Self-Employment and Labor Market Policies

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We develop a model of self-employment in the search and matching framework of Mortensen and Pissarides. We integrate two strands of theoretical literature: models of self-employment and models of unemployment. Our model explains many empirical findings which are not explained by the existing models of self-employment. In our model, higher minimum wage and unemployment benefits have negative effect on self-employment. These results are supported by empirical evidence. In addition, in our model self-employed earn less, on average, than wage employed workers in equilibrium due to frictions in the labor market. Thus our model provides a novel explanation to one of the key puzzles identified in the empirical literature. We also find that a higher business tax and a lower wage tax reduce self-employment.

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1 Introduction

We develop a model of self-employment and analyze the effects of taxes and labor market policies on self-employment.[1] Self-employed workers constitute an important segment of the labor force. In the United States, about a 10th of all workers are self-employed (Evans and Leighton 1989, Hipple 2004). They operate most of the businesses and employ about a tenth of all wage (salaried) workers (Evans and Leighton 1989, Hipple 2004). In other OECD countries, the proportion of self-employed varies between 8-30 percent (Blanchflower 2004).

There is substantial empirical evidence that labor market policies such as the minimum wage and unemployment benefits have a negative effect on self-employment. Bruce and Mohsin (2006) and Garrett and Wall (2006) find significant negative association between self-employment rate and the minimum wage in the United States. Similar to these studies, using PSID (the Panel Study of Income Dynamics) data of males in the age-group of 25-54 years for the period 1977-1996, we find that a higher minimum wage significantly reduced self-employment rate in the United States. In addition, we find that a higher minimum wage reduces the probability that an unemployed worker transits to self-employment. Empirical evidence also finds negative association between unemployment benefits and self-employment (Carrasco 1999 for Spain, Parker and Robson 2000 across OECD countries).

Empirical evidence also suggests that there is significant inflow of unemployed workers into self-employment. Our calculation using PSID data for the period 1977-96 shows that on average 10 percent of unemployed workers move to self-employment from one year to the next in the United States. In contrast, only 3.2 percent of wage employed workers move to self-employment. Other studies also find that unemployed workers are two to three times more likely to become self-employed than wage employed workers (see Evans and Leighton 1989 for the U.S., Kuhn and Schuetze 2001 for Canada, Carrasco 1999 for Spain). Indeed, there is a view that many individuals choose self-employment due to limited job opportunities in the wage sector and for many workers it is a transient state (Blanchflower 2004, Rissman 2003).

[1] The notion of self-employed workers applied in this paper is that they own and operate businesses. In the empirical literature number of self-employed workers is usually measured in two ways: (i) Business owners: if an individual or any other individual in his/her family owns a business then that individual is classified as self-employed (e.g. studies based on PSID and NLSY data sets). (ii) Particular class of workers: any employed person who is not employed by the government, private company or non-profit organization and does not own incorporated business is classified as self-employed (e.g. studies based on CPS data set).

Existing theoretical models of self-employment cannot explain the empirical evidence cited above. These models typically assume a perfectly competitive environment in the labor market in which there is no unemployment.[2] Workers choose between wage employment and self-employment. Absence of unemployment in these models and their static nature preclude the analysis of transitions between self-employment and other labor market states, particularly between self-employment and unemployment, and factors affecting them. In addition, as shown in section 5.4, a model with a competitive labor market predicts a positive association between the minimum wage and self-employment. The reason is that in these models wage workers who loose their jobs due to higher minimum wage become self-employed.

The main aim of the paper is to develop an equilibrium model of self-employment that allows for unemployment and transitions between unemployment and self-employment to explain these empirical findings. We integrate two strands of theoretical literature – models of self-employment and models of unemployment. In particular, we embed a model of occupational choice in the search and matching framework of Mortensen and Pissarides (Pissarides 2000). Search and matching framework is the dominant framework to model labor market flows and is widely used to address labor market issues. In these models, opportunities to trade in the labor market arise randomly and depend on the search-effort of firms and workers, the hiring strategies of firms, the job-acceptance strategies of workers, and luck. Unemployment arises endogenously due to search and matching frictions in the labor market.

Our model distinguishes among three labor market states: self-employment, wage employment, and unemployment. Agents can choose to be either self-employed workers or wage workers in any time-period. Wage workers can be unemployed or wage (or salary) employed. Self-employed who want to hire wage workers have to create vacancies or job openings and search for wage workers. Similarly, wage workers who want to find jobs have to search for suitable vacancies or job openings.

In the baseline model, we focus on the analysis of transitions between self-employment and unemployment. This can be viewed as a model of small firms/businesses, which do not have significant start-up cost and/or require substantial human capital. We focus on these transitions for a number of reasons. Firstly, as mentioned earlier empirical evidence suggests that unemployed workers are much more likely to become self-employed than wage employed workers. Secondly, existing models allow workers to choose only

[2]The examples of models we have in mind are (Lucas 1978, Kanbur 1979, 1981, Kihlstrom and Laffont 1979, Jovanovic 1994).

between self-employment and wage employment and ignore the flows between unemployment and self-employment. We view our model as shedding light on a very important, largely neglected, and an interesting component of self-employment. Finally, restricting attention to these flows allows us to clearly differentiate our approach and mechanisms from the existing models.

After analyzing the baseline model, we extend our model in two different directions. In the baseline model, we assume that *ex-ante* self-employed workers have sufficient managerial ability and knowledge of hiring rules and regulations to become employers. However, it is more realistic to assume that these abilities and knowledge are discovered and/or acquired over time. In addition, empirical evidence suggests that most of the self-employed workers do not hire wage workers (*i.e.*, only work on own account). In the first extension, we assume that managerial ability and knowledge of hiring rules and regulations are experience goods, which are acquired while working on own-account. This extension leads to emergence of two types of self-employed workers in equilibrium: own account workers and employers.

In the second extension, we incorporate dual labor markets: one with high wages and one with low wages. One can think of the low wage sector as consisting of small businesses and the high wage sector as consisting of big businesses.[3] Labor market policies such as minimum wage and unemployment benefits are more likely to affect (directly) the low wage sector. We introduce on the job search by both self-employed and employed wage workers engaged in the low wage sector. On the job search by self-employed workers allows us to incorporate transition from self-employment to wage employment. Empirical evidence suggests that the rate of transition from self-employment to wage employment is fairly high. Using PSID data we find that on average 18 percent of self-employed move to wage employment from one year to the next. In contrast, only 3 percent of self-employed move to unemployment (see also Rissman 2003).

In the models developed, we examine four policies – a wage tax, a business tax, unemployment benefits, and the minimum wage. While the models are flexible enough to allow the analysis of effects of other taxes and labor market policies (*e.g.* payroll taxes, sales tax, job creation subsidies, job-protection policies), we believe that the four policies analyzed are sufficient to illustrate the mechanisms developed in the paper.

Our primary findings regarding the effects of tax and labor market policies are as follows. First, we find that a lower wage tax and a higher business tax reduce the level of self-employment and the rate of inflow into self-

[3]Empirical literature suggests significant positive correlation between average wage of employees and size of firms (firm-size wage premium).

employment from unemployment. Second, our model suggests that higher unemployment benefits and minimum wage reduce the level and the rate of inflow into self-employment from unemployment, which is consistent with the empirical evidence cited above. Finally, in the two-sector model we also find that a lower wage tax and a higher business tax, unemployment benefit, and minimum wage reduce the number as well as the proportion of employed wage workers in the high wage sector.

The assumptions of our models that (i) the opportunity cost of being a self-employed worker is to become a wage worker and (ii) a wage worker must search before she finds a job have an interesting and important implication regarding earnings differential. Self-employed workers earn less, on average, than wage employed workers in equilibrium in our model. A self-employed worker accepts lower earnings in equilibrium in order to avoid the spell of unemployment.

Our paper provides a novel explanation based on frictions in the labor market for one of the key puzzles identified in the empirical literature, namely average earnings of self-employed workers is less than the average earnings of employed wage workers (Aronson 1991, Hamilton 2000). This puzzle arises because in the existing theoretical models, typically individuals with superior attributes (*e.g.* superior managerial ability (Lucas 1978, Jovanovic 1994), higher risk-taking ability (Knight 1921, Kanbur 1979, Kihlstorm and Laffont 1979), greater access to capital (Evans and Jovanovic 1989)) become self-employed (more precisely entrepreneurs). The existing literature attributes the earnings differential between these two groups either to non-pecuniary benefits of self-employment such as "*being your own boss*" (Hamilton 2000) or to a greater possibility of tax evasion in self-employment (Parker 2004).

Our paper is related to Fonseca, Lopez-Garcia, and Pissarides (2001), who study the effects of business start-up costs on employment in a search and matching model of agents with *ex-ante* heterogeneous managerial ability. Unlike them our focus is on the labor market and tax policies. In addition, in their model only high ability agents become self-employed (entrepreneurs) and thus average earnings of self-employed exceed the average earnings of wage employed workers.

The remainder of the paper is organized as follows. In sections 2 to 5, we develop and analyze our baseline model. In section 2 we describe the environment. In section 3, we analyze the optimal decisions of self-employed and wage workers. In section 4, we prove the existence and uniqueness of a stationary equilibrium and analyze its main characteristics. In section 5, we analyze the effects of taxes and labor market policies. In section 6, we develop extensions of our baseline model and analyze the effects of taxes and labor market policies. In section 7, we provide empirical evidence on the effects of minimum wage on self-employment. This is followed by a conclusion. All proofs are in an appendix.

2 Environment

To begin with we consider a one-sector model with flows between self-employment and unemployment and between unemployment and wage employment. In section 6, we develop the two-sector model which allows for inter-sectoral transitions and outflow from self-employment to wage employment as well.

Time is discrete. Consider a labor market consisting of a unit measure of infinitely-lived *ex-ante* identical individuals. These individuals discount the future at the common rate r. They are *risk-neutral* and can choose to be either a self-employed worker or a wage workers in any time period. No individual can be both at the same time. The assumption of risk-neutral individuals allows us to clearly characterize the *direct effects* of public policies on self-employment in an imperfect labor market.[4] Also assuming occupational choice as a discrete rather than continuous variable is standard in the literature (*e.g.* Lucas 1978, Kanbur 1979, 1981).

Wage workers can be employed or unemployed. Let E_t, N_t, and U_t be the measures of self-employed workers, wage employed workers, and unemployed workers respectively in the economy at time t. Then

$$E_t + N_t + U_t = 1 \ \forall \ t. \tag{2.1}$$

Note that total employment at time t is given by the sum of self-employed and wage employed workers, $E_t + N_t$.

Self-employed workers create and manage firms (or businesses) and organize (market) production. We assume that firms can be created costlessly, *i.e.*, the self-employed do not face any significant start-up cost and/or are not liquidity constrained. As discussed in section 4.2, the presence of significant start-up cost-up does not change the qualitative effects of taxes and labor market policies on equilibrium variables. However, it can potentially change the implications regarding the earnings differential.

[4] Attitude towards risk is one of the most important determinants of self-employment (Knight 1921, Kanbur 1979, Kihlstorm and Laffont 1979). The interaction between attitude towards risk and frictions in the labor market is of obvious importance as frictions in the labor market increase the risks for both self-employed workers (through randomness in hiring, hold-up problem etc.) and wage workers (through unemployment). These interactions also provide a natural setting to analyze the effects of social insurance policies on self-employment. These issues are left for future research.

The effect of liquidity constraints on self-employment has been the subject of controversy in the literature. A large literature has documented a positive relationship between initial wealth and subsequent business entry (Evans and Jovanovic 1989, Holtz-Eakin *et. al.* 1994a,b, Fairlie 1999). However, Hurst and Lusardi (2004) using the PSID data find that there is no relationship between wealth and business entry over most of the wealth distribution. They find positive relationship between the two only for the households in top five percent of the wealth distribution. There conclude that liquidity constraints are not empirically important deterrent in the formation of the majority of small businesses.[5]

Meyer (1990) also finds that business start-up cost is relatively low in the U.S. for most of the businesses and that liquidity constraints are not an important determinant of the racial differences in self-employment rates. Since we focus on the transitions between unemployment and self-employment and assume low start-up cost, our model can be thought of as a model for small firms/businesses/employers.

A self-employed worker is assumed to create and manage just one firm in any period of time. Thus, in any period of time the number of self-employed workers and firms are equal. In the remainder of the paper, we will be using the terms self-employed, firms, and businesses interchangeably. Production at a firm depends on the number of employees, n (which can be zero), and the effort by the owner/manager, e. The production function is assumed to be an increasing and concave function of its arguments. Owners/managers supply their effort inelastically. For production at a firm to take place, effort by the owner/manager is necessary. In what follows, we normalize $e = 1$. The production function is given by

$$F(n, 1) = f(n), \text{ with } f_n(n) > 0, f_{nn}(n) < 0, \lim_{n \to 0} n f_n(n) = 0 \tag{2.2}$$

where $f_n(n)$ and $f_{nn(n)}$ are first and second derivatives with respect to the number of employees.[6]

Self-employed workers face the possibility of business failure. In any period a self-employed receives an $i.i.d$ business failure shock with probability μ. In the case of business failure both the self-employed as well as the employees

[5]Hurst and Lusardi (2004) report that twenty-five percent of small businesses started between 1980 and 1988 in the U.S. started with less than \$5000 (in 1996 dollars) of capital. The median starting capital was \$22,700. They also find self-employment rates in the U.S. to be relatively high at both ends of the wealth distribution.

[6]Throughout the paper for any function, $F(,)$, F_i and F_{ii} denote the first and the second derivatives with respect to the ith argument. F_{ij} denotes its cross-derivative.

