Chapter 13
TEMPERATURE AND THE IDEAL GAS

Conceptual Questions

1. The development of standard temperature scales requires use of the zeroth law of thermodynamics. This law tells us that if a thermometer is in thermodynamic equilibrium with both a test object and an object used to define a standard, then the test and standard objects must be in thermodynamic equilibrium with each other. If this law did not hold, the temperature scale on a thermometer would have no relation to the actual temperature of the test object, and therefore, no standard could be defined.

2. Absolute zero is the temperature at which the motion of atoms in a substance is a minimum and the temperature can decrease no further. There is no fundamental significance to the zero degree point of the Celsius and Fahrenheit temperature scales—they are simply of historical origin.

3. A temperature difference of 1 K is the same as a temperature change of 1 °C. Thus, either scale may be used in applications dealing only with temperature differences.

4. Imagine a circle drawn on the plate instead of a hole cut into it. The drawn circle must expand in the same manner as the cut hole. Thus, the center of the plate expands outward whether or not a hole exists and the hole must therefore grow larger.

5. The thermal expansion coefficients of silver and brass only differ by about 5%. Thus, a bimetallic strip made from these materials would bend only slightly during expansion, contrary to its intended purpose.

6. Metals have thermal expansion coefficients several orders of magnitude larger than glass. Running the jar under hot water therefore facilitates its opening since the lid expands more than the jar as its temperature increases.

7. According to the ideal gas law, if the temperature of an ideal gas is negative, then either the pressure or the volume must likewise have a negative value. This requirement is nonsensical since absolute pressure and volume must be positive quantities. Unlike the Celsius and Fahrenheit temperature scales, temperatures in the Kelvin scale are always positive and therefore avoid the aforementioned problem.

8. Conversion of the price per cubic foot of natural gas into the price per mole requires knowledge of the number of moles contained within the given volume. From the macroscopic ideal gas law, the number of moles in a sample is proportional to the volume, pressure, and temperature of the gas. Using the known volume and the additional quantities of pressure and temperature, the number of moles may be calculated and the price may be converted. The conversion could also be made if either the number density or the mass density and the mass per molecule were known. The latter method works whether or not the gas is ideal.

9. The SI units of mass density and number density are kg/m³ and m⁻³, respectively. An equal number density does not imply an equal mass density because the mass of an individual atom may be different in each gas.

10. From the ideal gas law, two gases at equal temperature and pressure must have identical number densities. The mass of a nitrogen atom is greater than the mass of a helium atom—the mass density of nitrogen is therefore greater.

11. One mole of aluminum atoms has a mass of 27.0 g.

12. The pressure of the air inside the ball increases as it is heated, pushing outward on the dents.
13. The pressure of the air outside the balloon decreases as its distance above the Earth increases. The balloon expands until the pressures inside and outside are equal.

14. Hydrogen and helium molecules in the high-energy tail of the Maxwell-Boltzmann distribution have enough kinetic energy to escape from Earth’s atmosphere. Other molecules are gradually boosted into the vacated high-energy region and eventually escape themselves. Only a negligible number of hydrogen and helium molecules have enough kinetic energy to escape from Jupiter.

15. The kinetic energy is proportional to the velocity squared, so the molecules with large velocities count more in the average than those with small velocities.

16. Since the diameter of the molecules is considerably smaller than the average intermolecular distance, there is a lot of empty space for the molecules to move around in without hitting one another.

17. We could use any two of the quantities to decide whether the gas is dilute. The gas is dilute if the number density is low enough, or equivalently if the average intermolecular distance is large enough compared to the molecular diameter. For example, if the mean free path is much larger than the diameter, then we know that the intermolecular distance is large enough and the number density small enough so that the gas is dilute.

18. In this case the air would be a liquid or solid so there would be no sense in speaking of a mean free path; the molecules are so close together that they are never free of interactions with other molecules.

19. The passenger’s velocity changes more gradually as he sinks into the bag than if he comes into contact with a hard surface (dashboard or high-pressure bag). If the air pressure in the bag is too low, the passenger is not sufficiently slowed as the bag collapses.

