Bureau of Economic Analysis, Real Gross Domestic Product [GDPC1], FRED, Federal Reserve Bank of St. Louis.
   (d) US. Bureau of Economic Analysis, Gross Domestic Product: Implicit Price Deflator [GDPDEF], FRED, Federal Reserve Bank of St. Louis.
2. All Canadian series are taken from Statistics Canada's CANSIM database, available at http://dc1.chass.utoronto.ca/cansimdim/.
3. Natural logs are used for all series except for interest rates. The upper case notation denotes levels; lower case used in text of paper and tables denotes logarithms.
4. All estimation uses quarterly data. For the monthly data series, averages over monthly data were used to construct quarterly data.
5. US nominal GDP, N^*, is constructed from US real GDP Y^* and the GDP deflator P^*.
6. Data vintage is autumn 2015; data are available upon request.

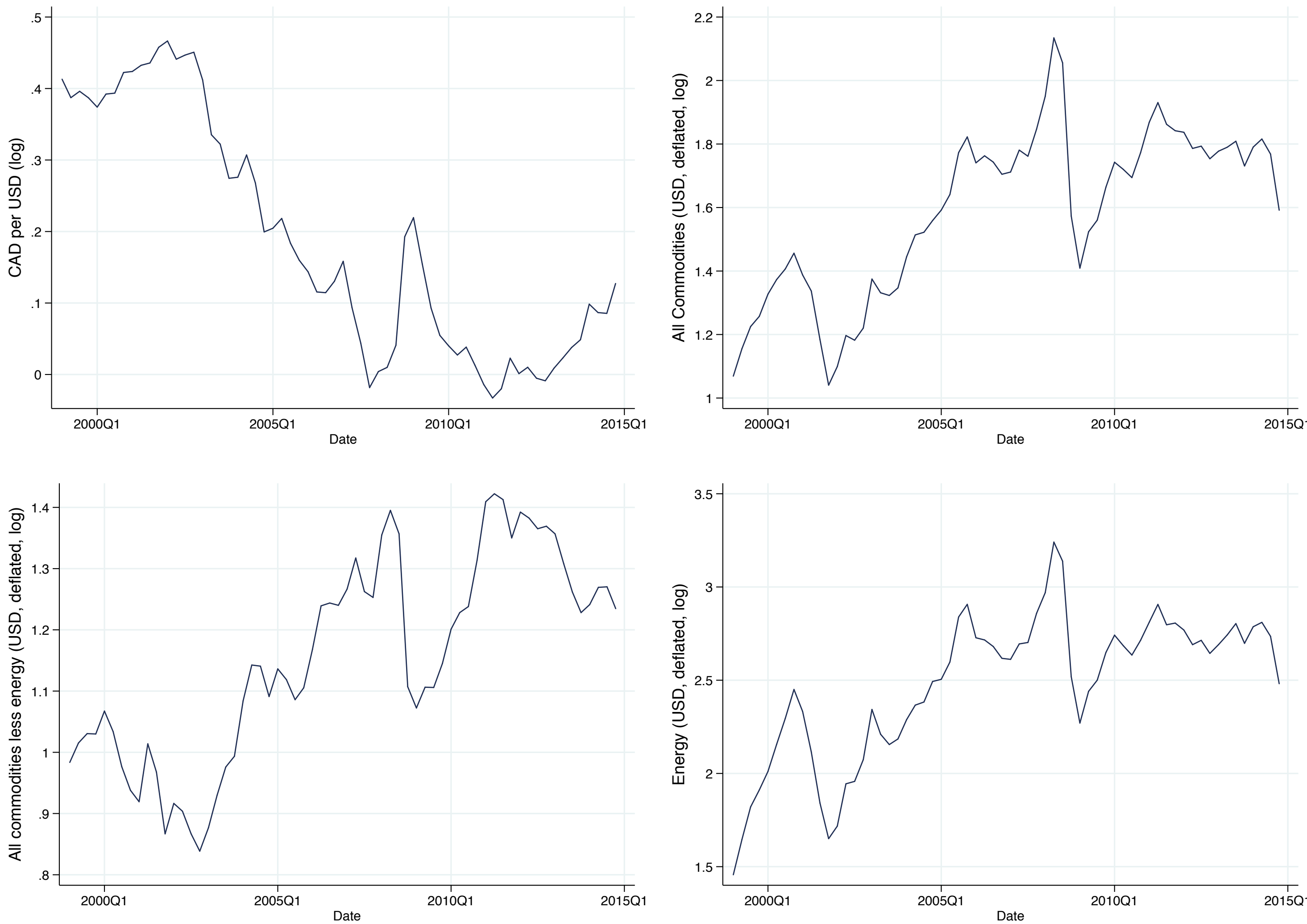

Figure 1 Canada–US Series for Estimation

Figure 2 Canada–US Series for Estimation

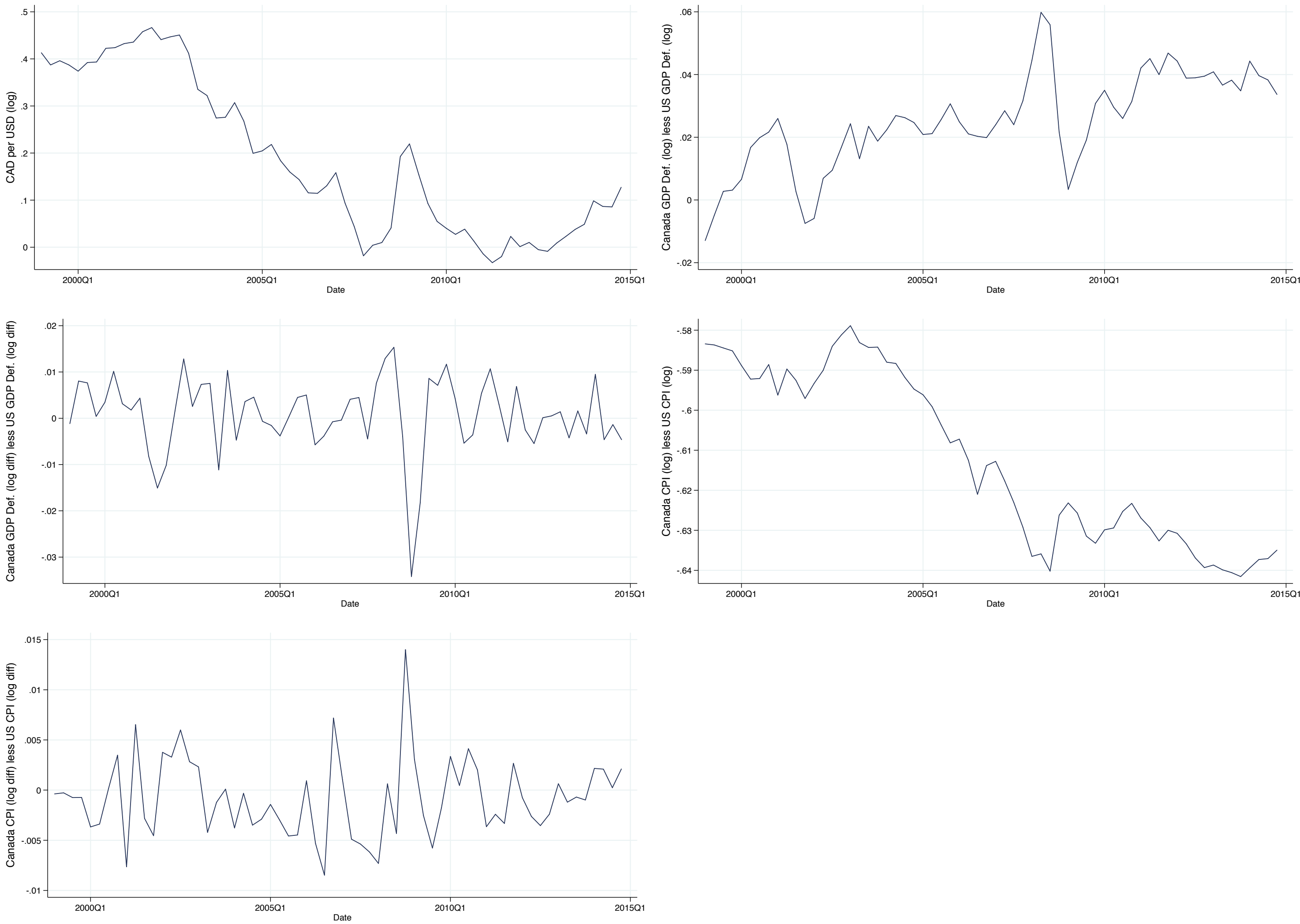

Figure 3 Canada–US Series for Estimation