(if any) become unemployed. The business failure shock is entirely temporary and a failed self-employed can start a business after a spell of unemployment. This assumption ensures that individuals are inherently identical. The idea is that if a self-employed fails in one business, it does not preclude her from starting another successful business.

The labor market is characterized by search frictions *i.e.*, opportunities to trade in the labor market arise randomly. A firm which wants to hire wage workers has to create vacancies or job openings and search for wage workers. Due to frictions, a firm may not be able to hire wage workers, even if it would like to do so. Thus, at any point in time in the model there will be two types of self-employed workers (or firms): own-account workers (or firms with no employees) and employers (or firms with employees). This distinction arises solely due to friction in the labor market. However, other factors that can create such distinction such as managerial ability, knowledge about employment rules and regulations etc. can be introduced. In section 6.1, we extend our model to incorporate these additional factors. The qualitative results do not change.

Similarly, a wage worker who wants to find a job has to search for suitable vacancies. Note that an individual who does not want to become a self-employed worker first joins the ranks of unemployed wage workers.[7] Thus the opportunity cost of being a self-employed worker is being an unemployed worker. In the two-sector model developed in section 6.2, we allow transition of self-employed workers engaged in the low wage sector to jobs in the high wage sector without any intervening unemployment spell.

Let ξ be the cost of creating and maintaining a vacancy per period in terms of goods. Denote the total number of vacancies created in the economy by V_t at time t. Vacancies and unemployed wage workers are brought together bilaterally by a matching function, $M(U_t, V_t)$. The matching function gives the total number of contacts among unemployed workers and vacancies or matches in any time period. The matching function is assumed to have constant returns to scale in U_t and V_t, be an increasing and concave function of its arguments, and be equal to zero if either U_t or V_t is zero. In addition, $M(U_t, V_t) \leq min(U_t, V_t)$, $\forall t$, which ensures the co-existence of unemployment and vacancies. These assumptions are standard in the search and matching literature.

[7] In the model, an individual can find a wage job only after search. Empirical evidence suggests that there is significant direct inflow to wage employment from inactivity (Blanchard and Diamond 1990). However, in these cases the unemployed and the inactive are distinguished on the basis of their search-intensities. Wage workers with low search-intensity are categorized as inactive. In our model, inactive workers would be workers who do not search at all.

Define labor market tightness $q_t \equiv \frac{V_t}{U_t}$. Then the matching probability of unemployed wage workers is given by

$$\frac{M(U_t,V_t)}{U_t} \equiv m^u(q_t), \text{ where } m^u_q(q_t) > 0, \lim_{q_t \to 0} m^u(q_t) = 0. \tag{2.3}$$

The matching probability of vacancies is given by

$$\frac{M(U_t,V_t)}{V_t} \equiv \frac{m^u(q_t)}{q_t} \equiv m^v(q_t), \text{ where } m^v_q(q_t) < 0, \lim_{q_t \to 0} m^v(q_t) = \infty. \tag{2.4}$$

Once a match is formed between an unemployed worker and a vacancy (*i.e.*, both the unemployed worker and the firm accept the match), the vacancy or job opening is filled. Assume that a newly filled job starts producing from the next period. This assumption implies that a new self-employed worker works on her own account for at least a period.

The filled job continues producing until it is terminated. A filled job can be terminated by an exogenous idiosyncratic job-separation shock or business failure shock. Assume that a filled job receives the job-separation shock with probability, σ, per period. Thus an employed wage worker becomes unemployed with probability, $\sigma + \mu$, while a self-employed worker becomes unemployed with probability, μ.[8] In any period, these shocks are realized after production has taken place.

Denote the fraction of unemployed workers who become self-employed or start businesses every period by γ_t. Anticipating an equilibrium in which the value function (expected life-time earnings/utility under optimal strategies) of a wage employed worker, λ^n_t, and a self-employed worker, $\pi_t(n_t)$, is greater than or equal to the value function of an unemployed worker, λ^u_t, the evolution of the number of unemployed workers, wage employed workers, and self-employed workers over time are given by

$$U_{t+1} = (\sigma + \mu)N_t + \mu E_t + (1 - m^u(q_t) - \gamma_t)U_t. \tag{2.5}$$

Similarly, the law of motion for wage employed workers is given by

$$N_{t+1} = m^u(q_t)U_t + (1 - \sigma - \mu)N_t. \tag{2.6}$$

The law of motion for self-employed workers is given by

$$E_{t+1} = \gamma_t U_t + (1 - \mu)E_t. \tag{2.7}$$

[8]Our calculation using PSID data shows that 3.7 percent of wage employed worker move to unemployment from one year to the next in the United States. The corresponding figure for the self-employed workers is 2.97 percent.

The left hand side of (2.5) is the number of unemployed workers at the beginning of period $t+1$. The first term on the right hand side is the number of wage employed workers who become unemployed. The second term is the number of self-employed who become unemployed due to business failures. The third term is the number of unemployed workers at the beginning of time t, who remain unemployed at the end of period t.

The left hand side of (2.6) is the number of wage employed workers at the beginning of period $t + 1$. The first term on the right hand side is the inflow to the wage employment pool. The second term is the number of wage employed workers at the beginning of time t, who remain employed at the end of period t. Similarly, the left hand side of (2.7) is the number of self-employed at the beginning of period $t + 1$. The first term on the right hand side is the inflow to the pool of self-employment. The second term is the number of self-employed at the beginning of time t, who continue to be self-employed at the end of period t.

Finally, there is a government which imposes taxes, a minimum wage, and pays unemployment benefits. We consider two taxes: a business tax, τ_d, and a wage tax, τ_w. The business tax is imposed on the income of self-employed workers and the wage tax on the income of wage workers (both unemployed and employed). Assume that $0 \leq \tau_d, \tau_w < 1$. Both tax rates are proportional and assumed to be constant over time. Also suppose that each unemployed worker receives unemployment benefit, b ($0 \leq b < \infty$), per period from the government as long as she is unemployed. In the next section, we analyze the optimal choices of self-employed workers and wage workers.

3 Optimal Decisions

We first describe the optimal choices of self-employed workers (or firms) and then of wage workers. The process of wage determination is analyzed in section 3.3 below.

3.1 Self-employed Workers or Firms

Denote the wage paid to an employee (if any) by a firm at time t by w_t and the number of vacancies created by v_t. Then the earnings of a firm net of business tax at time t is $(1 - \tau_d)[f(n_t) - w_t n_t - \xi v_t]$. A firm chooses the match acceptance strategy and a sequence of levels of employment, n_{t+1}, and vacancies, v_t, in order to maximize its expected life-time earnings (or inter-temporal profit). Denote the expectation operator conditional on time t information as $\mathcal{E}_t$. The value function of a firm at time t, $\pi_t(n_t)$, is given by

$$\pi_t(n_t) = \max_{v_t, n_{t+1}} (1-\tau_d)[f(n_t) - w_t n_t - \xi v_t] + \frac{1}{1+r}\mathcal{E}_t[(1-\mu)\pi_{t+1}(n_{t+1}) + \mu\lambda^u_{t+1}] \tag{3.1}$$

subject to

$$n_{t+1} \leq (1-\sigma)n_t + m^v(q_t)v_t, \text{ if the business does not fail} \tag{3.2}$$

given labor market tightness, q_t, and the strategies of wage workers, and other firms. (3.1) gives the maximal inter-temporal profit net of business tax and cost of creating v_t vacancies. The first expression on the right hand side is the net flow of profit at time t. The second expression is the discounted expected continuation value, which takes into account that the business can fail with probability μ. (3.2) gives the number of employees at the beginning of period $t+1$, if the business does not fail. The first term on the right hand side of (3.2) is the number of employees at the beginning of period t who remain with the firm at the end of period t. The second term is the expected number of new employees.

The first order condition for the optimal level of vacancies is

$$v_t: \ (1-\tau_d)\xi = m^v(q_t)\frac{1-\mu}{1+r}\mathcal{E}_t\pi_{nt+1}(n_{t+1}). \tag{3.3}$$

(3.3) equates the marginal cost of creating vacancies with its discounted expected marginal benefit. The expected marginal benefit is the product of the matching probability of a vacancy, $m^v(q_t)$, the probability of the business not failing, and the value of the expected future pay-offs from one additional employee (marginal value of an employee or a filled job), $\pi_{nt+1}(n_{t+1})$. From the envelope condition, we have

$$\pi_{nt}(n_t) = (1-\tau_d)(f_n(n_t) - w_t) + \frac{(1-\mu)(1-\sigma)}{1+r}\mathcal{E}_t\pi_{nt+1}(n_{t+1}). \tag{3.4}$$

(3.4) shows that one extra employee increases the net profit of the firm by $(1-\tau_d)(f_n(n_t)-w_t)$. The second term in the bracket is the expected continuation value (or the value of expected future pay-offs), which takes into account that a match may break up due to business failure and job separation shocks. In the paper, we will focus on the equilibrium in which $\pi_{nt}(n_t) > 0, \ \forall t$, and thus the inequality given in (3.2) will be binding.

For remainder of the paper, we focus on the steady state. In the steady state, the value functions and other endogenous variables are invariant with respect to time. Also inflows and outflows from any labor market state are equal. In order to denote variables in the steady state, we drop the subscript t.

In the steady state, (3.3) and (3.4) imply that the optimal level of vacancies is given by

$$\xi = (1-\mu)m^v(q)\left[\frac{f_n(n) - w}{r+\sigma+\mu-\sigma\mu}\right]. \tag{3.5}$$

The expected inter-temporal profit of a firm is given by

$$\pi(n) = (1-\tau_d)\frac{1+r}{r+\mu}\left[f(n) - wn - \xi v\right] + \frac{\mu}{r+\mu}\lambda^u. \tag{3.6}$$

We assume that the parameters of the model are such that $\pi(n) > (1-\tau_d)\frac{1+r}{r+\mu}f(0) + \frac{\mu}{r+\mu}\lambda^u$ and thus a firm is better-off hiring wage workers. This requires that the equilibrium flow of profit $f(n) - wn - \xi v > f(0)$. In section 6.1 we develop a model in which hiring wage workers requires sufficient managerial ability and knowledge about rules and regulations. Here the assumption is that any self-employed worker has sufficient managerial ability and knowledge to become an employer.

3.2 Wage Workers

A wage worker chooses job or match acceptance strategy in order to maximize her expected life-time earnings (utility); taking as given labor market tightness, q, and the strategies of firms and other wage workers. Let λ^n and λ^u be the value functions of employed wage workers earning wage w and unemployed workers respectively in the stationary state. Then λ^u and λ^n are given by

$$\lambda^u = b(1-\tau_w) + \frac{1}{1+r}\left[m^u(q)\lambda^n + (1-m^u(q))\lambda^u\right] \tag{3.7}$$

and

$$\lambda^n = w(1-\tau_w) + \frac{1}{1+r}\left[(1-\sigma-\mu)\lambda^n + (\sigma+\mu)\lambda^u\right]. \tag{3.8}$$

(3.7) reflects the fact that the current earnings of an unemployed worker net of the wage tax is, $b(1-\tau_w)$, and next period with probability, $m^u(q)$, she can become wage employed and with probability, $(1-m^u(q))$, she remains unemployed. (3.8) can be interpreted in a similar fashion. Current income of a wage employed worker net of the wage tax is $w(1-\tau_w)$. Next period she can become unemployed with probability, $\sigma+\mu$, and with probability, $(1-\sigma-\mu)$, she continues to be employed. The optimal job-acceptance strategy for an unemployed worker is to accept a job iff $\lambda^n \geq \lambda^u$.

3.3 Wage Determination

The wage determination process is modeled in the standard fashion (Pissarides 2000). A match between an unemployed wage worker and a vacancy generates surplus. The match surplus is $S = \lambda^n - \lambda^u + \pi_n(n)$.[9] The surplus is divided between the firm and the unemployed wage worker in a match through individual Nash bargaining. Let $\beta \in (0, 1)$ be the bargaining power of a firm. Thus the wage solves

$$\max_w (\lambda^n - \lambda^u)^{1-\beta} \pi_n^\beta(n) \tag{3.9}$$

subject to

$$S \geq 0. \tag{3.10}$$

The first order condition is

$$(1-\beta)(1-\tau_w)\pi_n(n) = \beta(\lambda^n - \lambda^u)(1-\tau_d). \tag{3.11}$$

As shown in the appendix, the wage function is given by

$$w = \frac{1}{1+A}\left[Af_n(n) + \frac{1-\beta}{\beta}\frac{\xi}{1-\mu}q + b\right], \text{ with } w_n < 0 \ \& \ w_q, w_b > 0 \tag{3.12}$$

where A is a positive constant given by

$$A \equiv \frac{r+\sigma+\mu}{r+\sigma+\mu-\sigma\mu}\frac{1-\beta}{\beta}.$$

(3.12) shows that the wage is increasing in the marginal product of labor, $f_n(n)$, labor market tightness, q, and the unemployment benefit, b, (which determines the outside option for unemployed workers)[10]. The return to a firm from accepting a match has two components: the marginal product of labor, $f_n(n)$, and the expected saving of hiring cost, ξq. The average hiring

[9] Since the level of vacancies is optimally chosen by firms, the marginal value of a vacancy is zero.

[10] When $\mu = 0$ or $\sigma\mu \approx 0$, $A = \frac{1-\beta}{\beta}$ and the wage function reduces to its standard form

$$w = (1-\beta)(f_n(n) + \xi q) + \beta b.$$

cost for each unemployed worker in the economy is $\frac{\xi V}{U} \equiv \xi q$. Wages go up as the return to a firm goes up. On the other hand, a higher unemployment benefit increases the wage by increasing the outside option of unemployed workers. Note that (3.4) and (3.12) imply that the net expected value of an additional employee is positive, $\pi_n(n) \geq 0$, only when the marginal product of labor exceeds the unemployment benefit, $f_n(n) > b$.