20. Water boils at a temperature below 100°C at altitudes above sea level because the air pressure is lower. Since the water temperature is less, the reactions responsible for cooking the egg proceed more slowly.

Problems

1. **Strategy** Use Eqs. (13-2b) and (13-3).

   **Solution** Convert the temperature.

   (a) \( T_C = \frac{T_F - 32^\circ F}{1.8^\circ F/\circ C} = \frac{84^\circ F - 32^\circ F}{1.8^\circ F/\circ C} = 29^\circ C \)

   (b) \( T = 29 K + 273.15 K = 302 K \)

2. **Strategy** Use Eqs. (13-2a) and (13-3).

   **Solution** Convert the temperature.

   (a) \( T_C = T - 273.15 K = 77 K - 273.15 K = -196^\circ C \)

   (b) \( T_F = (1.8^\circ F/\circ C)T_C + 32^\circ F = (1.8^\circ F/\circ C)(-196.15^\circ C) + 32^\circ F = -321^\circ F \)
3. (a) **Strategy** Set \( T_C = T_F = T \) in Eq. (13-2a) and solve for \( T \).

**Solution** Find the temperature.
\[
T = 1.8T + 32, \text{ so } T = \frac{32}{1 - 1.8} = -40.
\]

(b) **Strategy** Set \( T_C = T_K - 273.15 \) and \( T_K = T_F = T \) in Eq. (13-2a) and solve for \( T \).

**Solution** Find the temperature.
\[
T = 1.8(T - 273.15) + 32 = 1.8T - 459.67, \text{ so } T = \frac{-459.67}{1 - 1.8} = 575.
\]

4. **Strategy** Use Eq. (13-2a) and the fact that the kelvin has the same degree size as the Celsius scale.

**Solution** Find the temperature changes.

(a) \( \Delta T = \Delta T_C = -6.0 \text{ K} \)

(b) \( \Delta T_F = (1.8^\circ\text{F}/^\circ\text{C})\Delta T_C = (1.8^\circ\text{F}/^\circ\text{C})(-6.0^\circ\text{C}) = -11^\circ\text{F} \)

5. **Strategy and Solution** There are 78 + 114 = 192 degrees C and 144 degrees J between the freezing and boiling points of ethyl alcohol. Thus, the conversion factor is 144/192 = 0.750. So, \( T_j = (0.750\,^\circ\text{J}/^\circ\text{C})T_C + A \), where \( A \) is the offset to be determined. Find \( A \) by setting both temperatures equal to their respective boiling temperatures. 144\,^\circ\text{J} = (0.750\,^\circ\text{J}/^\circ\text{C})(78\,^\circ\text{C}) + A \), so \( A = 144\,^\circ\text{J} - (0.750\,^\circ\text{J}/^\circ\text{C})(78\,^\circ\text{C}) = 85.5\,^\circ\text{J} \).

Thus, the conversion from \(^\circ\text{J}\) to \(^\circ\text{C}\) is given by \( T_j = (0.750\,^\circ\text{J}/^\circ\text{C})T_C + 85.5\,^\circ\text{J} \).

6. **Strategy** Use Eq. (13-4).

**Solution** Find the amount the faucet rises.
\[
\frac{\Delta L}{L_0} = \alpha\Delta T, \text{ so } \Delta L = L_0\alpha\Delta T = (2.4 \text{ m})(16\times10^{-6} \text{ K}^{-1})(90.0^\circ\text{C} - 20.0^\circ\text{C}) = 2.7 \text{ mm}.
\]

7. **Strategy** Use Eq. (13-4).