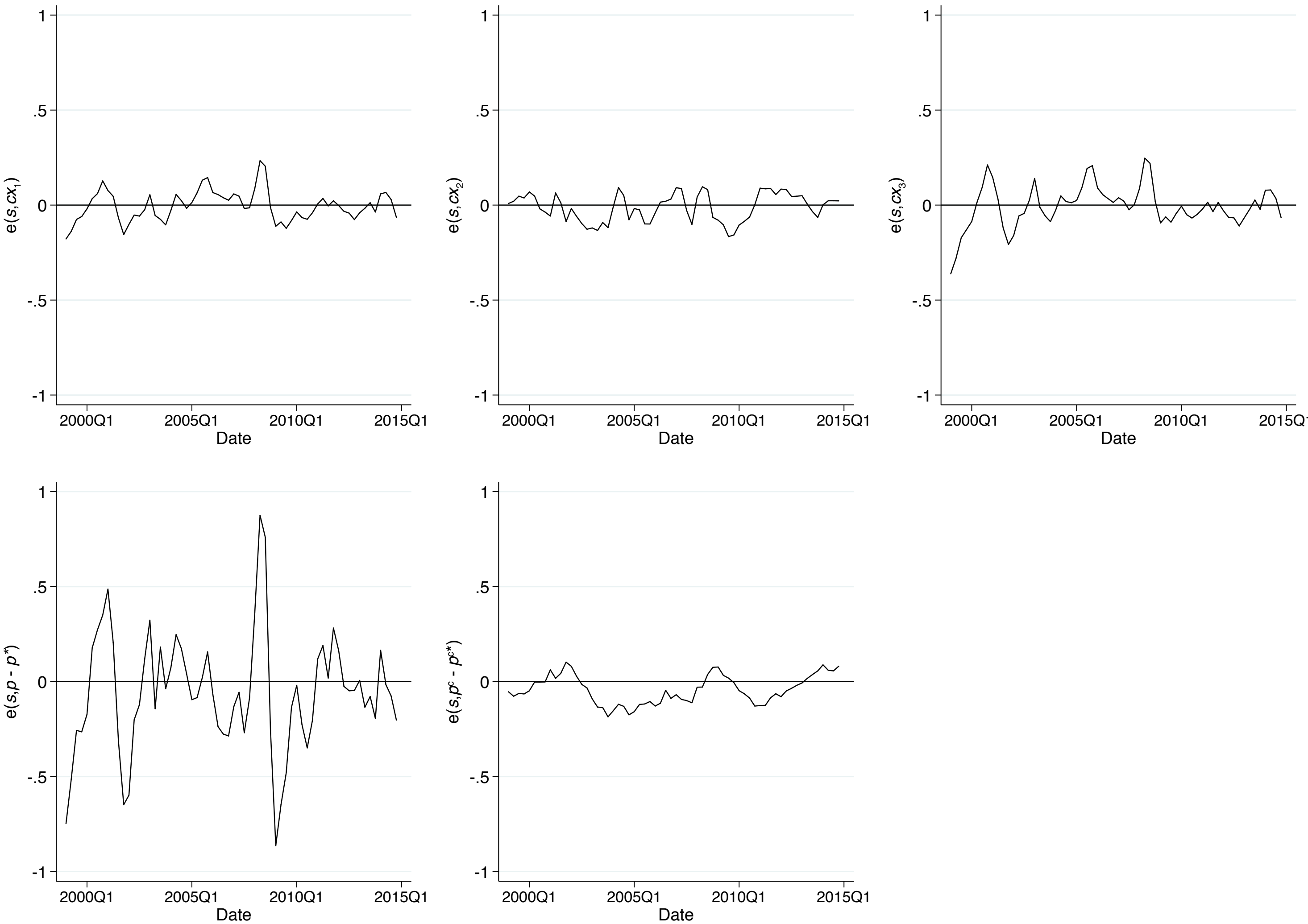

Figure 4 Error Correction Terms

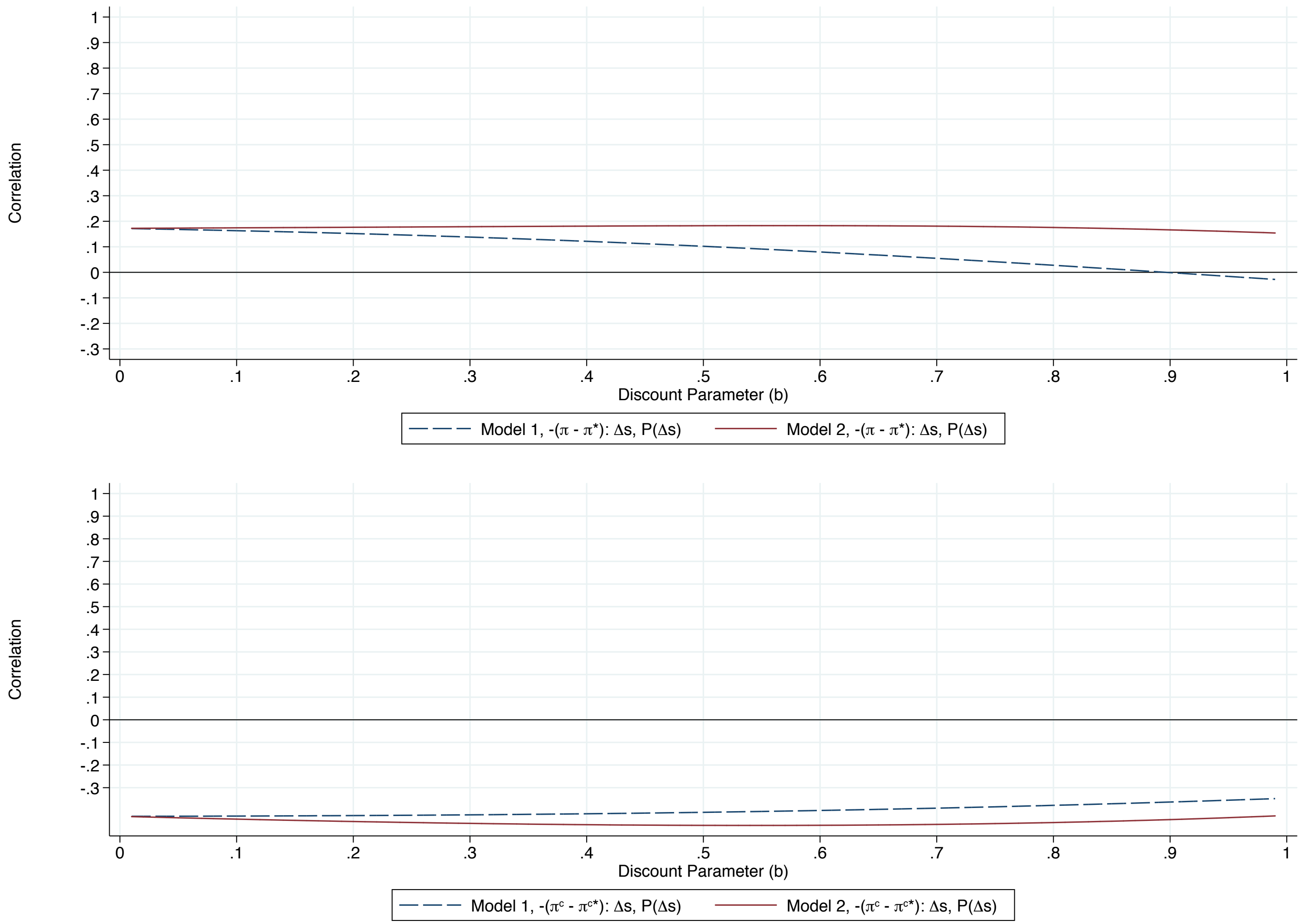

Figure 5 Correlations between Actual and Predicted Δs; I(0) Fundamentals

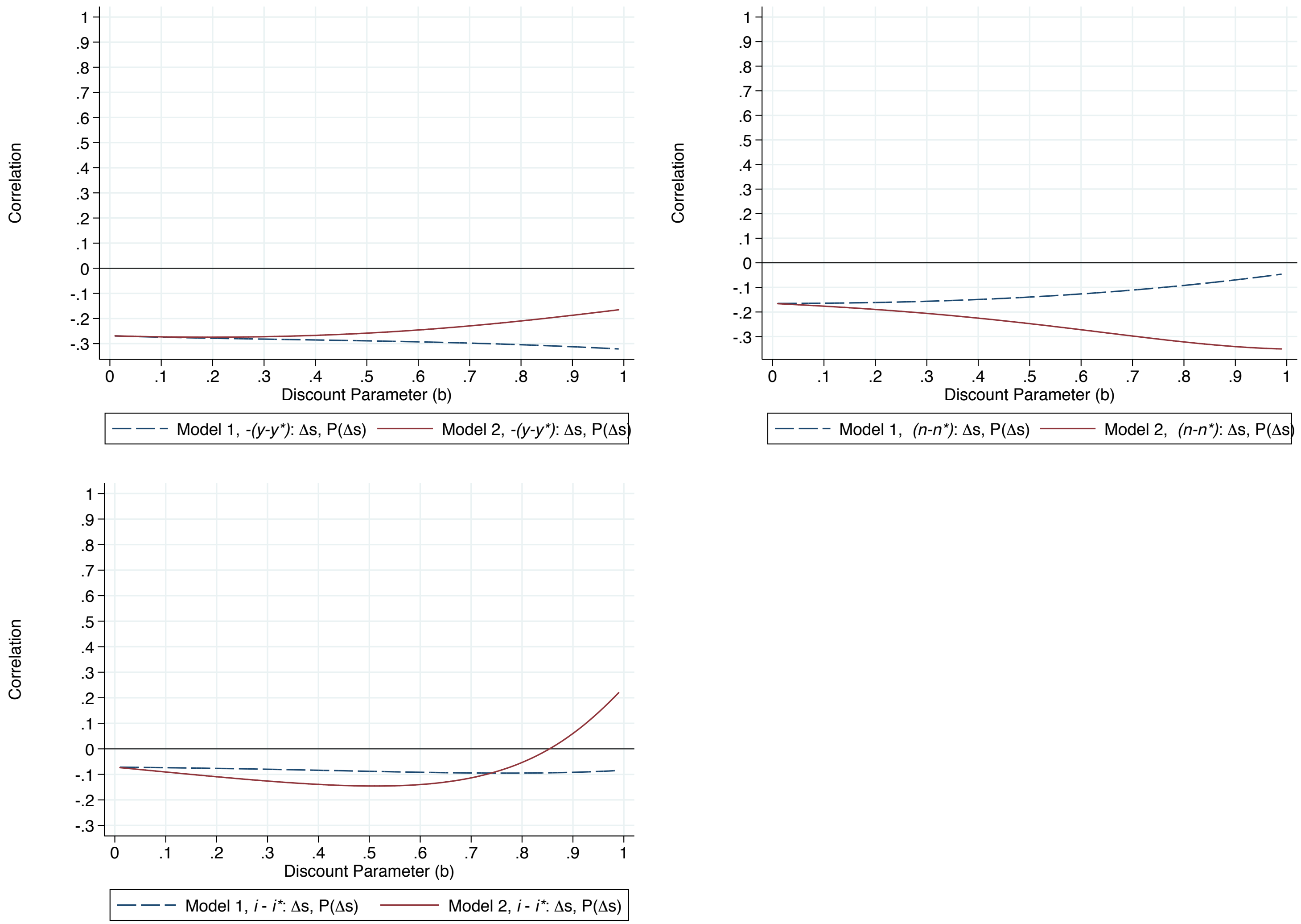

Figure 6 Correlations between Actual and Predicted Δs; I(1) Fundamentals w/o Cointegration