4 Equilibrium

In the steady state, the inflows to and outflows from any state are equal. Also the total number of wage employed workers is equal to the total number of employees, $N = nE$. Then utilizing (2.1) and (2.5)-(2.7), we can derive expressions for the equilibrium number of self-employed workers, E, wage employed workers, N, unemployed workers, U, and the fraction of unemployed workers who become self-employed every period, γ.

The equilibrium number of self-employed is given by

$$E = \frac{m^u(q)}{m^u(q) + n\left(\sigma + \mu + m^u(q)\right)} \text{ with } E_q > 0 \;\&\; E_n < 0. \tag{4.1}$$

(4.1) is a key equation of our model. It shows that the equilibrium number of self-employed, E, is increasing in labor market tightness, q, and decreasing in average firm-size, n. Intuitively, for a given average firm-size, n, a higher matching probability of unemployed workers requires that in equilibrium the number of self-employed should be larger. On the other hand, for a given q or matching probability of workers, larger average firm-size leads to a smaller number of self-employed workers in order to maintain equality between the inflow to and outflow from the unemployment pool. Similarly, one can derive expressions for the number of wage employed workers and unemployed workers, which are given by

$$N = \frac{nm^u(q)}{m^u(q) + n\left(\sigma + \mu + m^u(q)\right)} \text{ with } N_q > 0 \;\&\; N_n > 0 \tag{4.2}$$

and

$$U = \frac{n(\sigma + \mu)}{m^u(q) + n\left(\sigma + \mu + m^u(q)\right)} \text{ with } U_q < 0 \;\&\; U_n > 0. \tag{4.3}$$

(4.2) shows that the equilibrium number of wage employed workers, N, is increasing in both labor market tightness, q, and average size of a firm, n. (4.3) shows that the equilibrium number of unemployed workers, U, is decreasing in labor market tightness, q, and increasing in average firm-size,

n. Since total employment $N + E = 1 - U$, total employment is increasing in q and decreasing in n.

For a given average size of firm, n, wage employment is increasing and unemployment is decreasing in labor market tightness, q, due to higher matching probability of unemployed workers. On the other hand, for a given labor market tightness, q, both wage employment and unemployment are increasing in average firm-size, n, since higher n implies a smaller number of firms and, thus, a larger number of wage workers. For a given q and thus matching probability it leads to both higher wage employment and unemployment.

Finally, the expression for the fraction of unemployed workers who become self-employed (or start a business) every period, γ, is given by

$$\gamma = \frac{\mu m^u(q)}{(\sigma + \mu)n} \text{ with } \gamma_q > 0 \ \& \ \gamma_n < 0. \tag{4.4}$$

Intuitively for a given average firm-size, n, higher labor market tightness, q, implies that the outflow from unemployment to wage employment is higher. Thus, the number of firms should also be higher, which implies higher γ. Similarly, for a given labor market tightness, q, a higher average firm-size, n, implies a smaller number of firms and thus lower γ.

4.1 Existence of an Equilibrium

(3.5-3.8), (3.12), and (4.1-4.4) show that value functions, the levels of vacancies, wage, profit, wage employment, unemployment, self-employment, and the fraction of unemployed workers who become self-employed are functions of two endogenous variables: labor market tightness, q, and average firm-size, n. Once we find equilibrium values of these two variables, we can derive other endogenous variables.

By combining the first-order condition for the vacancy creation given in (3.5) and the expression for the wage given in (3.12), we get an expression which gives a relationship between labor market tightness, q, and average firm-size, n.

$$(1 + A)\frac{\xi}{1-\mu}\frac{r + \sigma + \mu - \sigma\mu}{m^v(q)} + \frac{1-\beta}{\beta}\frac{\xi}{1-\mu}q = f_n(n) - b. \tag{4.5}$$

We call the relationship between q and n traced by (4.5) as the *job-creation curve.* The properties of this curve are summarized in lemma 1 below.

Lemma 1 For any average firm-size, $n \in (0, \overline{n})$, where $\overline{n}$ solves $f_n(n) = b$, there exists a unique labor market tightness, $q \in (0, \infty)$, which solves equation (4.5). In addition, (4.5) implies a strictly negative association between labor market tightness, q, and average firm-size, n, *i.e.*, $\frac{dq}{dn} < 0$. Therefore the job creation curve is downward sloping in (n, q) space.

The intuition for a negative association between labor market tightness, q, and average firm-size, n, is quite simple. A higher n implies a lower marginal product of labor and thus a lower expected return from creating vacancies. Thus firms create smaller number of vacancies; reducing labor market tightness, q.

Because the opportunity cost of being a self-employed worker is to become an unemployed worker, the equilibrium requires that an individual be indifferent between these two states at the margin. Thus,

$$\pi(n) \equiv (1-\tau_d)\frac{1+r}{r+\mu}\left[f(n) - wn - \xi v\right] + \frac{\mu}{r+\mu}\lambda^u = \lambda^u. \tag{4.6}$$

(4.6) pins down the distribution of individuals between self-employed workers and wage workers and thus the number of wage workers available per self-employed worker or firm. As shown in the proof of Lemma 2, (4.6) can be written as

$$(1-\tau_d)\Big[f(n) - wn - \frac{\xi(\sigma+\mu)n}{m^v(q)}\Big] = (1-\tau_w)\Big[b + \frac{\xi}{1-\mu}\frac{1-\beta}{\beta}q\Big]. \tag{4.7}$$

(4.7) gives another relationship between q and n. We call this relation the *firm-size curve*. The properties of the firm-size curve are summarized in lemma 2 below.

Lemma 2 Under the condition that $f(\overline{n}) - \overline{n}b > \frac{1-\tau_w}{1-\tau_d}b$, for any average firm-size, $n \in (\underline{n}, \overline{n})$, where $\underline{n}$ satisfies

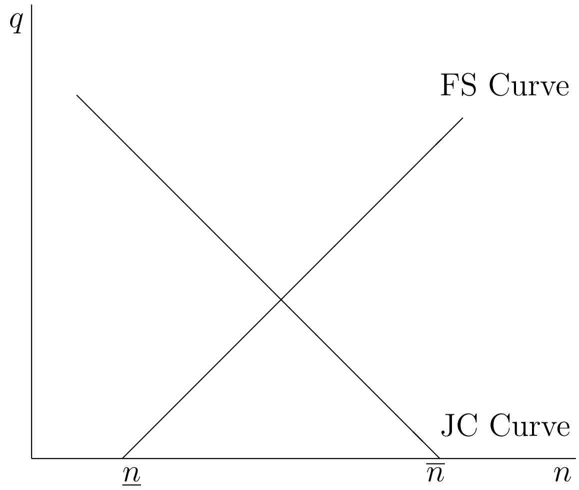
$$(1-\tau_d)\left[f(\underline{n}) - \frac{n}{1+A}[Af_n(\underline{n}) + b]\right] = (1-\tau_w)b$$

there exists a unique labor market tightness, q, which solves (4.7). In addition, (4.7) implies a strictly positive association between labor market tightness, q, and average firm-size, n, *i.e.*, $\frac{dq}{dn} > 0$. Therefore, the firm-size curve is upward sloping in (n, q) space.

At $\underline{n}$ the value function of a self-employed worker is equal to the value function of an unemployed worker when labor market tightness, $q = 0$, *i.e.* $\lim_{q\to 0}(\pi(\underline{n}) - \lambda^u) = 0$. For any $n \in (\underline{n}, \overline{n})$, $\lim_{q\to 0}(\pi(n) - \lambda^u) > 0$. The intuition for an upward sloping firm-size curve is that for a given labor market tightness, q, a higher average firm-size, n, implies that firms earn more profit, which induces them to create more vacancies. The result is that labor market tightness, q, increases. The intersection of the job-creation curve and the firm-size curve determines equilibrium labor market tightness, q, and average firm-size, n (see figure 1 below).

Proposition 1 Under the conditions specified in lemmas 1 and 2, there exists a steady state equilibrium characterized by equations (3.12) and (4.1-4.7).

Figure 1
Graphic Portrait of Equilibrium



Below we discuss the implication of our model for the earnings of self-employed workers and employed wage workers.

4.2 Earnings Differential

In equilibrium, $\lambda^n > \lambda^u = \pi(n)$. In other words, the expected life-time earnings of wage employees is strictly greater than the expected life-time earnings of self-employed workers. To see this, consider (3.7) and (3.8) which imply

$$\lambda^n - \lambda^u = (1 - \tau_w)(1 + r) \left[\frac{w - b}{r + \sigma + \mu + m^u(q)} \right]. \tag{4.8}$$

Since in equilibrium, $n < \overline{n}$, the marginal product of labor exceeds the unemployment benefit, $f_n(n) > b$. This implies that the equilibrium wage given in (3.12), $w > b$ and thus $\lambda^n > \lambda^u = \pi(n)$.

The model also implies that the average current earnings of a wage employed worker exceeds the average current earnings of a self-employed worker *i.e.*, $w(1-\tau_w) > (1-\tau_d)[f(n) - wn - \xi v]$. Since, $\lambda^u = \pi(n)$, in equilibrium, it follows that

$$\frac{r}{1+r}\lambda^u = (1-\tau_d)\left[f(n) - wn - \xi v\right]. \tag{4.9}$$

(3.7) and (4.8) imply that[11]

$$\frac{r}{1+r}\lambda_u = b(1-\tau_w) + m^u(q)(1-\tau_w)\left[\frac{w-b}{r+\sigma+m^u(q)}\right]. \tag{4.10}$$

Thus from (4.9) and (4.10), $w(1-\tau_w) > (1-\tau_d)[f(n) - wn - \xi v]$ requires that

$$w - b > \frac{m^u(q)}{r+\sigma+\mu+m^u(q)}[w-b] \tag{4.11}$$

which always holds.

The result that the expected wage earnings of an employed wage worker exceeds the expected earnings of a self-employed workers is an implication of our assumptions that the labor market is characterized by search frictions, the opportunity cost of being a self-employed is being an unemployed worker, and there are no start-up costs for businesses. In this environment, a self-employed worker is willing to accept lower earnings in order to avoid the spell of unemployment. In the case of significant start-up costs, the expected earnings of a self-employed worker may be higher or lower than the expected wage earnings depending on the size of the start-up cost (see also section 6.1).[12]

Our model (with low start-up cost) is consistent with one of the key puzzles in the empirical literature. The literature finds that on average self-employed workers earn less than wage and salaried employees (Aronson 1991,

[11] From (4.10), it follows that

$$\lambda^u = (1-\tau_w)\frac{1+r}{r}\left[b + \frac{m^u(q)}{r+\sigma+m^u(q)}(w-b)\right] > (1-\tau_w)\frac{b(1+r)}{r} > 0.$$

$(1-\tau_w)\frac{b(1+r)}{r}$ is the expected life-time earnings of an unemployed worker, who chooses to continue as unemployed and does not accept any match/job-offer. The inequality shows that an unemployed worker is better-off by accepting a job offer/match.

[12] In the presence of start-up cost, ζ, the job-creation curve remains unaffected, but the firm-size curve is given by $\pi(n) - \zeta = \lambda^u$. In this case, $\pi(n)$ may exceed λ^n for high enough start-up cost, ζ. The presence of start-up costs, though, do not change the effects taxes and labor market policies have on equilibrium variables.

Hamilton 2000). Hamilton (2000) using the 1984 panel of the Survey of Income and Program Participation (SIPP) estimates the median earnings differential to be 35 percent in the United States, which he argues may be the lower bound as he does not take into account employer financed health and other fringe benefits.

In the current literature, this earnings differential is attributed to either non-pecuniary benefits of self-employment such as "*being your own boss*" (Hamilton 2000) or greater opportunities to avoid or evade taxes in self-employment (Scheutze 2000). The first explanation is likely to be more applicable to relatively high income and wealthier self-employed, who can *afford* a substantial reduction in their incomes. Regarding the second, the evidence suggests that there is significant under-reporting of earnings by the self-employed. In the U.S., Kesselman (1989) using US Internal Revenue Service's Taxpayer Compliance Measurement Program (TCMP) data estimate that income under-reporting by the self-employed is roughly 20 percent of the reported income. Even if we assume that the extent of under-reporting is same in both data sets, the tax-avoidance explanation still leaves roughly half of the earnings differential unexplained.

Our model offers a novel explanation for the earnings differential between these two occupations; namely frictions in the labor market and low start-up cost for small businesses. The extent to which these two features account for the observed earnings differential is beyond the scope of this paper and left for future research. In the next section, we analyze the effects of public policies on equilibrium variables.

5 Effects of Public Policies

5.1 Taxes

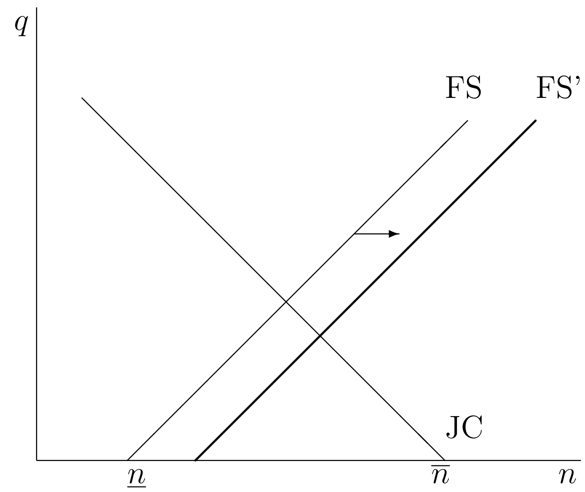
We first discuss the effects of business and wage taxes. These two taxes affect equilibrium variables through their effects on the firm-size curve (see equation 4.7). The effects of these two taxes are summarized in the following proposition. Proofs are in the appendix.