**Solution** Find the expansion of the rods to determine how far the unfixed end moves.
\[
\frac{\Delta L}{L_0} = \alpha\Delta T, \text{ so } \Delta L = L_0\alpha\Delta T. \text{ For the system of rods, we have }
\Delta L_{Cu} + \Delta L_{Al} = L_0\alpha_{Cu}\Delta T + L_0\alpha_{Al}\Delta T = L_0\Delta T(\alpha_{Cu} + \alpha_{Al}) = (0.350 \text{ m})(16\times10^{-6} \text{ K}^{-1} + 32.5\times10^{-6} \text{ K}^{-1})(150^\circ\text{C} - 0.0^\circ\text{C}) = 2.0 \text{ mm}.
\]

8. **Strategy** Since each section of track expands along its entire length, only half of the expansion is considered for a particular gap. Two sections meet at a gap, so the gap should be as wide as the expansion of one section of track. Use Eq. (13-4).

**Solution** Find the amount of space that should be left between the track sections.
\[
\frac{\Delta L}{L_0} = \alpha\Delta T, \text{ so } \Delta L = L_0\alpha\Delta T = (18.30 \text{ m})(12\times10^{-6} \text{ K}^{-1})(50.0^\circ\text{C} - 10.0^\circ\text{C}) = 8.8 \text{ mm}.
\]
9. Strategy Since each concrete slab expands along its entire length, only half of the expansion is considered for a particular gap. Two sections meet at a gap, so the gap should be as wide as the expansion of one concrete slab. Use Eq. (13-4).

Solution Find the sizes of the expansion gaps.

(a) \[ \Delta L = L_0 \alpha \Delta T = (15 \text{ m})(12 \times 10^{-6} \text{ K}^{-1})(40.0^\circ \text{C} - 20.0^\circ \text{C}) = 3.6 \text{ mm} \]

(b) \[ \Delta L = L_0 \alpha \Delta T = (15 \text{ m})(12 \times 10^{-6} \text{ K}^{-1})(-20.0^\circ \text{C} - 20.0^\circ \text{C}) = -7.2 \text{ mm} \]

gap width = 7.2 mm + 3.6 mm = 10.8 mm

10. Strategy Form a proportion with \( \Delta L_{\text{pb}} \) and \( \Delta L_{\text{glass}} \) (which are set equal) to solve for \( T_{\text{glass}} \) when \( T_{\text{pb}} = 50.0^\circ \text{C} \). Use Eq. (13-4).

Solution Find the final temperature of the glass.

\[ 1 = \frac{\Delta L_{\text{pb}}}{\Delta L_{\text{glass}}} = \frac{L_0 \alpha_{\text{pb}} \Delta T_{\text{pb}}}{L_0 \alpha_{\text{glass}} \Delta T_{\text{glass}}} \]

so

\[ T_{\text{glass}} = \frac{29 \times 10^{-6} \text{ K}^{-1}}{9.4 \times 10^{-6} \text{ K}^{-1}} (50.0^\circ \text{C} - 20.0^\circ \text{C}) + 20.0^\circ \text{C} = 113^\circ \text{C} \]

11. Strategy The hole expands just as if it were a solid brass disk. Use Eq. (13-6).

Solution Find the increase in area of the hole.

\[ \Delta A = 2\alpha A_0 \Delta T = 2(1.9 \times 10^{-5} \text{ K}^{-1})(1.00 \text{ mm}^2)(30.0^\circ \text{C} - 20.0^\circ \text{C}) = 3.8 \times 10^{-4} \text{ mm}^2 \]

12. Strategy According to Eq. (13-4), the change in diameter is given by \( \Delta d = d_0 \alpha \Delta T = d - d_0 \).

Solution Find the diameter of the rivets.

\[ d = d_0 + \Delta d = d_0 + d_0 \alpha \Delta T = d_0(1 + \alpha \Delta T), \text{ so} \]

\[ d_0 = \frac{d}{1 + \alpha \Delta T} = \frac{0.6350 \text{ cm}}{1 + (22.5 \times 10^{-6} \text{ K}^{-1})(-78.5^\circ \text{C}-20.5^\circ \text{C})} = 0.6364 \text{ cm} \]