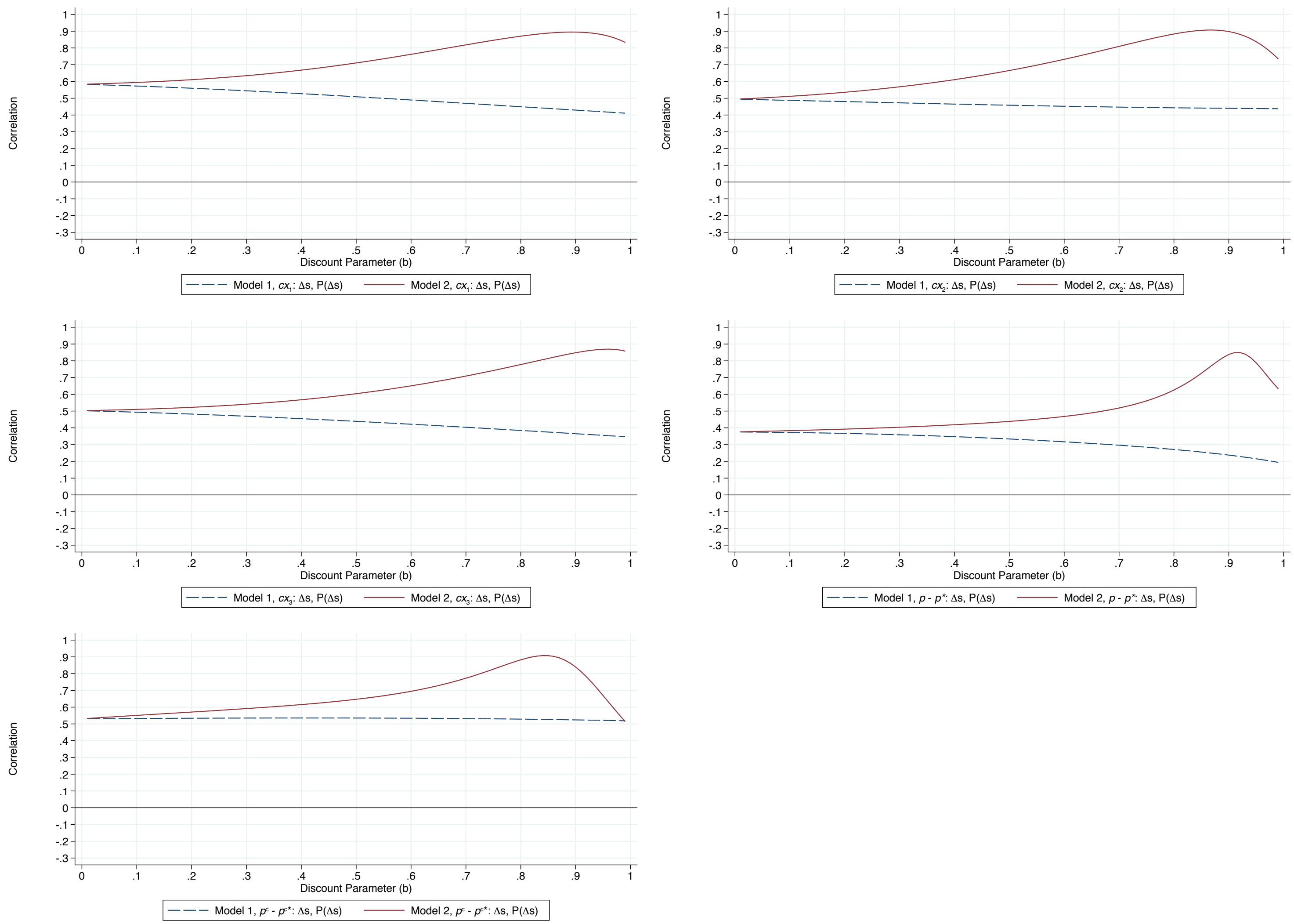

Figure 7 Correlations between Actual and Predicted Δs; I(1) Fundamentals w/ Cointegration