Proposition 2 A higher business tax, τ_d, and a lower wage tax, τ_w, increase average firm-size, n, and lower labor market tightness, q. They reduce the number of self-employed, E, and the rate of inflow to self-employment from unemployment, γ. In addition, they reduce total employment, $E + N$, and the wage, w, and increase unemployment, U.

The intuition for these results are as follows. A higher business tax and a lower wage tax reduce the relative return from self-employment. Thus the number of self-employed as well as inflow to self-employment from unemployment fall and the number of wage workers rises. A larger number of wage workers leads to higher unemployment and average firm-size. Higher unemployment leads to lower total employment. The effect of a higher business tax and a lower wage tax on wage employment is ambiguous, since the number of firms fall and average firm-size increases.

Turning to the effects on wages, a larger average firm-size reduces the marginal product of labor. In addition, lower labor market tightness reduces the expected savings on hiring costs. Both of these lead to lower wages. Another way to think about these results is that a higher business tax and a lower wage tax shift the firm-size curve downward to the right in (n, q) space. Since the job-creation curve is unaffected, the equilibrium requires a higher average firm-size and a lower labor market tightness. The effects of taxes are illustrated in figure 2 below.

Figure 2
Effects of a Higher Business Tax and a Lower Wage Tax



Next we analyze the effects of unemployment benefits and the minimum wage. To keep things simple, we set both the tax rates to be zero, $\tau_d,\ \tau_w = 0$ in what follows.

5.2 Unemployment Benefits

An unemployment benefit, b, affects both the job-creation and the firm-size curves. A higher b shifts the job-creation curve down to the left, while it shifts the firm-size curve down to the right. However, as shown in the appendix, it shifts the firm-size curve more than the job-creation curve. Therefore, labor market tightness falls and average firm-size rises. The intuition is that a higher unemployment benefit *directly* increases the value of unemployment. In addition, it reduces the return from self-employment *indirectly* by raising wages. Therefore, it affects the firm-size curve both directly and indirectly (see equation 4.7). On the other hand it affects the job-creation curve only indirectly through an increase in the wage (see equation 4.5); resulting in a smaller shift in the job-creation curve. This is illustrated in figure 3 below. The effects of a higher unemployment benefit, b, are summarized below.

Proposition 3 A higher unemployment benefit, b, reduces labor market tightness, q, and increases average firm-size, n. It reduces the number of self-employed workers, E, the rate of inflow to self-employment from unemployment, γ, and total employment, $E + N$, and increases unemployment, U.

The effect of unemployment benefits on wage employment is ambiguous as the number of self-employed falls and average firm-size rises. A higher unemployment benefit may raise or lower wages in equilibrium. It directly raises wages, but the fall in the marginal product of labor (due to larger average firm-size) and the expected saving on hiring costs (due to fall in labor market tightness) indirectly reduce wages.

The prediction of our model regarding the effect of the unemployment benefit on self-employment is supported by empirical studies. Using panel data on Spain, Carrasco (1999) estimates the impact of unemployment benefits on the transition from unemployment to self-employment. Consistent with our model, she finds that the receipt of unemployment benefits reduces the probability of a transition from unemployment to self-employment. Parker and Robson (2000) using aggregate data on OECD countries between 1972 and 1993 find a significant negative effect of the "replacement ratio" (the ratio of the average of all benefits to non-workers, including unemployment benefits, to average earnings) on aggregate self-employment. Next, we analyze the effects of a minimum wage.

5.3 Minimum Wage

In order to analyze the effects of the minimum wage, w^m, we assume that it is binding in the sense that it is higher than the equilibrium wage given in (3.12) for a given set of parameters. With the binding minimum wage, the job-creation condition is modified to

$$f_n(n) - w^m = \frac{\xi}{1-\mu}\frac{(r+\sigma+\mu-\sigma\mu)}{m^v(q)}. \quad (5.1)$$

The value of unemployment, λ^u, is given by[13]

$$\lambda^u = \frac{1+r}{r}\left[b + \frac{m^u(q)}{r+\sigma+\mu+m^u(q)}(w^m - b)\right]. \quad (5.2)$$

Equating λ^u to $\pi(n)$, we derive the firm-size curve which satisfies

$$f(n) - \frac{\xi(\sigma+\mu)n}{m^v(q)} = \left[n + \frac{m^u(q)}{r+\sigma+\mu+m^u(q)}\right]w^m + \frac{r+\sigma+\mu}{r+\sigma+\mu+m^u(q)}b. \quad (5.3)$$

It can easily be shown that for any $n > 0$ such that $f_n(n) > w^m > b$, the job-creation curve is downward sloping and the firm-size curve is upward sloping. Using arguments analogous to proposition 1, it can be established that there exists a unique stationary equilibrium.

An increase in the minimum wage affects both the job-creation curve and the firm-size curve. A higher minimum wage reduces the return from creating jobs and, thus, firms reduce the number of vacancies. Consequently, the job-creation curve shifts down to the left in (n, q) space. A higher minimum wage also reduces the profits of firms and raises the return of unemployment for a given average firm-size, n, and labor market tightness, q. Consequently, for a given average firm-size, n, labor market tightness, q, falls, which shifts the firm-size curve down to the right. The result is that equilibrium labor market tightness, q, unambiguously falls.

The effect of an increase in the minimum wage on equilibrium average firm-size, n, depends on the relative size of the shift of these two curves. Under the condition that at the initial equilibrium, $n + \frac{m^u(q)}{r+\sigma+\mu+m^u(q)} > 1$, the firm-size curve shifts more than the job-creation curve, and equilibrium average firm-size, n, rises. In this case, a higher minimum wage reduces the number of self-employed and total employment and increases unemployment. The above condition is automatically satisfied if the average number of employed workers per firm or firm-size, $n \geq 1$ at the initial equilibrium. The effects of changes in the minimum wage are summarized below.

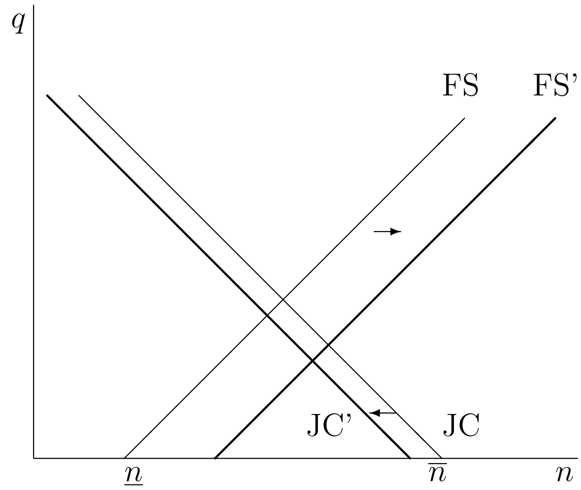
Proposition 4 If at the initial equilibrium, $n + \frac{m^u(q)}{r+\sigma+\mu+m^u(q)} > 1$, a higher minimum wage, w^m, reduces labor market tightness, q, and increases average

[13] The value of an employed worker is given by

$$\lambda^n = w^m + \frac{1}{1+r}[(1-\sigma-\mu)\lambda^n + (\sigma+\mu)\lambda^u].$$

firm-size, n. It reduces the number of self-employed workers, E, the rate of inflow to self-employment from unemployment, γ, and total employment, $E + N$, and increases unemployment, U.

Figure 3
Effects of a Higher Unemployment Benefit and Minimum Wage



A higher minimum wage may increase or reduce wage employment, N, as labor market tightness, q, falls but average firm-size, n, rises. The disemployment effect of minimum wage has been subject of controversy in the empirical labor literature. This analysis suggests that one has to be careful about interpreting these results as a higher minimum wage may have different effects on total employment and wage employment. In addition, the surviving firms may hire more wage workers.

If $n + \frac{m^u(q)}{r+\sigma+\mu+m^u(q)} < 1$, a higher minimum wage reduces equilibrium average firm-size, n. In this case, a higher minimum wage may increase or reduce the number of self-employed and total employment. It also has an ambiguous effect on unemployment.

In the next section, we show that the negative relationship between the minimum wage and the level of self-employment, suggested by our model, contrasts with the result obtained in a competitive labor market model. In part, the mechanism by which the level of self-employment falls in our model when the minimum wage rises is the resulting reduction in the flow of workers from unemployment to self-employment.

Previous empirical studies find support for the negative association between the minimum wage and the unemployment rate (Bruce and Mohsin 2006, Garrett and Wall 2006).[14] We test the results further through an empirical examination to determine whether increases in the minimum wage are

[14]In the empirical literature, the self-employment rate is usually defined as the ratio of

associated with a reduction in individual transitions from unemployment to self-employment in the United States.

5.4 Effects of Taxes and Minimum Wage in a Competitive Labor Market

In this section, we analyze the effects of taxes and the minimum wage in a competitive labor market. This exercise is included to allow for a comparison between our model and that which is common in the literature. In the competitive labor market, a firm hires employees up to the level at which the marginal product of labor equals the wages.

$$f_n(n) = w. \tag{5.4}$$

As an individual can either become a self-employed worker or an employee, in equilibrium the incomes of the self-employed and employees should be the same, which implies

$$w(1-\tau_w) = (1-\tau_d)\left[f(n) - wn\right]. \tag{5.5}$$

(5.5) highlights one of the key differences between our model and the competitive model. In our model, due to search frictions, an individual can either be self-employed or an *unemployed* worker. Since a wage worker cannot become an employee without search, this implies that the average earnings of employed wage workers will be greater than that of self-employed.

Combining (5.4) and (5.5), we have one equation in one unknown, n,

$$\frac{1-\tau_w}{1-\tau_d} f_n(n) = f(n) - nf_n(n). \tag{5.6}$$

Under the assumption that $\lim_{n\to 0} \frac{1-\tau_w}{1-\tau_d} f_n(n) > f(0)$ as shown in the appendix, there exists a unique n which solves (5.6). Finally the labor market clears, $E + N = 1$. Since $nE = N$, we have

$$E = \frac{1}{1+n} \tag{5.7}$$

and

self-employed workers to total employment $\frac{E}{E+N}$. Using (4.1) and (4.2) we have $\frac{E}{E+N} = \frac{1}{1+n}$. The qualitative effects of taxes and labor market policies are the same whether we consider E or the self-employment rate, $\frac{E}{E+N}$. A lower wage tax and higher business tax, unemployment benefits, and minimum wage reduce E as well as $\frac{E}{E+N}$.

$$N = \frac{n}{1+n}. \tag{5.8}$$

From (5.7) and (5.8), it is clear that higher average firm-size, n, reduces the number of firms and increases wage employment.

Given that the labor market clears, one immediate implication is that public policies affect only the *composition* of total employment and not its *size*. In our model, public policies affect both the *composition* as well as the *size* of total employment.

As shown in the appendix, in a competitive market a higher business tax increases average firm-size and wage employment and reduces the number of self-employed and wages. A higher wage tax, on the other hand, has the opposite effects. Thus, the predictions of our model regarding average firm-size, number of self-employed, and wages are the same as that of the competitive model. But the predictions regarding wage employment differ as our model suggests higher taxes may increase or reduce wage employment.

The predictions of our model differ from that of the competitive model with respect to the minimum wage. With a binding minimum wage, average firm-size in the competitive market is completely determined by

$$f_n(n) = w^m. \tag{5.9}$$

A higher minimum wage reduces average firm-size since $f_{nn}(n) < 0$. Then from (5.7) and (5.8) it follows that it increases the number of self-employed and reduces wage employment. However, our model predicts that under reasonable conditions, a higher minimum wage increases average firm-size and unemployment, reduces the number of self-employed and total employment, and may raise or lower wage employment.

6 Extensions

In this section, we extend our previous model in two different directions. In the first extension, we assume that hiring and managing workers require a minimum level of managerial ability and sufficient knowledge of employment rules and regulations. These abilities/knowledge are acquired over time while working on own-account. In the second extension, we develop a model with dual labor markets: one with high wages and another with low wages.

6.1 Own Account Workers and Employers

Empirical evidence suggests that most self-employed workers are own account workers. For instance, in the U.S., only 20 percent of self-employed workers (unincorporated businesses) are employers (Hipple 2004). As mentioned earlier in the model, the distinction between own account workers and employers arise solely due to frictions in the labor market. Given that the matching rate of vacancy is fairly high (average duration of vacancy is reported to be less than a month days in the U.S. (Blanchard and Diamond 1989)), labor market frictions alone are unlikely to explain such a high proportion of own account workers. There are other factors such as managerial ability, learning employment rules and regulations etc. which play an important role in the decision to become an employer. One can introduce these realistic aspects in the current model as shown below.

Assume that a self-employed worker or firm hires wage workers only when she has sufficient managerial ability and/or knowledge about rules and regulations. One simple way to introduce these factors is to assume that individuals acquire managerial ability and knowledge about rules and regulations while working as own account self-employed workers. Assume that there is some randomness in acquiring these abilities and knowledge.

Suppose that a self-employed worker without ability/knowledge to be an employer produces y units of good per unit of time. Further suppose that at any point in time with probability α a self-employed worker without ability/knowledge to be an employer acquires managerial ability and/or knowledge about rules and regulations.[15] Assume that businesses run by the self-employed workers without ability/knowledge to be an employer do not fail. This assumption makes the derivations a bit simpler. The rest of the environment remains as in the baseline model. We focus on an equilibrium in which the pay-off as an employer (once sufficient ability and knowledge are acquired) is higher than the pay-off as an own account worker, *i.e.*, $\pi(n) > \frac{1+r}{r}(1-\tau_d)y$.