13. Strategy A decrease in volume for a fixed mass increases the density, and vice versa. Use Eq. (13-7).

Solution

(a) \[ \frac{\Delta V}{V_0} = \beta \Delta T \text{ and } \frac{\Delta \rho}{\rho} = -\frac{\Delta V}{V_0}. \text{ Thus, } \frac{\Delta \rho}{\rho} = -\beta \Delta T, \text{ so } \Delta \rho = -\beta \rho \Delta T. \]

(b) Compute the fractional change in density.

\[ \frac{\Delta \rho}{\rho} = -\beta \Delta T = -(57 \times 10^{-6} \text{ K}^{-1})(-10.0^\circ \text{C}-32^\circ \text{C}) = 2.4 \times 10^{-3} \]
14. **Strategy** Determine how much more the volume of the water expands than that of the brass container. Use Eq. (13-7).

**Solution** Find the amount of water that overflows.

$$\frac{\Delta V}{V_0} = \beta \Delta T$$, so

$$\Delta V_{\text{water}} - \Delta V_{\text{brass}} = V_0(\beta_{\text{water}} - \beta_{\text{brass}})\Delta T$$

$$= (75.0 \text{ cm}^2)(20.0 \text{ cm})(207 \times 10^{-6} \text{ K}^{-1} - 57 \times 10^{-6} \text{ K}^{-1})(95.0^\circ \text{C} - 25.0^\circ \text{C}) = 15.8 \text{ cm}^3$$.

15. **Strategy** Determine how much the volume of the water expands. Use Eq. (13-7).

**Solution** Find the amount of water that will have spilled.

$$\frac{\Delta V}{V_0} = \beta \Delta T$$, so

$$\Delta V = V_0 \beta \Delta T = (268.4 \text{ mL})(207 \times 10^{-6} \text{ K}^{-1})(32.0^\circ \text{C} - 2.0^\circ \text{C}) = 1.67 \text{ mL}$$.

16. (a) **Strategy** Both the glass and the water expand, but the water expands much more than the glass. Use Eq. (13-7).

**Solution** Find the difference in volume expansions to find the amount of water spilled.

$$\Delta V_{\text{H}_2\text{O}} - \Delta V_{\text{glass}} = V_0 \beta_{\text{H}_2\text{O}} \Delta T - V_0 \beta_{\text{glass}} \Delta T = V_0(\beta_{\text{H}_2\text{O}} - \beta_{\text{glass}})\Delta T$$

$$= (268.4 \text{ mL})(207 \times 10^{-6} \text{ K}^{-1} - 28.2 \times 10^{-6} \text{ K}^{-1})(32.0^\circ \text{C} - 2.0^\circ \text{C}) = 1.44 \text{ mL}$$

Less water spilled compared to the situation in Problem 15.

(b) **Strategy** Refer to Problem 15 and part (a).

**Solution** Compute the percent change.

$$\frac{1.44 \text{ mL} - 1.67 \text{ mL}}{1.67 \text{ mL}} \times 100\% = -14\%$$

So, 14% less water was spilled than when we don’t consider the expansion of the glass.

17. **Strategy** Use Eq. (13-4). The internal radius of the ring expands as if it were a solid piece of brass.

**Solution** Find the temperature at which the internal radius of the ring is 1.0010 cm.

$$\frac{\Delta L}{L_0} = \alpha \Delta T$$, so

$$\frac{\Delta L}{\alpha L_0} = \Delta T = T - T_0$$, Compute the temperature.

$$T = \frac{\Delta L}{\alpha L_0} + T_0 = \frac{1.0010 \text{ cm} - 1.0000 \text{ cm}}{(19 \times 10^{-6} \text{ K}^{-1})(1.0000 \text{ cm})} + 22.0^\circ \text{C} = 75^\circ \text{C}$$

18. **Strategy** For a physical pendulum, \( T \propto \sqrt{\ell} \). Form a proportion. Use Eq. (13-5).