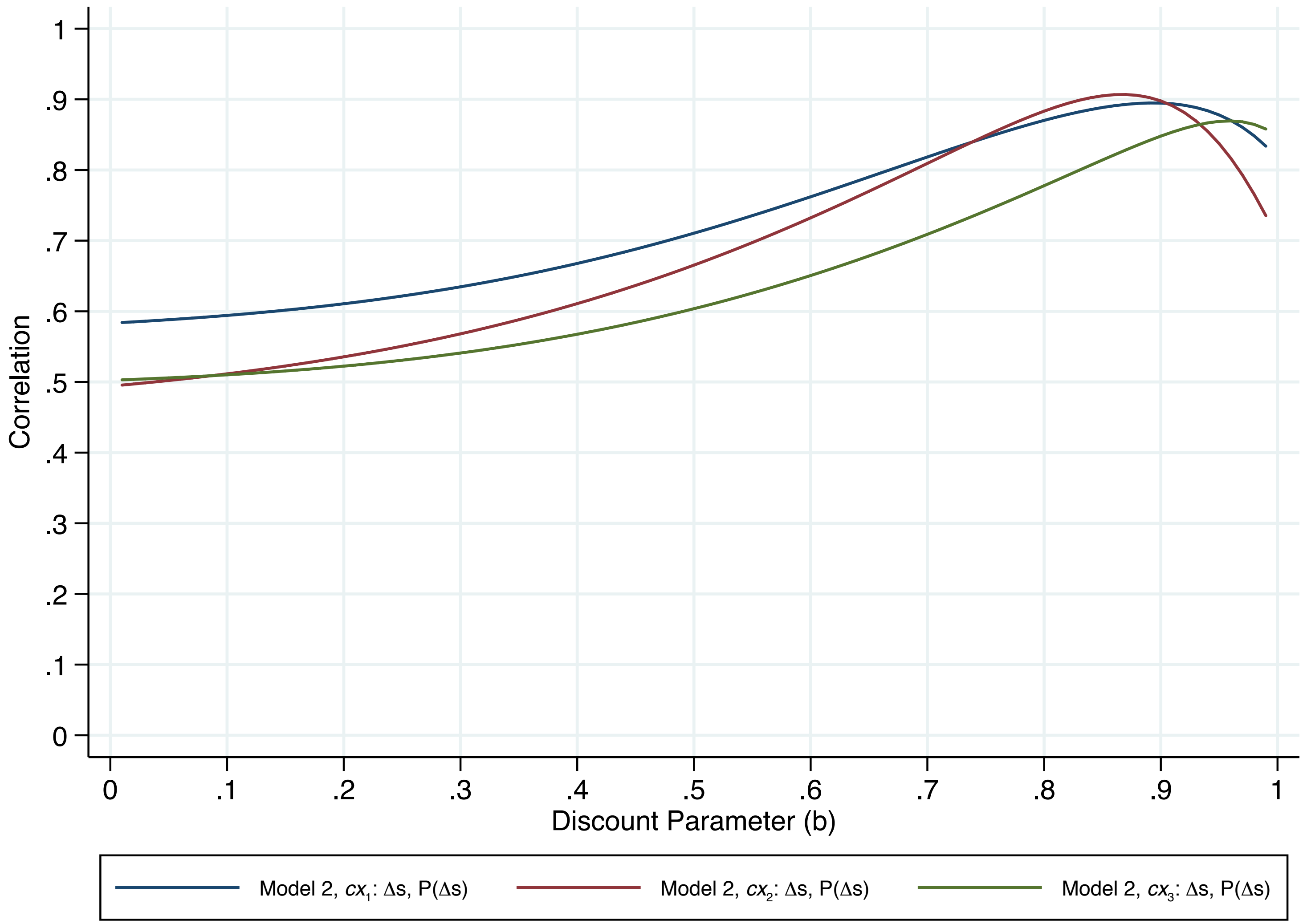

Figure 8 Correlations between Actual and Predicted Δs; Comparison of Commodity Price Models