Let E_t^N and E_t^O denote the number of self-employed workers with and without the ability/knowledge to become employers respectively at time t. At any point in time, the total number of self-employed workers, $E_t = E_t^O + E_t^N$. The laws of motion for self-employed workers with and without the

[15] It is possible to give other interpretations to the model. For example, one can assume that a self-employed worker starts with some business ideas. Her ideas become successful with probability α, in which case she hires wage workers. The current model reduces to the previous model in which we assumed that every agent has the ability/knowledge to become an employer if we set $\alpha = \infty$. Also α can be endogenized by making it a function of effort/resources put in by own account workers.

ability/knowledge to become employers are given by

$$E_{t+1}^{O}=\gamma_t U_t+(1-\alpha)E_t^{O} \tag{6.1}$$

and

$$E_{t+1}^{N}=\alpha E_t^{O}+(1-\mu)E_t^{N}. \tag{6.2}$$

The interpretation of (6.1) and (6.2) is straight-forward. The left hand side of (6.1) is the total number of self-employed workers without the ability/knowledge to become employers at the beginning of time $t+1$. The first term on the right hand side is the number of unemployed workers who become self-employed. The second term is the total number of self-employed workers who do not acquire the ability/knowledge to become employers at the end of period t.

The left hand side of (6.2) is the total number of self-employed workers with the ability/knowledge to hire wage workers at the beginning of time $t+1$. The first term on the right hand side is the number of self-employed workers who acquire the ability/knowledge to become employers. The second term is the total number of self-employed workers with the ability/knowledge to become employers at the beginning of time t who remain in the same pool at the end of time t.

The law of motion for unemployed workers gets slightly modified as only businesses run by self-employed workers with the ability/knowledge to become an employer fail

$$U_{t+1}=(\sigma+\mu)N_t+\mu E_t^{N}+(1-m^u(q_t)-\gamma_t)U_t. \tag{6.3}$$

The law of motion for employed wage workers continues to be given by (2.6). Using (2.1), (2.6), and (6.1-6.3), and the condition that $N=nE^N$, one can derive steady-state numbers of $E^O, E^N, N, U \& \gamma$ which are given by

$$E^O=\frac{\mu}{\alpha}E^N, \tag{6.4}$$

$$E^N=\frac{m^u(q)}{\left(\frac{\mu}{\alpha}+1\right)m^u(q)+(\sigma+\mu+m^u(q))n}, \tag{6.5}$$

$$N=nE^N, \tag{6.6}$$

$$U=\frac{(\sigma+\mu)n}{\left(\frac{\mu}{\alpha}+1\right)m^u(q)+(\sigma+\mu+m^u(q))n}\ \& \tag{6.7}$$

$$\gamma = \frac{\mu m^u(q)}{(\sigma + \mu)n}. \tag{6.8}$$

The expression for γ is same as before. It is easy to show that $E_q^O > 0,\ E_n^O < 0,\ E_q^N > 0,\ E_n^N < 0,\ N_q > 0,\ N_n > 0,\ U_q < 0,\ U_n > 0,\ \gamma_q > 0,\ \&\ \gamma_n < 0$. The intuitions for these effects are the same as before.

In the steady state, the value function of a self-employed worker without the ability/knowledge to become an employer, $\hat{\pi}$, can be written as

$$\hat{\pi} = (1 - \tau_d)y + \frac{1}{1+r}[\alpha\pi(n) + (1-\alpha)\hat{\pi}] \tag{6.9}$$

where $\pi(n)$ continues to be given by (3.6). The interpretation of (6.9) is straightforward. In the current period, a self-employed worker without the ability/knowledge to become employer produces y units of goods. In the next period, she can become an employer with probability, α, the value of which is, $\alpha\pi(n)$, and with probability, $1-\alpha$, she remains without the ability/knowledge to become an employer, the value of which is $(1-\alpha)\hat{\pi}$.

The job-creation curve continues to be given by (4.5). The firm-size curve is slightly modified and is given by the condition that

$$\hat{\pi} \equiv (1 - \tau_d)\frac{1+r}{r+\alpha}y + \frac{\alpha}{r+\alpha}\pi(n) = \lambda^u \tag{6.10}$$

where the value function of an unemployed worker, λ^u, continues to satisfy (3.7).

It is easy to show that the introduction of learning of managerial ability or knowledge about rules and regulations does not change the qualitative effects of taxes and labor market policies. The implications regarding earnings differential are modified. The self-employed workers without the knowledge/ability to become employers earn less than wage employed workers on average. But the average earnings of self-employed workers with the knowledge/ability to become employers may be higher or lower than the average earnings of wage employed workers. This can be shown as follows. Using (3.6) and (6.9), we can rewrite (6.10) as

$$\frac{r}{1+r}\lambda^u = (1-\tau_d)\frac{r+\mu}{r+\mu+\alpha}y + (1-\tau_d)\frac{\alpha}{r+\mu+\alpha}[f(n) - wn - \xi v]. \tag{6.11}$$

(4.10) gives another expression for $\frac{r}{1+r}\lambda^u$. Since $(1-\tau_w)w > \frac{r}{1+r}\lambda^u$ and $f(n) - wn - \xi v > y$, (4.10) and (6.11) imply that $(1-\tau_w)w > (1-\tau_d)y$. However, the net average earnings of an employer, $(1-\tau_d)[f(n) - wn - \xi v]$, may be higher or lower than the net average earnings of a wage employee,

$(1-\tau_w)w$. Empirical evidence suggests that the average earnings of employers is higher than the average earnings of non-employers. Alba-Ramirez (1994) finds that own-account workers earn 22 percent less, whereas self-employed with over five employees earn 26 percent more than wage and salaried workers in Spain.

6.2 Two Sector Model

In this section, we extend the model to incorporate a dual labor market – one with high wages and the other with low wages. As discussed in the introduction, this is done for variety of reasons. First, labor market policies such as the minimum wage and unemployment benefits are more likely to affect (directly) the low wage sector. Second, it allows us to incorporate transitions from self-employment to wage employment, which is quantitatively very significant. Our calculation shows that on average 18 percent of self-employed move to wage employment from one year to the next in the United States. Third, this extension allows us to study the interaction between the high and the low wage sectors. Finally, it also implies that wage workers need not just be employed by small businesses and firms. Empirical evidence suggests that most of the wage workers are employed by large (incorporated) businesses and firms.

After developing the model, we once again analyze the effects of taxes and labor market policies. The focus is on how changes in taxes and labor market policies for the low wage sector or small businesses affect equilibrium variables. We also analyze the effects of changes in wages in the high wage sector.

We assume that there are a *fixed* number of firms (or self-employed) who operate in the high wage sector. One can assume that starting and operating firms in the high wage sector requires higher ability or a significant amount of capital, and/or access to costly technology, which is possessed only by a fraction of the population.[16] Suppose that the nature of jobs in this sector is such that they can only be performed by agents who are working in the low wage sector either as employed wage workers or self-employed and not by unemployed workers. Such restriction can arise if jobs in the high wage sector require recent work experience or if a stigma is attached to unemployment. We also assume that the wages in the high wage sector are fixed. The endogenization of wages as well as the number of firms or self-employed in the high wage sector is left for future research. As we will see below, despite

[16] With these restrictions, the earnings of self-employed in the high wage sector are likely to exceed those of wage employed workers.

fixed wages and the number of firms in the high wage sector, employment in this sector is endogenous and responds to policy variables.

Suppose that both self-employed workers and employed wage workers in the low wage sector receive a job-opportunity to work in the high wage sector with probability, δ, in any time period t. Assume that high wage jobs are destroyed with probability, ρ, in any time period. We continue to model the low wage sector as in the baseline model, with one exception. To keep expressions simple, we assume that there is no possibility of business failure in the sense that the self-employed workers in the low wage sector do not become unemployed. However, an employed wage worker in the low wage sector can become unemployed either due to job-separation shock or because the employer receives a high wage offer, *i.e.*, with probability, $\sigma + \delta$.

Let w^h and w^l denote wages in the high wage and the low wage sectors respectively. Wages in the low wage sector, w^l, continue to be determined by Nash bargaining. We will assume that the parameters of the model are such that $w^h > w^l$.

Denote the number of wage workers employed (or filled jobs) in the high wage sector at time t by H_t. E_t and N_t denote the number of self-employed and employed wage workers respectively in the low wage sector at time t. U_t denotes the number of unemployed workers and γ_t the rate of inflow to self-employment from unemployment in the low wage sector at time t. q_t and n_t denote labor market tightness and average firm-size, respectively, at time t in the low wage sector.

The laws of motion for employed wage workers in the high wage sector, employed wage workers and self-employed in the low wage sector, and unemployed workers are as follows:

$$H_{t+1} = (1 - \rho)H_t + \delta(E_t + N_t), \tag{6.12}$$

$$E_{t+1} = (1 - \delta)E_t + \gamma_t U_t, \tag{6.13}$$

$$N_{t+1} = m^u(q_t)U_t + (1 - \sigma - 2\delta)N_t, \tag{6.14}$$

and

$$U_{t+1} = (1 - m^u(q_t) - \gamma_t)U_t + (\sigma + \delta)N_t + \rho H_t. \tag{6.15}$$

We provide interpretations of (6.12), (6.14), and (6.15). (6.13) can be interpreted as before. The term on the left hand side of (6.12) is the number of employed wage workers in the high wage sector at the beginning of period $t + 1$. The first term on the right hand side is the number of employed wage workers at the beginning of time t who continue to be employed in the high wage sector at the end of time t. The second term is the number of new employed wage workers in the high wage sector.

The term on the left hand side of (6.14) is the number of employed wage workers in the low wage sector at the beginning of period $t+1$. The first term on the right hand side is the number of new employed wage workers in the low wage sector. The second term is the number of employed wage workers at the beginning of time t who continue to be employed in the low wage sector at the end of time t. Note that an employed wage worker in the low wage sector leaves this sector either due to a job-separation shock or if the employer or employee receives a high wage offer.

The first term on the right hand side of (6.15) is the number of unemployed workers at the beginning of period t, who remain unemployed at the end of period t. The second and third terms together give the number of wage employed workers who become unemployed at time t. Finally at any point in time, the economy satisfies the following identity:

$$E_t + H_t + N_t + U_t = 1. \tag{6.16}$$

Combining (6.12) to (6.16), one can show that in the steady state

$$E = \frac{m^u(q)}{(\sigma+2\delta)n + \frac{\rho+\delta}{\rho} m^u(q)(1+n)}, \tag{6.17}$$

$$N = \frac{m^u(q)n}{(\sigma+2\delta)n + \frac{\rho+\delta}{\rho} m^u(q)(1+n)}, \tag{6.18}$$

$$U = \frac{1}{1 + \frac{\rho+\delta}{\rho}\frac{m^u(q)}{\sigma+2\delta}\frac{1+n}{n}}, \tag{6.19}$$

$$H = \frac{\delta m^u(q)(1+n)}{\rho(\sigma+2\delta)n + (\rho+\delta)m^u(q)(1+n)}, \tag{6.20}$$

and

$$\gamma = \frac{\delta}{\sigma+2\delta}\frac{m^u(q)}{n}. \tag{6.21}$$

Total employment in the low wage sector, $E+N$, is given by

$$E + N = \frac{m^u(q)(1+n)}{(\sigma+2\delta)n + \frac{\rho+\delta}{\rho} m^u(q)(1+n)}, \tag{6.22}$$

and the ratio of wage employed workers in the low and high wage sectors, $\frac{N}{H}$, is given by

$$\frac{N}{H} = \frac{\rho}{\delta} \frac{n}{1+n}. \tag{6.23}$$

The effects of changes in labor market tightness, q, and average firm-size, n, on the number of self-employed, E, unemployed workers, U, employees in the low wage sector, N, and the rate of inflow to self-employment from unemployment, γ, are the same as discussed earlier ($E_q > 0,\ E_n < 0,\ U_q < 0,\ U_n > 0,\ N_q > 0,\ N_n > 0,\ \gamma_q > 0$ & $\gamma_n < 0$). The intuition for these effects are also the same as before.

(6.20) is new and shows that the number of employed workers in the high wage sector, H, is increasing in labor market tightness, q, and decreasing in average firm-size, n, *i.e.*, $H_q > 0,\ H_n < 0$. Similarly, (6.22) shows that total employment in the low wage sector, $E + N$, is increasing in labor market tightness, q, and decreasing in average size of firms in the low wage sector, n. Finally, (6.23) shows that the ratio of wage employed workers in the low and high wage sectors, $\frac{N}{H}$, is increasing in average firm-size in the low wage sector, n.

The intuition for these effects is quite simple. Total employment in the low wage sector, $E+N$, is increasing in labor market tightness, q, and decreasing in average firm-size, n, for reasons discussed earlier. As for the number of employed wage workers in the high wage sector, their number depends on total employment in the low wage sector, $E + N$. Thus, it is increasing in q and decreasing in n. Finally, a higher average firm-size in the low wage sector, n, reduces H and increases N, and thus the ratio of wage employed workers in the low and high wage sectors, $\frac{N}{H}$, is increasing in average firm-size in the low wage sector, n.

Denote the value function of an employed high wage worker at time t as λ_t^h. Anticipating an equilibrium in which $\lambda_t^h > \pi_t(n_t)$, the value function of a self-employed worker in the low wage sector, $\pi_t(n_t)$, is given by

$$\pi_t(n_t) = \max_{v_t, n_{t+1}} (1-\tau_d)[f(n_t) - w_t^l n_t - \xi v_t] + \frac{1}{1+r}\mathcal{E}_t[(1-\delta)\pi_{t+1}(n_{t+1}) + \delta\lambda_{t+1}^h] \tag{6.24}$$

subject to

$$n_{t+1} \leq (1-\sigma-\delta)n_t + m^v(q_t)v_t, \text{ if the business continues.} \tag{6.25}$$

The first expression on the right hand side of (6.24) is the net flow of profit at time t. The second expression is the expected continuation value, which takes into account the possibility that a self-employed worker in the low wage sector receives an opportunity to work in the high wage sector with probability δ. The interpretation of (6.25) is as before. The first term on the right hand side is the number of employees at time t who do no leave the firm at the end of time t. The employee leaves the firm either due to a job-separation shock or if the employer receives a high wage offer. The second term is the expected number of new employees.