**Solution** Find the fractional change in the period of the oscillating steel rod.

$$\frac{T_{\text{new}}}{T_0} = \sqrt{\frac{L_{\text{new}}}{L_0}} = \sqrt{\frac{L_0(1 + \alpha \Delta T)}{L_0}} = \sqrt{1 + \alpha \Delta T}$$, so

$$\frac{\Delta T}{T_0} = \frac{T_{\text{new}} - T_0}{T_0} = \frac{T_{\text{new}}}{T_0} - 1 = \sqrt{1 + \alpha \Delta T} - 1 = \sqrt{1 + (12 \times 10^{-6} \text{ K}^{-1})(0^\circ \text{C} - 25^\circ \text{C})} - 1 = -1.5 \times 10^{-4}$$
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Solution Find the maximum change in the length of the span over an entire year.

\[ \Delta L = \alpha L_0 \Delta T = (12 \times 10^{-6} \text{ K}^{-1})(1.6 \times 10^3 \text{ m})[105^\circ \text{F} - (-15^\circ \text{F})]\left\{ \frac{1 \text{ K}}{1.8^\circ \text{F}} \right\} = 1.3 \text{ m} \]

20. Strategy The area of the hole expands as if it were a solid disk of brass. Use Eq. (13-6).

Solution Find the temperature at which the area of the hole is 4.91000 cm².

\[ \frac{\Delta A}{A_0} = 2\alpha \Delta T = 2\alpha(T - T_0), \text{ so } T = \frac{\Delta A}{2\alpha A_0} + T_0 = \frac{4.91000 \text{ cm}^2 - 4.90874 \text{ cm}^2}{2(19 \times 10^{-6} \text{ K}^{-1})(4.90874 \text{ cm}^2)} + 20.0^\circ \text{C} = 26.8^\circ \text{C} \]

21. Strategy The diameter of the hole expands as if it were a solid piece of copper. Use Eq. (13-4).

Solution Find the temperature at which the diameter of the hole is 1.0000 cm.

\[ \frac{\Delta L}{L_0} = \alpha \Delta T = \alpha(T - T_0), \text{ so } T = \frac{\Delta L}{\alpha L_0} + T_0 = \frac{1.0000 \text{ cm} - 0.9980 \text{ cm}}{(16 \times 10^{-6} \text{ K}^{-1})(0.9980 \text{ cm})} + 20.0^\circ \text{C} = 150^\circ \text{C} \]

22. Strategy Find the temperature at which the diameters of the washer and bolt are the same. Use Eq. (13-5).

Solution Set the final diameters equal and solve for the final temperature.

\[ L_{c_0} + L_{c_0} \alpha_c \Delta T = L_{s_0} + L_{s_0} \alpha_s \Delta T \]
\[ L_{c_0} \alpha_c (T_f - T_i) = L_{s_0} - L_{c_0} + L_{s_0} \alpha_s (T_f - T_i) \]
\[ T_f = \frac{L_{s_0} - L_{c_0} + T_i (L_{c_0} \alpha_c - L_{s_0} \alpha_s)}{L_{c_0} \alpha_c - L_{s_0} \alpha_s} \]
\[ T_f = \frac{1.0000 \text{ cm} - 0.9980 \text{ cm}}{(0.9980 \text{ cm})(16 \times 10^{-6} \text{ K}^{-1}) - (1.0000 \text{ cm})(12 \times 10^{-6} \text{ K}^{-1})} + 20.0^\circ \text{C} = 520^\circ \text{C} \]

23. Strategy Use Eqs. (13-4) and (13-5).

Solution Find the new scale of the rule.

\[ \frac{\Delta L_f}{L_{r_0}} = \frac{L_f - L_{r_0}}{L_{r_0}} = \frac{L_f}{L_{r_0}} - 1 = \alpha_i \Delta T, \text{ so } \frac{L_f}{L_{r_0}} = 1 + \alpha_i \Delta T. \]

Divide the new length of the brick by the new scale.