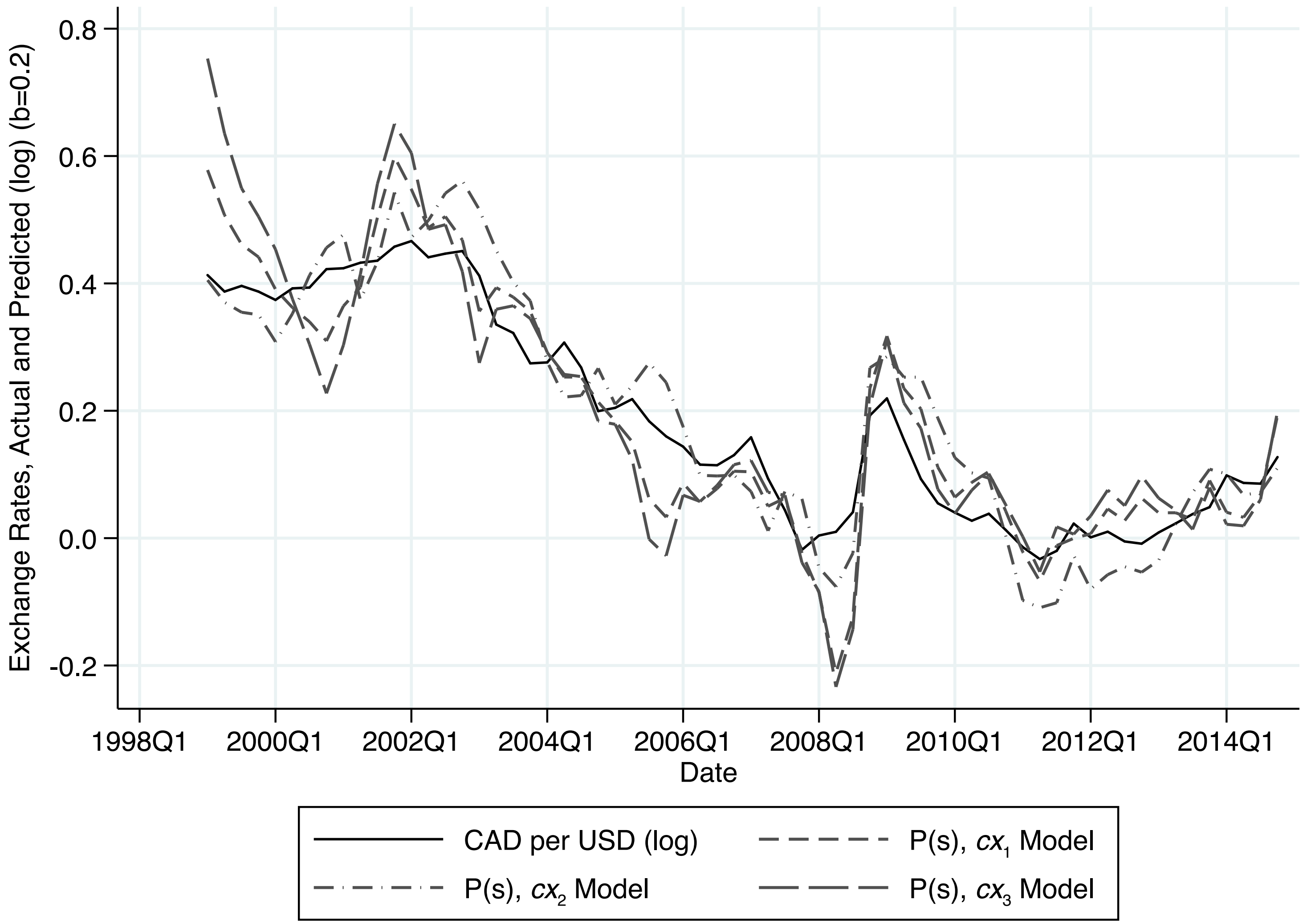

Figure 9 Actual and Predicted s; Commodity Price Models, $b = 0.2$