In the steady state, the level of vacancies, v, in the low wage sector satisfy:

$$\xi = (1-\delta)m^v(q)\left[\frac{f_n(n) - w^l}{r+\sigma+2\delta-\delta(\sigma+\delta)}\right]. \tag{6.26}$$

(6.26) can be interpreted as before.

In the steady state the value functions of employed wage workers in the high and low wage sectors and unemployed workers satisfy the following functional equations:

$$\lambda^h = (1-\tau_w)w^h + \frac{1}{1+r}[(1-\rho)\lambda^h + \rho\lambda^u] \tag{6.27}$$

$$\lambda^n = (1-\tau_w)w^l + \frac{1}{1+r}[(1-\sigma-2\delta)\lambda^n + \delta\lambda^h + (\sigma+\delta)\lambda^u] \tag{6.28}$$

and

$$\lambda^u = (1-\tau_w)b + \frac{1}{1+r}[m^u(q)\lambda^n + (1-m^u(q))\lambda^u]. \tag{6.29}$$

The first term on the right hand side of (6.27) is the net wage received by an employed wage worker in the high wage sector. The second expression is the discounted continuation value, which takes into account that an employed wage worker in the high wage sector can become unemployed with probability, ρ. Similarly, the first term on the right hand side of (6.28) is the net wage received by an employed wage worker in the low wage sector. The second expression is the discounted continuation value, which takes into account that an employed wage worker can become unemployed with probability, $\sigma+\delta$, and receive a high wage offer with probability, δ. (6.29) can be interpreted similarly.

As shown in the appendix under the condition that $\delta\rho \approx 0$, Nash bargaining between a matched employer and a employee in the low wage sector implies that the wages in the low wage sector, w^l, are given by

$$w^l = \frac{1}{1+B}\left[Bf_n(n) - C + \frac{r+\delta}{r}\left\{b + \frac{1-\beta}{\beta}\frac{\xi}{1-\delta}q\right\}\right] \tag{6.30}$$

where B and C are two positive constants given by

$$B \equiv \frac{1-\beta}{\beta} \frac{r+\sigma+2\delta}{r+\sigma+2\delta-\delta(\sigma+\delta)} \ \& \ C \equiv \frac{\delta}{r+\rho} w^h.$$

In the rest of the paper, we will assume that $\delta\rho \approx 0$. (6.30) reduces to the standard form when $\delta = 0$. In this case $B = \frac{1-\beta}{\beta}$ and $C = 0$. This wage function is decreasing in average firm-size, n, and increasing in labor market tightness, q, and the unemployment benefit, b, as before. One additional property of this wage function is that it is decreasing in the wages of the high wage sector, w^h (or C). This we call the *foot-in-the-door effect.* In the model, one can get a job in the high wage sector only by working in the low wage sector. Thus, when w^h goes up workers employed in the low wage sector are willing to accept lower wages.

(6.26) and (6.30) together give the *job-creation curve*, which satisfies

$$f_n(n) + C - \frac{r+\delta}{r} b - \frac{r+\delta}{r} \frac{1-\beta}{\beta} \frac{\xi}{1-\delta} q = \frac{\xi(r+\sigma+2\delta-\delta(\sigma+\delta))}{1-\delta} \frac{1+B}{m^v(q)}. \tag{6.31}$$

As shown in the appendix, for any $n \in (0, n^{**})$ where n^{**} solves $f_n(n) + C = \frac{r+\delta}{r} b$, (6.31) traces a downward sloping job-creation curve in (n, q) space as before. The firm-size curve satisfies

$$\pi(n) \equiv \frac{(1-\tau_d)(1+r)}{r+\delta} \left[f(n) - w^l n - \frac{\xi(\sigma+2\delta)n}{m^v(q)} \right] + \frac{\delta}{r+\delta} \lambda^h = \lambda^u. \tag{6.32}$$

Since $\lambda^h > \lambda^u$ and the flow of profit is strictly positive, for a well-defined firm-size curve to exist we require that $\lambda^u > \frac{\delta}{r+\delta} \lambda^h$. Intuitively, if the probability of receiving a high wage offer through self-employment, δ, is too high, then no individual would choose to remain unemployed in order to search for wage jobs. Our calculation using PSID data shows that the transition rate from unemployment to wage employment (55 percent) is three times higher than the transition rate from self-employment to wage employment (18 percent) from one year to the next. We assume that the parameter values of the model are such that the above condition is satisfied.

(6.32) can be written as (see Appendix)

$$\frac{1-\tau_d}{1-\tau_w} \left[f(n) - w^l n - \frac{\xi(\sigma+2\delta)n}{m^v(q)} \right] + C = \frac{r+\delta}{r} \left[b + \frac{1-\beta}{\beta} \frac{\xi}{1-\delta} q \right]. \tag{6.33}$$

In the appendix, we show that (6.33) traces an upward sloping firm-size curve in (n, q) space, as before, for any $n \in (n^*, n^{**})$ where n^* satisfies

$$\frac{1-\tau_d}{1-\tau_w}\left[f(n^*) - \frac{n^*}{1+B}[Bf_n(n^*) - C + \frac{r+\delta}{r}b]\right] + C = \frac{r+\delta}{r+\rho}b. \qquad (6.34)$$

At n^* the value function of a self-employed worker is equal to the value function of an unemployed worker when labor market tightness, $q = 0$, *i.e.*, $\lim_{q\to 0}(\pi(n^*) - \lambda^u) = 0$. For any $n \in (n^*, n^{**})$, $\lim_{q\to 0}(\pi(n) - \lambda^u) > 0$. The intersection of the job-creation curve and the firm-size curve gives the equilibrium (n, q). Hence, we have the following proposition.

Proposition 5: Under conditions that $\delta\rho \approx 0$ and $f(n^{**}) - n^{**}f_n(n^{**}) > \frac{1-\tau_w}{1-\tau_d}\left[\frac{r+\delta}{r+\rho}b - C\right]$, there exists a unique equilibrium characterized by equations (6.17-6.21), (6.26-6.30), (6.31), and (6.33).

Next, we analyze the effects of taxes on equilibrium variables. As before changes in taxes affect equilibrium variables through the firm-size curve. A higher business tax and a lower wage tax reduce equilibrium labor market tightness, q, and increase average firm-size, n, by reducing the relative return to self-employed. Thus changes in taxes have the following effects:

Proposition 6: A higher business tax, τ_d, and a lower wage tax, τ_w, reduce the number of self-employed, E, and the rate of inflow to self-employment from unemployment, γ. In addition, they reduce total employment in the low wage sector, $E + N$, wage employment in the high wage sector, H, and wages in the low wage sector, w^l. They also increase unemployment, U, and the ratio of employed wage workers in the low wage sector to employed wage workers in the high wage sector, $\frac{N}{H}$.

Wage employment in the low wage sector, N, can rise or fall, since N is increasing in both labor market tightness, q, and average firm-size, n. Next, we analyze the effects of unemployment benefits, b, and the minimum wage, w^m. For analyzing the effects of the minimum wage, w^m, we assume that it is binding only for the low wage sector, *i.e.*, $w^h > w^m > w^l$. The job-creation and firm-size curves in the case of a binding minimum wage are derived in the appendix.

Unemployment benefits and the minimum wage affect both the job-creation and firm-size curves. As before the firm-size curve is more responsive to the unemployment benefit than the job-creation curve. Therefore, a higher unemployment benefit reduces equilibrium labor market tightness, q, and increases average firm-size, n. The sufficient condition for the higher minimum wage to have the same effects as a higher unemployment benefit is that average firm-size, $n \geq 1$, at the initial equilibrium. The effects of unemployment benefit and the minimum wage are summarized in the following proposition:

Proposition 7:

a. A higher unemployment benefit, b, reduces the number of self-employed, E, and the rate of inflow to self-employment from unemployment, γ. In addition, it reduces total employment in the low wage sector, $E + N$, and wage employment in the high wage sector, H. It also increases unemployment, U, and the ratio of employed wage workers in the low wage sector to the employed wage workers in the high wage sector, $\frac{N}{H}$.

b. If at the initial equilibrium average firm-size in the low wage sector, n, is such that

$$n + \frac{r + \delta}{r(r + \sigma + 2\delta) + m^u(q)(r + \delta)} m^u(q) > 1 \tag{6.35}$$

then a higher minimum wage, w_m, has same effects as an increase in the unemployment benefit.

The effect of the unemployment benefit on wages in the low wage sector, w^l, is ambiguous. This happens because higher unemployment benefits increases w^l. However, a lower marginal product of labor and labor market tightness reduce w^l. Higher unemployment benefit and minimum wage may raise or lower wage employment in the low wage sector, N, as they reduce labor market tightness, q, and increase average firm-size, n.

To further shed light on the interactions between the high and the low wage sectors, next we consider the effects of an increase in the wage in the high wage sector, w^h. As discussed earlier, a higher w^h reduces wages in the low wage sector, w^l, through the foot-in-the-door effect for a given labor market tightness, q, and average firm-size, n. The decline in w^l increases the marginal return from creating a vacancy and the earnings of self-employed in the low wage sector. This shifts both curves up to the left in (n, q) space. However, as shown in the appendix, the firm-size curve shifts up more than the job-creation curve resulting in higher equilibrium labor market tightness, q, and lower average firm-size, n. The effects of an increase in the wage in the high wage sector, w^h, is summarized below:

Proposition 8: A rise in the wage in the high wage sector, w^h, increases the number of self-employed, E, and the rate of inflow to self-employment from unemployment, γ. In addition, it increases total employment in the low wage sector, $E+N$, and wage employment in the high wage sector, H. It also reduces unemployment, U, and the ratio of employed workers in the low wage sector to employed wage workers in the high wage sector, $\frac{N}{H}$.

An increase in w^h may raise or lower the wage in the low wage sector, w^l. This happens because higher q and lower n raise w^l but a higher w^h reduces w^l.

7 Minimum Wage and Self-Employment: Empirical Evidence

In this section, we test the predictions of our model that a higher (real) minimum wage reduces the self-employment rate and the transition probability of unemployed workers to self-employment. We focus on the effects of the minimum wage since it allows us to discriminate between our model and the competitive model. Recall that the competitive model predicts a positive association between (real) minimum wage rate and the self-employment rate. In addition, a test of the effect of the (real) minimum wage on the transition probabilities sheds new light on the mechanism through which the (real) minimum wage influences self-employment outcomes. As shown below empirical evidence supports both the predictions of our model.

As mentioned earlier, there are other empirical studies which find a negative association between the minimum wage and the self-employment rate (Bruce and Mohsin 2006, Garrett and Wall 2006) similar to us. The examination of the relationship between the minimum wage and the transition probability of unemployed workers to self-employment is new to the literature. Thus we focus on this aspect of the effects of the minimum wage. We examine the effect of the (real) minimum wage on individual transitions between unemployment and self-employment using micro panel data covering the period 1977-96. An individual panel approach allows us to address individual-level decisions to enter self-employment.

We start with a description of the transition analysis. Similar to a number of previous self-employment transition analyses we estimate transition equations of the following type by random effects probit:

$$S_{i,t+1} = G'X_{i,t} + Dw^m_{s,t} + \kappa_i + \chi_{i,t+1} \tag{7.1}$$

where S is an indicator variable; equal to one if individual i transitions from unemployment in year t to self-employment in year $t+1$ and zero if the individual remains unemployed in both years.

X is a vector of individual and household characteristics from year t and includes a constant term. G' is the associated vector of coefficients. w^m is the state level (as indicated by the subscript s) legislated minimum wage expressed in real terms in year t. D is the associated coefficient, which is our main object of interest. The error term is comprised of two components. κ_i is a time-invariant, individual specific random disturbance (capturing factors such as unobserved managerial ability) and $\chi_{i,t+1}$ is *i.i.d.* with mean zero and constant variance.

Because of the need for information on transitions, the data for this study are primarily drawn from 1977-1996 waves of the Panel Study of Income Dynamics (PSID). The PSID began in 1968 with a representative random sample of 4,800 American households, who were followed through time and asked to respond to similar surveys annually up until 1997. After 1997 the PSID began surveying individuals every two years; changing the nature of any labor market transitions observed in the data post 1996. These years have been omitted for this reason. New respondents have been brought into the sample over time as members of the original households have formed new households of their own. As of 1997, the PSID included data on over 60,000 individuals.

We restrict our attention to male heads of households between the ages of 25 and 54 who did not live outside the U.S. at the time of the survey. We confine our attention to heads of households, because the PSID provides self-employment status and other key variables for household heads and their spouses only. The age and gender restrictions are imposed to avoid the confounding effects of schooling/retirement and changes in labor force participation, respectively.

For the purposes of the empirical analysis, workers are considered to be self-employed on the basis of responses to the question of whom they primarily work for: someone else, themselves, or both. The latter two categories are included in our definition of self-employment, but less than one percent of each year's workers report working for both themselves and someone else. Workers are considered to be unemployed if they are not working at the time of the survey but looking for work.

In order to estimate the random effects probit model specified in equation 7.1, the sample must be further restricted to male heads of households who were unemployed in year one and either transition to self-employment or remain unemployed in year two. Because of this restriction, sample sizes in any given year become small enough to warrant pooling the data. Therefore, we pool the 1977-1996 waves of data but remove individuals with previous self-employment experience back as far as 1968 (the first year of the PSID). This yields a sample of 1129 person-years of data for the transition analysis.

