Simultaneous Equations

- $y_1 = a_1 y_2 + b_1 z_1 + u_1$
- $y_2 = a_2 y_1 + b_2 z_2 + u_2$

Simultaneity

- Simultaneity is a specific type of endogeneity problem
- Here, the explanatory variable is jointly determined with the dependent variable
- As with other types of endogeneity, OLS estimates would be biased and inconsistent
- IV estimation can be used to solve this problem
- There are some special issues to consider with simultaneous equations models (SEM)

Supply and Demand Example

- Let's start with an equation you'd like to estimate, say a labor supply function
- $h_s = a_1 w + b_1 z + u_1$
- Where w is the wage and z is a supply shifter (e.g. non-labor income or number of children)
- We call this a structural equation – it's derived from economic theory and has a causal interpretation where w directly affects h_s

Example (cont)

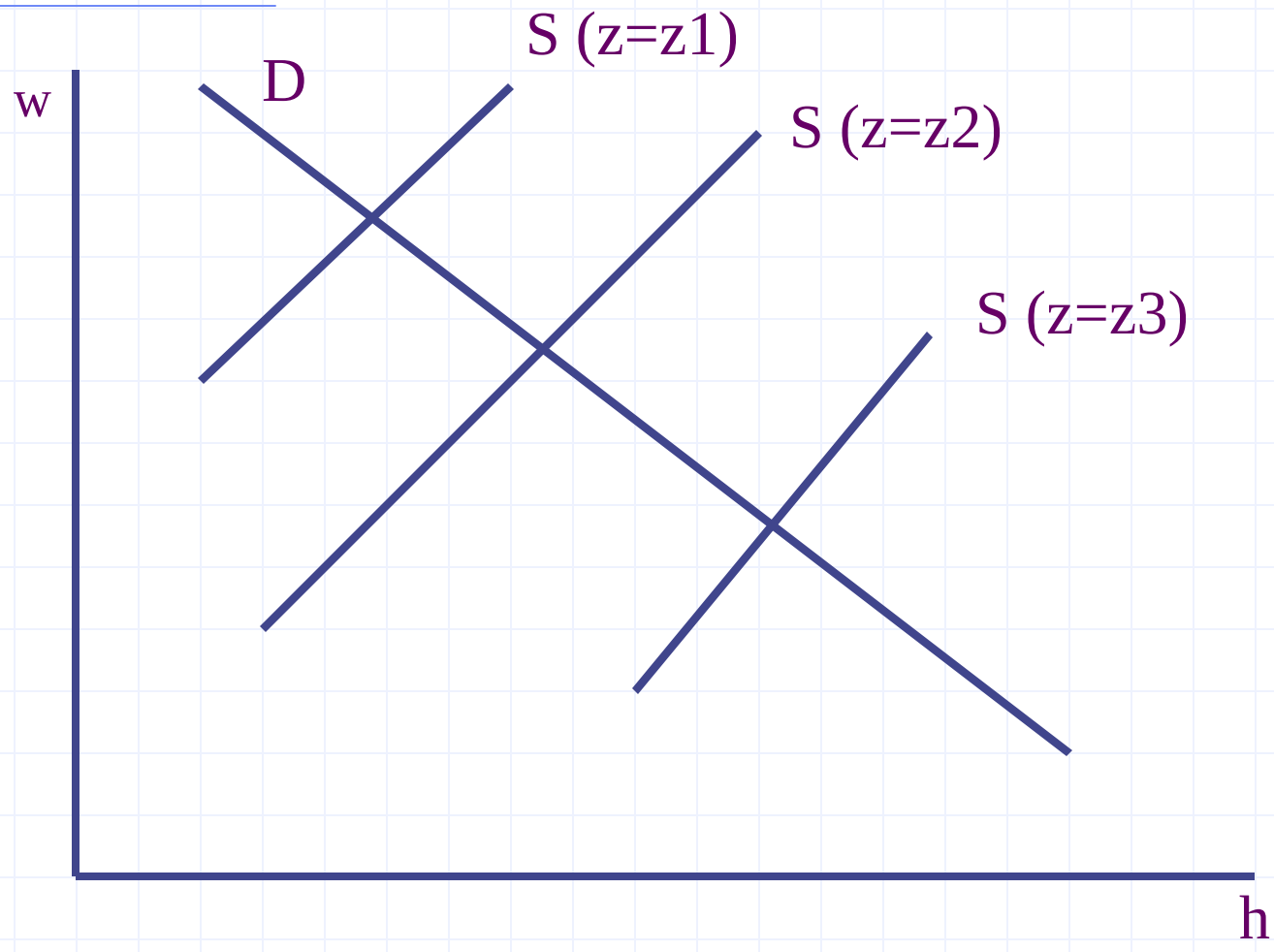
Problem:

- We can't just regress observed hours on wage, because observed hours and wages are determined by the equilibrium of supply and demand
- i.e. we only observe equilibrium wages
- Thus, we must also consider a second structural equation -- the labor demand function
- $h_d = a_2 w + u_2$
- May also have shift variables (e.g. price of capital)
- So hours are determined by a SEM

Example (cont)

- Notice that both *h* and *w* are endogenous because they are determined by the equilibrium of supply and demand
- However, *z* is exogenous
- We need this exogenous supply shifter to allow us to identify **the structural demand equation**
- With no observed demand shifters, supply is not identified and cannot be estimated
- We can show why this is the case graphically

Identification of Demand Equation



Using IV to Estimate Demand

- We can, therefore, estimate the structural demand equation, using z as an instrument for w
- First stage equation is $w = p_0 + p_1 z + v_2$
- Second stage equation is $h = a_2 \quad + u_2$
- Thus, 2SLS provides a consistent estimator of a_2, the slope of the demand curve
- We cannot estimate a_1, the slope of the supply curve unless we can also find a demand shifter that doesn't belong in the supply equation

The General SEM

- More generally, suppose you want to estimate the structural equation: $y_1 = a_1 y_2 + b_1 z_1 + u_1$
- where, $y_2 = a_2 y_1 + b_2 z_2 + u_2$
- Thus, $y_2 = a_2(a_1 y_2 + b_1 z_1 + u_1) + b_2 z_2 + u_2$
- So, $(1 - a_2 a_1) y_2 = a_2 b_1 z_1 + b_2 z_2 + a_2 u_1 + u_2$,
- We can rewrite this as the reduced form equation: $y_2 = p_1 z_1 + p_2 z_2 + v_2$

The General SEM (continued)

- Now, since v_2 is a linear function of u_1, y_2 is correlated with the error term(u_1) in the structural equation (i.e. y_2 is endogenous)
- Thus, estimating the structural equation for y_1 by OLS will lead to a biased estimate of a_1 – called simultaneity bias
- The sign of the bias is complicated, but can use the simple regression case as a rule of thumb
- In the simple regression case, the sign of the bias is the same as $a_2/(1 - a_2 a_1)$

Identification of General SEM

- Let $\boldsymbol{z_1}$ be all the exogenous variables in the first equation, and $\boldsymbol{z_2}$ be all the exogenous variables in the second equation
- It's okay for there to be overlap in $\boldsymbol{z_1}$ and $\boldsymbol{z_2}$
- To identify equation 1, there must be some variables in $\boldsymbol{z_2}$ that are not in $\boldsymbol{z_1}$
- To identify equation 2, there must be some variables in $\boldsymbol{z_1}$ that are not in $\boldsymbol{z_2}$
- We refer to this as the "order condition"

Rank and Order Conditions

- Also, in order to get identification we also need to satisfy the rank condition which says more than the order condition
- The exogenous variable excluded from the first equation must also have a non-zero coefficient in the second equation for the rank condition to hold
- Note that the order condition clearly holds if the rank condition does – there will be an exogenous variable for the endogenous one

Estimation of the General SEM

- Estimation of SEM is straightforward
- The instruments for 2SLS are the exogenous variables from **both equations**
- Can extend the idea to systems with more than 2 equations
- For a given identified equation, the instruments are all of the exogenous variables in the whole system