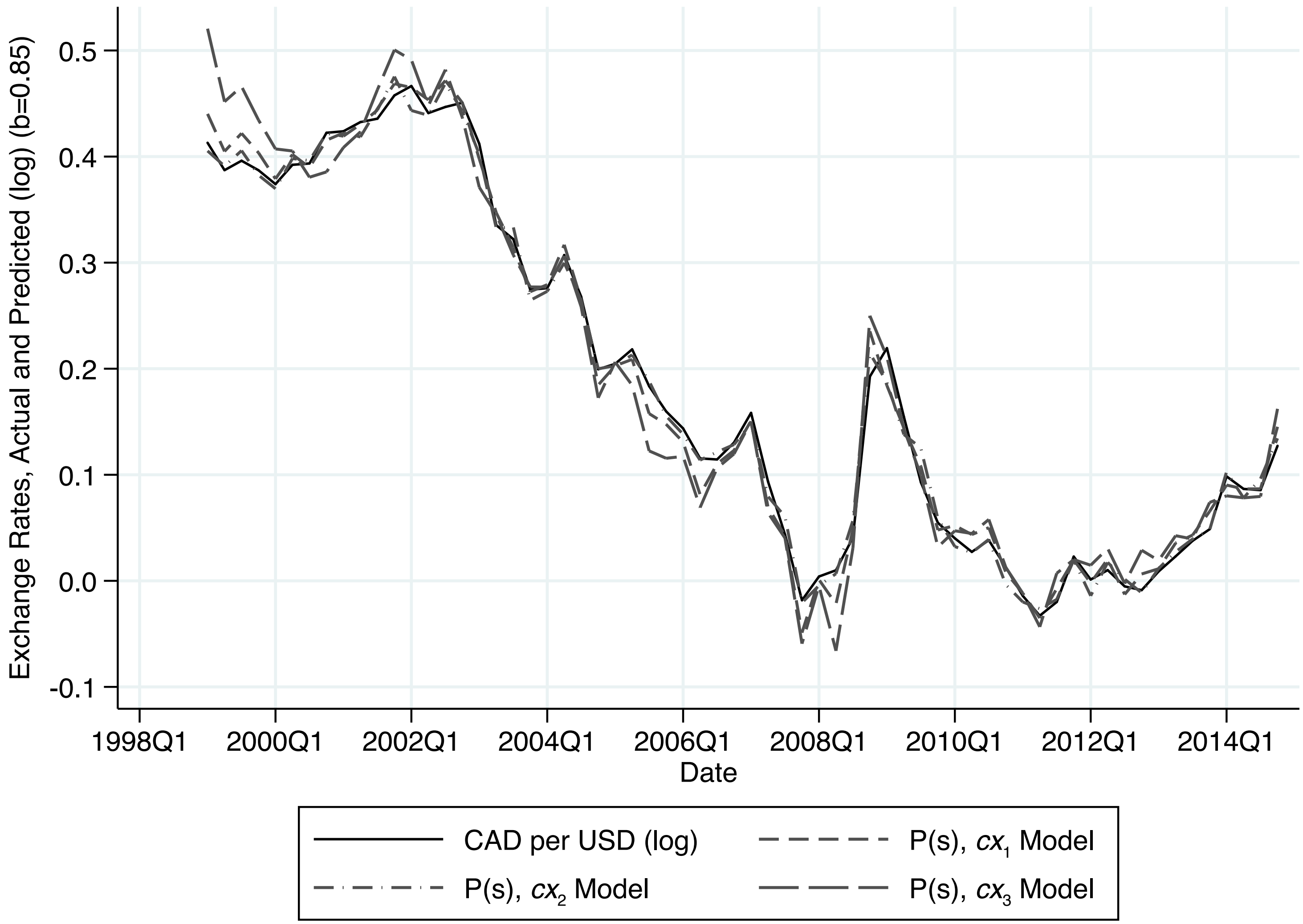

Figure 10 Actual and Predicted s; Commodity Price Models, $b = 0.85$