The PSID data are supplemented with data on the state level statutory (nominal) minimum wage. Many states in the U.S. do not have a minimum wage. In these states, the federal minimum wage applies. Many states impose a minimum wage higher than the federal level. Data suggests substantial variation in the nominal minimum wage across states and time. We convert nominal minimum wages to real values using the state level GDP deflator (base year = 2000), which is our primary object of interest. The GDP deflator is only available back to 1977; thus, we were unable to include years prior to 1977 in the analysis.

We follow previous studies in choosing other control variables. Individual characteristics include a series of education indicators (Less Than High School, High School Graduate, Some College, College Graduate, and some Post-Graduate, where Less Than High School serves as the reference category), an indicator for Black race, Age, and a quadratic specification of Age. Household characteristics consist of an indicator for whether the worker is Married (with spouse present) and the Number of Children under age 18 living in the household. Also included are indicator variables for region of residence (North-Central, North-East, South, and West, where North-East is the reference category).

In some specifications we also include a measure of personal tax rate as one of the control variables. Since, it is paid by both self-employed and wage employed workers, it contains elements of both wage and business taxes. Thus according to our model, a higher personal tax may have positive or negative effect on self-employment (rate as well as transition). Separate estimation of the wage tax and the business tax is a formidable data exercise, and thus the empirical examination of their effects is left for future research. For the similar reason we do not study the effects of unemployment benefits.

The measure of tax we use is the average of the sum of state and federal marginal income tax rates on earnings; where the earnings used to calculate the tax rates are the same nationally representative sample from 1995 (properly deflated) for each state and year.[17] Holding earnings constant in real terms over the period yields a measure of tax rates that captures changes in tax law rather than a combination of changes in earnings and deductions with changes in tax law (*i.e.*, is exogenous). Table 1 below provides summary statistics for the regression sample.

[17] These data were obtained through the NBER's Taxsim program. For more details see http://www.nber.org/∼taxsim/state-marginal/.

Table 1
Summary Statistics: Mean (Standard Deviation)

Variable	All	Males Remaining in Unemployment	Males Transiting to Self-Employment
Real Minimum Wage	5.08 (0.60)	5.11 (0.62)	4.94 (0.44)
Tax Rate	29.70 (3.53)	29.87 (3.51)	28.77 (3.50)
Education:			
< High School	0.42 (0.49)	0.44 (0.50)	0.31 (0.46)
High School	0.39 (0.49)	0.38 (0.49)	0.43 (0.50)
Some College	0.12 (0.32)	0.12 (0.32)	0.12 (0.33)
College Graduate	0.04 (0.19)	0.03 (0.18)	0.07 (0.25)
Some Post-Graduate	0.02 (0.12)	0.01 (0.11)	0.03 (0.18)
Other Characteristics:			
Age	34.94 (8.95)	34.80 (9.00)	35.67 (8.68)
Married	0.52 (0.50)	0.51 (0.50)	0.57 (0.50)
No. of Children (<18)	1.18 (1.50)	1.19 (1.55)	1.14 (1.27)
Black	0.63 (0.48)	0.66 (0.47)	0.42 (0.50)
North-Central	0.13 (0.34)	0.13 (0.34)	0.14 (0.35)
Northeast	0.29 (0.46)	0.30 (0.46)	0.26 (0.44)
South	0.41 (0.49)	0.40 (0.49)	0.42 (0.50)
West	0.17 (0.38)	0.17 (0.38)	0.18 (0.39)
No. of Observations	1129	951	178

Table 1 shows that males making a transition from unemployment to self-employment appear to live in jurisdictions with slightly lower real minimum wages, on average, than those who remain unemployed. In addition, those exiting unemployment were older, had higher levels of education, were more likely to be married and less likely to be black and had fewer children under the age of 18 living with them, on average, compared to those staying in unemployment.

The pooled transition probit results for several specifications are included in Table 2. The entries are the random effects probit coefficients with bootstrapped robust standard errors in parentheses.[18] The three specifications presented in Table 2 are intended to highlight the robustness of our primary finding.[19] Column one includes just age and education as regressors, column

[18]Our regressions are fitted to micro observations using both aggregate (state level) and microdata as explanatory variables, which can result in a downward bias of the standard errors (Moulton 1990). To account for this the standard errors reported in Table 2 are bootstrapped robust (for clustered samples by state/year) standard errors.

[19]Several other specifications were also estimated (including specifications with years of unemployment, an indicator for location in an MSA, and the state labor force participation

two adds the remaining individual and household characteristics, and column three includes all variables in column two plus the tax environment variable. To provide the reader with a better sense of the magnitude of the effects we include estimated marginal effects of an increase in the real minimum wage on the probability of a transition into self-employment (in square brackets).[20]

Table 2
Pooled Probit Results

Variable	(1)	(2)	(3)
Real Minimum Wage	-0.520(0.182)**	-0.428(0.147)**	-0.362(0.176)**
Age	0.156(0.80)**	0.110(0.062)*	0.105(0.082)
Age^2	-0.002(0.001)**	-0.001(0.001)*	-0.001(0.001)
High School	0.252 (0.199)	0.242 (0.152)	0.242 (0.155)
Some College	0.266 (0.244)	0.201(0.266)	0.185(0.184)
College Graduate	0.929(0.441)**	0.591(0.355)*	0.594(0.370)
Some Post-Graduate	1.421(1.336)	1.053(0.729)	1.019(1.585)
Married		0.070(0.173)	0.079(0.194)
No. of Children (<18)		-0.059(0.070)	-0.059(0.062)
Black		-0.682(0.177)**	-0.663(0.171)**
North-Central		0.033(0.235)	0.043(0.234)
South		0.210(0.233)	0.190(0.229)
West		-0.095(0.226)	-0.074(0.254)
Tax Rate			-0.019(0.027)
Constant	-1.882(1.715)	-0.799(1.455)	-0.477(1.456)
No. of Observations	1129	1129	1129
Marginal Effect of Real Minimum Wage	[-0.083] (0.028)	[-0.074] (0.025)	[-0.063] (0.028)

Note: *Bootstrapped standard errors in parenthesis.* $*$ *and* $**$ *indicate statistical significance at 10% and 5% respectively.*

Before analyzing the effects of the real minimum wage on the transition probability into self-employment, note that the effects of the other explanatory variables are consistent with findings in the existing literature (see Parker 2004 for review). The probability of a transition into self-employment is increasing at a decreasing rate with age, increases with the level of education, is higher (though not statistically significantly here) among married men and is substantially lower for blacks.

Our primary focus is on the effect of the real minimum wage on tran-

rate), but the primary results are unchanged. These are available from the authors upon request.

[20] These were generated using the mfix post-estimation command in STATA and assuming that the random effect is zero.

sitions into unemployment. In all three specifications the coefficient on the real minimum wage is negative and statistically significant. The estimated marginal effects suggest that a 1 dollar increase (year 2000 dollars) in the real minimum wage results in a 6 to 8 percentage point reduction in the probability of a transition into self-employment among the unemployed, which is quite large.

Finally, as noted above, we also check whether the negative relationship between the self-employment rate and the real minimum wage (found in previous studies) is observed using the PSID data. To do this we regress the state level self-employment rate on the real minimum wage, a set of region indicator variables, and a vector of state average demographic characteristics (including average age, percent of labor force by education category, percent married, and percent black) and the personal tax rate. As in previous studies, we define self-employment rate as the ratio of self-employed workers to total employment (see footnote 14). We use the sample of male heads from the PSID as in the transition analysis but restrict attention to those who were either wage employed or self-employed in any given year from 1977 to 1997 (instead of those unemployed in the initial year). This sample of 73,291 men is then used to calculate the self-employment rates and demographic characteristics by year and state. The regression result is given below.

Table 3

Dependent Variable: Self-Employment Rate

Variable	Coefficient (s.d.)
Real Minimum Wage	-0.012 (.007)*
Average Age	0.003 (.001)**
% High School	0.147 (0.036)**
% Some College	0.069 (.031)*
% College Graduate	0.042 (0.039)
% Some Post-Graduate	0.029 (0.051)
% Married	0.075 (0.033)
% Black	-0.020 (0.023)
% North-central	-0.010 (0.013)
% South	-0.034 (0.014)
% West	-0.018 (0.012)
Average Tax Rate	-0.002 (0.001)
Constant	0.020 (0.082)
No of Observations	1050
R^2	0.08
Adjusted R^2	0.07

Note: $*$ *and* $**$ *indicate statistical significance at 10% and 5% respectively.*

The table shows that there is a negative statistically significant relationship between the self-employment rate and the real minimum wage.

To conclude, the empirical evidence supports the predictions of our model that a higher real minimum wage reduces both the self-employment rate as well as the transition of unemployed workers to self-employment. An increase in the real minimum wage has a large negative effect on the transition from unemployment to self-employment. This result is robust to several model specifications.

8 Conclusion

In the paper, we developed a theory of self-employment in a search and matching framework. The major contribution of the paper is to incorporate unemployment and transitions between self-employment and unemployment in a model of occupational choice. The paper integrates two major strands of literature: namely, economics of self-employment and economics of unemployment. Existing models of self-employment typically ignore unemployment, while the models of unemployment focus on transitions between unemployment and wage employment.

Our model is able to explain many empirical regularities, particularly with regard to the effects of labor market policies, which are not explained by the existing theoretical models of self-employment. In our model, a higher unemployment benefit and minimum wage reduce self-employment rate and the rate of inflow to self-employment from unemployment. These results are supported by empirical evidence. Empirical evidence also suggests that self-employed earn less than wage employed workers on average. Seen through the lens of existing theoretical models, such earnings differential is considered to be a puzzle. Our model shows that such earnings differential can arise due to frictions in the labor market and low start-up cost of businesses.

We conclude by discussing some future areas of research. In the two-sector model, we assumed that number of firms and wages in the high productivity/wage sector are fixed. The endogenization of wages and the opportunities to work in the high wage sector will lead to much richer interaction between low and high productivity/wage sectors. In addition, endogenization of number of firms in the high wage sector would allow us to incorporate heterogeneous nature of self-employment as well as outflow from wage employment to self-employment.

Appendix

The Derivation of the Wage Function in a One-sector Model (Equation 3.12)

The wage function solves

$$\max_w (\lambda^n - \lambda^u)^{1-\beta}\pi_n(n)^\beta. \tag{A1}$$

The first-order condition is

$$(1-\beta)(1-\tau_w)\pi_n(n) = \beta(1-\tau_d)(\lambda^n - \lambda^u). \tag{A2}$$

(A2), (3.3), and (3.7) imply that

$$\frac{r}{1+r}\frac{1}{1-\tau_w}\lambda^u = b + \frac{1-\beta}{\beta}\frac{\xi}{1-\mu}q. \tag{A3}$$

(3.8) implies that

$$\lambda^n - \lambda^u = \frac{1}{r+\sigma+\mu}[(1+r)(1-\tau_w)w - r\lambda^u]. \tag{A4}$$

(A2) and (A4) imply that

$$\frac{1-\beta}{\beta}\frac{1-\tau_w}{1-\tau_d}\pi_n(n) = \frac{1}{r+\sigma+\mu}[(1+r)(1-\tau_w)w - r\lambda^u]. \tag{A5}$$

Combining (3.4), (A3), and (A5) we have the wage function

$$w = \frac{1}{1+A}\left[Af_n(n) + b + \frac{1-\beta}{\beta}\frac{\xi}{1-\mu}q\right] \tag{A6}$$

where

$$A \equiv \frac{r+\sigma+\mu}{r+\sigma+\mu-\sigma\mu}\frac{1-\beta}{\beta}. \blacksquare$$

Lemma 1: The job-creation curve is given by

$$(1+A)\frac{\xi}{1-\mu}\frac{r+\sigma+\mu-\sigma\mu}{m^v(q)} + \frac{1-\beta}{\beta}\frac{\xi}{1-\mu}q = f_n(n) - b. \tag{A7}$$

The RHS of (A7) is independent of q. Given $\frac{dm^v(q)}{dq} < 0$, the LHS is increasing in q. In addition, as $\lim_{q\to 0} m^v(q) = \infty$, $\lim_{q\to 0} LHS = 0$. Since $f_{nn}(n) < 0$, then for a given $n \in (0, \overline{n})$ there exists a unique $q \in (0,\infty)$, which solves (A7). Finally, the RHS of (A7) is decreasing in n, and the LHS is independent of

n, which implies that $\frac{dq}{dn} < 0$. Note that (A7) implies that labor market tightness, $q = 0$, at $n = \overline{n}$ and for any $n < \overline{n}$, $q > 0$. ■

Lemma 2: Using the identity $q \equiv \frac{vE}{U}$ and (4.1) and (4.3), we have $v = \frac{(\sigma+\mu)n}{m^v(q)}$. Equation 4.6 then can be written as

$$\frac{(1+r)(1-\tau_d)}{r+\mu}\left[f(n) - wn - \frac{(\sigma+\mu)n}{m^v(q)}\right] = \frac{r}{r+\mu}\lambda_u. \tag{A8}$$

Putting (A3) in (A8), we have equation for the firm-size curve

$$(1-\tau_d)\big[f(n) - wn - \frac{\xi(\sigma+\mu)n}{m^v(q)}\big] = (1-\tau_w)\big[b + \frac{\xi}{1-\mu}\frac{1-\beta}{\beta}q\big]. \tag{A9}$$