\[ \frac{L_b}{1 + \alpha_i \Delta T} = \frac{L_{b_0}(1 + \alpha_b \Delta T)}{1 + \alpha_i \Delta T} = \frac{(25.00 \text{ cm})(1 + (0.75 \times 10^{-6} \text{ K}^{-1})(60.00 \text{ K}))}{1 + (12 \times 10^{-6} \text{ K}^{-1})(60.00 \text{ K})} = 24.98 \text{ cm} \]

24. Strategy Find the temperature change required to increase the circumference of the A340 by 26 cm. Use Eq. (13-4).

Solution Find the required increase in temperature.

\[ \frac{\Delta L}{L_0} = \alpha \Delta T, \text{ so } \Delta T = \frac{\Delta L}{\alpha L_0} = \frac{0.26 \text{ m}}{(22.5 \times 10^{-6} \text{ K}^{-1})(17.72 \text{ m})} = 650 \text{ K} \]

The melting point of aluminum is 660°C.
25. **Strategy** Use Eqs. (13-4) and (13-6) and the given initial and final areas.

**Solution** Find the fractional change.

\[
\frac{\Delta A}{A_0} = \frac{A - A_0}{A_0} = \frac{(s_0 + \Delta s)^2 - s_0^2}{s_0^2} = \frac{s_0^2 + 2s_0\Delta s + (\Delta s)^2 - s_0^2}{s_0^2} = \frac{2s_0\Delta s + (\Delta s)^2}{s_0^2} = \frac{\Delta s(2s_0 + \Delta s)}{s_0^2}
\]

Now, since \( s_0 \gg \Delta s \), we have

\[
\Delta A = \frac{s_0(2s_0 + \Delta s)}{s_0^2} = \frac{2s_0\Delta s}{s_0} = \frac{2\Delta s}{s_0} = 2\alpha \Delta T \quad \text{since} \quad \frac{\Delta s}{s_0} = \alpha \Delta T.
\]

26. **Strategy** Use Eqs. (13-4) and (13-7) and the given initial volume.

**Solution** Find the fractional change.

\[
\frac{\Delta V}{V_0} = \frac{V - V_0}{V_0} = \frac{(s_0 + \Delta s)^3 - s_0^3}{s_0^3} = \frac{s_0^3 + 3s_0^2\Delta s + 3s_0(\Delta s)^2 + (\Delta s)^3 - s_0^3}{s_0^3} = \frac{3s_0^2\Delta s + 3s_0(\Delta s)^2 + (\Delta s)^3}{s_0^3}
\]

Now, since \( \Delta s << s_0 \), \( 3s_0^2\Delta s \gg s_0(\Delta s)^2 \) and \( (\Delta s)^3 \). Thus, we have

\[
\frac{\Delta V}{V_0} = \frac{3s_0^2\Delta s}{s_0^3} = \frac{3}{s_0} = 3\alpha \Delta T \quad \text{since} \quad \frac{\Delta s}{s_0} = \alpha \Delta T.
\]

27. **Strategy** Use the definition of one mol and Avogadro’s number.

**Solution** Find the conversion between atomic mass units u and kg.

\[
\frac{12 \text{ g}}{12 \text{ mol}} = \frac{0.001 \text{ kg}}{6.022 \times 10^{23}}, \quad \text{so} \quad 1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}.
\]

28. **Strategy** Add the molar masses of each element in ammonia.

**Solution** Find the molar mass.

\[
m_{\text{NH}_3} = m_N + 3m_H = 14.00674 \text{ g/mol} + 3(1.00794 \text{ g/mol}) = 17.03056 \text{ g/mol}
\]

29. **Strategy** Add the molecular masses of each element in carbon dioxide.

**Solution** Find the mass of carbon dioxide in kg.

\[
\text{mass of CO}_2 \text{ in kg} = m_{\text{C}} + 2m_{\text{O}} = [12.011 \text{ u} + 2(15.9994 \text{ u})](1.6605 \times 10^{-27} \text{ kg/u}) = 7.31 \times 10^{-26} \text{ kg}
\]

30. (a) **Strategy and Solution** By the definition of the atomic mass unit, 13.003 g/mol = 13.003 u.

