Simultaneous Equations

- $y_1 = \alpha_1 y_2 + \beta_1 z_1 + u_1$
- $y_2 = \alpha_2 y_1 + \beta_2 z_2 + u_2$

Simultaneity

- Simultaneity is a specific type of endogeneity problem
- Here, the explanatory variable is jointly determined with the dependent variable
- As with other types of endogeneity, OLS estimates would be biased and inconsistent
- IV estimation can be used to solve this problem
- There are some special issues to consider with simultaneous equations models (SEM)

Supply and Demand Example

- Let's start with an equation you'd like to estimate, say a labor supply function
- $h_s = \alpha_1 w + \beta_1 z + u_1$
- Where w is the wage and z is a supply shifter (e.g. non-labor income or number of children)

Example (cont)

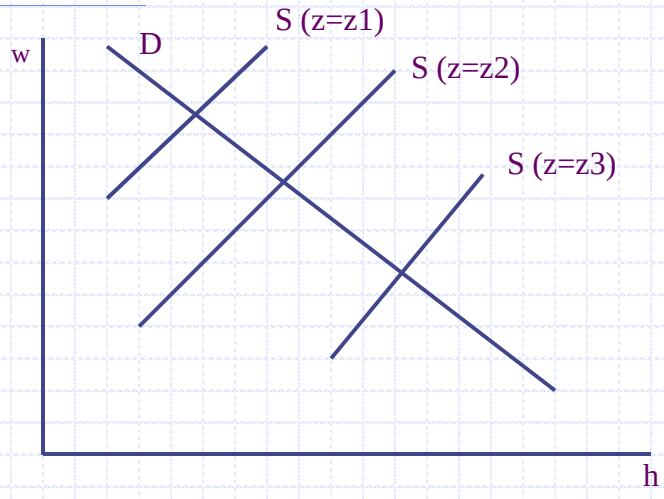
Problem:

- Thus, we must also consider a second structural equation -- the labor demand function
- $h_d = \alpha_2 w + u_2$
- May also have shift variables (e.g. price of capital)
- So hours are determined by a SEM

Example (cont)

- Notice that both *h* and *w* are endogenous because they are determined by the equilibrium of supply and demand
- However, *z* is exogenous

Identification of Demand Equation



Using IV to Estimate Demand

- We can, therefore, estimate the structural demand equation, using z as an instrument for w
- First stage equation is $w = \pi_0 + \pi_1 z + v_2$
- Second stage equation is $h = \alpha_2 \hat{w} + u_2$
- Thus, 2SLS provides a consistent estimator of α_2, the slope of the demand curve

The General SEM

- More generally, suppose you want to estimate the structural equation: $y_1 = \alpha_1 y_2 + \beta_1 z_1 + u_1$
- where, $y_2 = \alpha_2 y_1 + \beta_2 z_2 + u_2$
- Thus, $y_2 = \alpha_2(\alpha_1 y_2 + \beta_1 z_1 + u_1) + \beta_2 z_2 + u_2$
- So, $(1 - \alpha_2\alpha_1)y_2 = \alpha_2 \beta_1 z_1 + \beta_2 z_2 + \alpha_2 u_1 + u_2$,
- We can rewrite this as the reduced form equation:

The General SEM (continued)

- Now, since v_2 is a linear function of u_1, y_2 is correlated with the error term(u_1) in the structural equation (i.e. y_2 is endogenous)
- The sign of the bias is complicated, but can use the simple regression case as a rule of thumb
- In the simple regression case, the sign of the bias is the same as

Identification of General SEM

- Let $\boldsymbol{z_1}$ be all the exogenous variables in the first equation, and $\boldsymbol{z_2}$ be all the exogenous variables in the second equation
- It's okay for there to be overlap in $\boldsymbol{z_1}$ and $\boldsymbol{z_2}$
- To identify equation 1,
- To identify equation 2,
- We refer to this as the "order condition"

Rank and Order Conditions

- Also, in order to get identification we also need to satisfy the rank condition which says more than the order condition
- Note that the order condition clearly holds if the rank condition does – there will be an exogenous variable for the endogenous one

Estimation of the General SEM

- Estimation of SEM is straightforward
- The instruments for 2SLS are the exogenous variables from **both equations**
- Can extend the idea to systems with more than 2 equations
- For a given identified equation, the instruments are all of the exogenous variables in the whole system