For a given $n > 0$ a simple differentiation of (A9) shows that $\frac{dLHS}{dq} = -(1-\tau_d)\left[n\frac{1-\beta}{\beta}\frac{\xi}{1-\mu}\frac{1}{1+A} - \xi(\sigma+\mu)n\frac{dm^v(q)/dq}{m^v(q)^2}\right] < 0$ and $\frac{dRHS}{dq} = (1-\tau_w)\frac{\xi}{1-\mu}\frac{1-\beta}{\beta} > 0$. In addition,

$$\lim_{q\to 0} LHS = (1-\tau_d)\left[f(n) - \frac{n}{1+A}[Af_n(n) + b]\right]. \tag{A10}$$

$$\lim_{q\to 0} RHS = b(1-\tau_w). \tag{A11}$$

Now we derive the condition under which $\lim_{q\to 0} LHS > \lim_{q\to 0} RHS$. Taking the derivative of (A10) with respect to n, we have

$$\lim_{q\to 0}\frac{dLHS}{dn} = \frac{f_n(n) - b}{1+A} - \frac{A}{1+A}nf_{nn}(n) > 0 \;\forall\; n \in (0, \overline{n}]. \tag{A12}$$

Now suppose that the parameters of the model are such that $\lim_{q\to 0} LHS > \lim_{q\to 0} RHS$ evaluated at $\overline{n}$. This requires the following parametric restriction

$$f(\overline{n}) - \overline{n}b > \frac{1-\tau_w}{1-\tau_d}b. \tag{A13}$$

Given (A12), (A13), and $\lim_{n\to 0} nf_n(n) = 0$, there exists $\underline{n} \in (0, \overline{n})$, where $\underline{n}$ satisfies

$$(1-\tau_d)\left[f(\underline{n}) - \frac{n}{1+A}[Af_n(\underline{n}) + b]\right] = (1-\tau_w)b. \tag{A14}$$

Then for any given $n \in (\underline{n}, \overline{n})$, there exists a unique $q \in (0, \infty)$, which solves (A9). Note that in the limit (A9) implies that $q = 0$ when $n = \underline{n}$ and for any $n \in (\underline{n}, \overline{n})$, $q > 0$. The RHS of (A9) is independent of n. For a given q, differentiation of the LHS of (A9) gives

$$\frac{dLHS}{dn} = (1 - \tau_d)\left[f_n(n) - w - \frac{\xi(\sigma + \mu)}{m^v(q)} - n\frac{dw}{dn}\right]. \tag{A15}$$

As $\frac{dw}{dn} < 0$ for a given q, and (3.5) implies that $f_n(n) - w - \frac{\xi(\sigma+\mu)}{m^v(q)} > 0$, it follows that $\frac{dLHS}{dn} > 0$. This implies that $\frac{dq}{dn} > 0$. ∎

Proposition 1 Follows from lemmas 1 and 2. ∎

Proposition 2 Follows from the arguments given in the text. ∎

Proposition 3: Simple differentiation of the job-creation curve (4.5) shows that a higher unemployment benefit, b, shifts down the job-creation curve to the left in (n, q) space by -1. Similarly, it shifts down the firm-size curve to the right by $-\left[1 + \frac{n}{1+A}\right]$. Thus the firm-size curve shifts more than the job-creation curve and q falls and n rises.∎

Proposition 4: A higher minimum wage shifts down the job-creation condition by -1 (see equation 5.1). On the other hand, it shifts down the firm-size condition by $-\left[n + \frac{m^u(q)}{r+\sigma+\mu+m^u(q)}\right]$ (see equation 5.3). Then for any $n \geq 1$, q falls and n rises. ∎

Competitive Market and Public Policies: Firm size (equation 5.6) is given by

$$\frac{1 - \tau_w}{1 - \tau_d} f_n(n) = f(n) - nf_n(n). \tag{A16}$$

Under the assumption that $f_{nn} < 0$, the left hand side of (A16) is downward sloping. The RHS of (A16) is upward sloping. Also under the assumptions that $\lim_{n\to 0} \frac{1-\tau_w}{1-\tau_d} f_n(n) > f(0)$ and $\lim_{n\to 0} nf_n(n) = 0$, $\lim_{n\to 0} LHS > \lim_{n\to 0} RHS$. Thus there exists a unique n which solves (A16).

From (A16), it is clear that the LHS is an increasing function of τ_d and a decreasing function of τ_w for a given n. The RHS is independent of these taxes. Thus in (n, q) space, a higher τ_d shifts the LHS up to the right increasing equilibrium n and decreasing E. On the other hand, a higher τ_w shifts the LHS down to the left decreasing equilibrium n and increasing E. ∎

Derivation of the Wage Function in the Two-sector Model (Equation 6.30): Combining (6.29) with the first order conditions associated with Nash bargaining and the creation of vacancy, we derive an expression for λ^u, which is identical in form to (A3). From (6.27) we have

$$\lambda^h = \frac{1}{r+\rho}\left[(1-\tau_w)(1+r)w^h + \rho\lambda^u\right]. \tag{A17}$$

Under the assumption that $\delta\rho \approx 0$, (6.28) and (A17) imply that

$$\lambda^n = \frac{1}{r+\sigma+2\delta}\left[(1-\tau_w)(1+r)w^l + \frac{\delta}{r+\rho}(1+r)(1-\tau_w)w^h + (\sigma+\delta)\lambda^u\right]. \tag{A18}$$

From (A18) we have

$$\lambda^n - \lambda^u = \frac{1}{r+\sigma+2\delta}\left[(1-\tau_w)(1+r)w^l + \frac{\delta}{r+\rho}(1+r)(1-\tau_w)w^h - (r+\delta)\lambda^u\right]. \tag{A19}$$

Let $C \equiv \frac{\delta}{r+\rho}w^h$. Then the first order condition for Nash bargaining along with the expression for the marginal value of employees given by $\pi_n(n) = \frac{(1-\tau_d)(1+r)}{r+\sigma+2\delta-\delta(\sigma+\delta)}(f_n(n) - w^l)$ and (A19) imply that

$$\frac{1-\beta}{\beta}\frac{r+\sigma+2\delta}{r+\sigma+2\delta-\delta(\sigma+\delta)}(f_n(n)-w^l) = w^l + C - \frac{r+\delta}{1+r}\frac{\lambda_u}{1-\tau_w}. \tag{A20}$$

Letting $B \equiv \frac{1-\beta}{\beta}\frac{r+\sigma+2\delta}{r+\sigma+2\delta-\delta(\sigma+\delta)}$ and putting (A3) in (A20), we derive the expression for the wage function:

$$w^l = \frac{1}{1+B}\left[Bf_n(n) - C + \frac{r+\delta}{r}\left\{b + \frac{1-\beta}{\beta}\frac{\xi}{1-\delta}q\right\}\right]. \blacksquare \tag{A21}$$

Shape of the Job-Creation Curve in the Two-Sector Model (Equation 6.31): The job-creation curve is given by

$$f_n(n) + C - \frac{r+\delta}{r}b = \frac{\xi(r+\sigma+\delta+\delta(\sigma+\delta))}{1-\delta}\frac{1+B}{m^v(q)} + \frac{r+\delta}{r}\frac{1-\beta}{\beta}\frac{\xi}{1-\delta}q. \tag{A22}$$

The RHS of (A22) is independent of average firm-size, n. Given that $m^v(q)$ is decreasing in q, $\frac{dRHS}{dq} > 0$. Also, $\lim_{q\to 0} m^v(q) = \infty$. This implies that $\lim_{q\to 0} RHS = 0$.

The LHS of (A22) is independent of q. Also for any $n \in (0, n^{**})$ where n^{**} satisfies

$$f_n(n^{**}) + C = \frac{r+\delta}{r}b \tag{A23}$$

The LHS of (A22) > 0. Thus there exists a unique $q \in (0, \infty)$ for any $n \in (0, n^{**})$ which solves (A22). Also $\frac{dLHS}{dn} < 0$. This implies that the job-creation curve traces an inverse relationship between q and n. (A22) implies that at n^{**}, $q = 0$ and for any $n \in (0, n^{**})$, $q > 0$. ∎

Shape of the Firm-size Curve in the Two-Sector Model (Equation 6.33): By putting (A3) and (A17) in (6.32) we derive the firm-size curve which satisfies

$$\frac{1-\tau_d}{1-\tau_w}\left[f(n) - w^l n - \frac{\xi(\sigma+2\delta)n}{m^v(q)}\right] + C = \frac{r+\delta}{r+\rho}\Big[b + \frac{1-\beta}{\beta}\frac{\xi}{1-\delta}q\Big]. \quad \text{(A24)}$$

Differentiation of both sides of (A24) shows that $\frac{dRHS}{dq} > 0$ and given that $\frac{dw_l}{dq} > 0$ for any $n > 0$, $\frac{dLHS}{dq} < 0$. Now we derive the condition under which $\lim_{q\to 0} LHS > \lim_{q\to 0} RHS$.

$$\lim_{q\to 0} LHS = \frac{1-\tau_d}{1-\tau_w}\left[f(n) - \frac{n}{1+B}[Bf_n(n) - C + \frac{r+\delta}{r}b]\right] + C. \quad \text{(A25)}$$

Differentiating (A25) with respect to n, we have

$$\lim_{q\to 0}\frac{dLHS}{dn} = \frac{1-\tau_d}{1-\tau_w}\left[f_n(n) + C - \frac{r+\delta}{r}b - \frac{n}{1+B}Bf_{nn}(n)\right] > 0 \forall n \in (0, n^{**}). \quad \text{(A26)}$$

Now we derive the parametric restriction such that that $\lim_{q\to 0} LHS > \lim_{q\to 0} RHS = \frac{r+\delta}{r+\rho}b$ evaluated at n^{**}. This condition is

$$f(n^{**}) - n^{**}f_n(n^{**}) > \frac{1-\tau_w}{1-\tau_d}\left[\frac{r+\delta}{r+\rho}b - C\right]. \quad \text{(A27)}$$

If (A27) is satisfied then from (A26) it follows that there exists $n^* \in (0, n^{**})$ such that $\lim_{q\to 0} LHS(n^*) = \lim_{q\to 0} RHS(n^*)$ *i.e.*,

$$\frac{1-\tau_d}{1-\tau_w}\left[f(n^*) - \frac{n^*}{1+B}[Bf_n(n^*) - C + \frac{r+\delta}{r}b]\right] + C = \frac{r+\delta}{r+\rho}b. \quad \text{(A28)}$$

Then for any $n \in (n^*, n^{**})$ there exists a unique $q \in (0, \infty)$ which solves (A24). Since $\frac{dLHS}{dn} > 0$, it implies that there is positive relationship between q and n. (A24) implies that at n^*, $q = 0$ and for any $n \in (n^*, n^{**})$, $q > 0$. ∎

Proposition 5: Follows from the discussion in the text.∎

Proposition 6: Follows from the discussion in the text.■

Proposition 7:

a. **Unemployment Benefit**: Simple differentiation of (A22) with respect to b shows that it shifts down the job-creation curve in (n, q) space by $\frac{r+\delta}{r}$. Similarly, simple differentiation of (A24) with respect to b shows that it shifts down the firm-size curve by $\frac{r+\delta}{r}\left[1+\frac{n}{1+B}\right]$. Thus the firm-size curve shifts down more. ■

b. **Minimum Wage**: In the case of a binding minimum wage the job creation curve is given by

$$f_n(n) - w^m = \frac{\xi}{1-\delta}\frac{r+\sigma+2\delta-\delta(\sigma+\delta)}{m^v(q)}. \tag{A29}$$

For any n such that $f_n(n) > w_m$, it is easy to show that there exists a unique q which solves (A29) and the job-creation curve is downward sloping in (n, q) space. Simple differentiation of (A29) with respect to w_m shows that an increase in the minimum wage shifts down the job creation curve to the left by 1 in (n, q) space.

The value function of an employed worker in the low wage sector, λ^n and the firm-size curve continues to satisfy (6.28) and (6.32) respectively with w^l replaced by w^m. Under the condition that $\delta\rho \approx 0$, using (6.28), (6.29), and(6.30), we derive the expression for the value function of unemployed workers:

$$\lambda^u = \frac{1+r}{r(r+\sigma+2\delta)+(r+\delta)m^u(q)}\left[(r+\sigma+2\delta)b + m^u(q)(w^m + C)\right]. \tag{A30}$$

Putting (A30) in (6.32), we derive the equation for the firm-size curve given by

$$f(n) - \frac{\xi(\sigma+2\delta)n}{m^v(q)} = \left[n + \frac{m^u(q)(r+\delta)}{r(r+\sigma+2\delta)+(r+\delta)m^u(q)}\right]w^m +$$

$$\frac{(r+\sigma+2\delta)(r+\delta)b}{r(r+\sigma+2\delta)+(r+\delta)m^u(q)} - \left[1 - \frac{(r+\delta)m^u(q)}{r(r+\sigma+2\delta)+(r+\delta)m^u(q)}\right]C. \tag{A31}$$

Again one can show that for a given $n > 0$ there exists a unique q which solves (A31) and the firm-size curve is upward sloping in (n, q) space. Simple differentiation of (A31) with respect to w^m shows that a higher minimum wage shifts down the firm-size curve to the right by $n + \frac{m^u(q)(r+\delta)}{r(r+\sigma+2\delta)+(r+\delta)m^u(q)}$. Thus if

$$n + \frac{m^u(q)(r+\delta)}{r(r+\sigma+2\delta)+(r+\delta)m^u(q)} > 1 \tag{A32}$$

then the firm-size curve shifts down more than the job-creation curve leading to lower labor market tightness, q, and higher firm-size, n. ■

Proposition 8: Simple differentiation of (A22) and (A24) with respect to w^h show that it shifts the job-creation curve by $\frac{\delta}{r+\rho}$ up to the right and the firm-size curve by $\left[1 + \frac{1-\tau_d}{1-\tau_w}\frac{n}{1+B}\right]\frac{\delta}{r+\rho}$ up to the left. Since, it shifts up the firm-size curve more than the job-creation curve, q rises and n falls. ■

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