(b) **Strategy** Multiply the mass of one carbon-13 atom by the conversion factor 1.6605 \times 10^{-27} \text{ kg/u}.

**Solution** Find the mass in u.

\[
m_{13} = \frac{m}{N_A} = (13.003 \text{ u})(1.6605 \times 10^{-27} \text{ kg/u}) = 2.1591 \times 10^{-26} \text{ kg}
\]

31. **Strategy** Divide the mass of the water in the human by the mass of a water molecule.

**Solution** Estimate the number of water molecules.

\[
N = \frac{\text{mass of water}}{\text{molecular mass}} = \frac{0.62(80.2 \text{ kg})}{[2(1.0 \text{ u}) + 16.0 \text{ u}](1.66 \times 10^{-27} \text{ kg/u})} = 1.7 \times 10^{27}
\]
32. **Strategy** Use Eq. (13-10).

**Solution** Find the number density of carbon atoms.

\[
\text{number per unit volume} = \frac{N}{V} = \frac{\rho}{m} = \frac{(3500 \text{ kg/m}^3)(10^{-6} \text{ m}^3)}{(12.011 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 1.8 \times 10^{23} \text{ atoms/cm}^3
\]

33. **Strategy** Divide the total mass by the molar mass of sucrose to find the number of moles. Then use Eq. (13-11) to find the number of hydrogen atoms.

**Solution**

\[
m_{C_{12}H_{22}O_{11}} = 12(12.011 \text{ g/mol}) + 22(1.00794 \text{ g/mol}) + 11(15.9994 \text{ g/mol}) = 342.30 \text{ g/mol}
\]

There are 342.30 grams of sucrose per mole, so there are \(\frac{684.6 \text{ g}}{342.30 \text{ g/mol}} = 2.000 \text{ mol of sucrose.}\)

There are 2.000(22) = 44.00 moles of hydrogen. Find the number of hydrogen atoms.

\[
N = nN_A = (44.00 \text{ mol})(6.022 \times 10^{23} \text{ mol}^{-1}) = 2.650 \times 10^{25} \text{ atoms}
\]

34. **Strategy** Divide the total mass of He by its molar mass.

**Solution** Find the number of moles.

\[
\frac{\text{mass of He}}{\text{molar mass of He}} = \frac{13 \text{ g}}{4.00260 \text{ g/mol}} = 3.2 \text{ mol}
\]

35. **Strategy** Divide the total mass of methane by its molar mass.

**Solution** Find the number of moles.

\[
n_{CH_4} = \frac{\text{mass of CH}_4}{\text{molar mass of CH}_4} = \frac{144.36 \text{ g}}{12.011 \text{ g/mol} + 4(1.00794 \text{ g/mol})} = \frac{8.9985 \text{ mol}}{}
\]

36. **Strategy** Divide the molar mass of gold by Avogadro’s number.

**Solution** Find the mass of one gold atom.

\[
\text{mass of one gold (Au) atom} = \frac{\text{molar mass}}{N_A} = \frac{(196.96654 \text{ g/mol})(10^{-3} \text{ kg/g})}{6.022 \times 10^{23} \text{ atoms}} \left( \frac{1 \text{ mol}}{6.022 \times 10^{23} \text{ atoms}} \right) = 3.271 \times 10^{-25} \text{ kg}
\]

37. **Strategy** Use Eq. (13-10).

**Solution** Find the number of air molecules.

\[
N = \frac{\rho V}{m} = (1.2 \text{ kg/m}^3)(1.0 \text{ cm}^3) \left( \frac{1 \text{ m}^3}{10^6 \text{ cm}^3} \right) \left( \frac{1 \text{ u}}{29.0 \text{ u}} \right) \left( \frac{1 \text{ u}}{1.66 \times 10^{-27} \text{ kg}} \right) = 2.5 \times 10^{19} \text{ molecules}
\]
38. (a) **Strategy** Use Eq. (13-11).

**Solution** Find the number density.

\[ N = \frac{nN_A}{V} = \frac{(1.00 \, \text{mol})(6.022 \times 10^{23} \, \text{mol}^{-1})}{0.0224 \, \text{m}^3} = 2.69 \times 10^{25} \, \text{m}^{-3} \]

(b) **Strategy** Assume that each molecule is at the center of a sphere of radius \( r \).

**Solution** The volume of the sphere is

\[ \frac{V}{N} = \frac{1}{nN_A} = \frac{1}{2.69 \times 10^{25} \, \text{atoms/m}^3} = 3.72 \times 10^{-26} \, \text{m}^3 \text{ per molecule.} \]

Then

\[ \frac{V}{N} = \frac{4}{3} \pi r^3 = 4r^3, \text{ since } \pi = 3. \]

The distance separating molecules is approximately the diameter of the spheres. Solve for \( d = 2r \).

\[ d = 2r = 2 \left( \frac{V}{4N} \right)^{1/3} = 2 \left( \frac{3.72 \times 10^{-26} \, \text{m}^3}{4} \right)^{1/3} = 4 \, \text{nm} \]

(c) **Strategy** Multiply the number of moles by the molar mass and use the definition of mass density.

**Solution** Find the total mass and mass density.

\[ M = (1.00 \, \text{mol})[2(14.00674 \, \text{g/mol})] = 28.0 \, \text{g} \text{ and } \rho = \frac{M}{V} = \frac{28.0 \, \text{g}}{0.0224 \, \text{m}^3} \left( \frac{1 \, \text{kg}}{10^3 \, \text{g}} \right) = 1.25 \, \text{kg/m}^3. \]

39. **Strategy** The number of SiO\(_2\) molecules (and the number of Si atoms) \( N \) is roughly equal to the volume of a sand grain \( V_g \) divided by the volume of a SiO\(_2\) molecule \( V_m \).

**Solution** Find the order of magnitude of the number of silicon atoms in a grain of sand.

\[ N = \frac{V_g}{V_m} \approx \frac{4}{3} \pi r_g^3 = \left( \frac{d_g}{d_m} \right)^3 = \left( \frac{0.5 \times 10^{-3} \, \text{m}}{0.5 \times 10^{-9} \, \text{m}} \right)^3 = 10^{18} \, \text{atoms} \]

40. **Strategy** Use Gay-Lussac’s law.

**Solution** Compute the fractional change in pressure required to increase the cabin temperature from 18\(^\circ\)C to 24\(^\circ\)C.

\[ P \propto T, \text{ so } \frac{P_f - P_i}{P_i} = \frac{T_f - T_i}{T_i} = \frac{24 - 18}{18 + 273.15} = 0.02. \]

41. **Strategy** Use the macroscopic form of the ideal gas law, Eq. (13-16).

**Solution** Find the new temperature of the air.

\[ PV = nRT, \text{ so } \frac{T_f}{T_i} = \frac{P_fV_f}{nRT} = \frac{P_fV_f}{P_iV_i}, \text{ or} \]

\[ T_f = \frac{P_fV_f}{P_iV_i}T_i = \frac{20.0P_i(0.111W_i)T_i}{P_iV_i} = 20.0(0.111)(30 \, \text{K} + 273.15 \, \text{K}) = 673 \, \text{K} = 400\,\text{°C}. \]
42. **Strategy** Use the macroscopic form of the ideal gas law, Eq. (13-16).

**Solution** Find the final volume in terms of the initial volume.

\[ PV = nRT, \quad \text{so} \quad V_f = \frac{nRT}{P_f}, \quad \text{or} \quad V_f = \frac{P_i}{P_f}V_i. \]

The volume outside of the tire is equal to the difference between the total final volume and the initial volume.

\[ V_f - V_i = \frac{P_i}{P_f}V_i - V_i = V_i \left( \frac{P_i}{P_f} - 1 \right) = \left( 0.0250 \, \text{m}^3 \right) \left( \frac{36.0}{14.70} - 1 \right) = 0.0362 \, \text{m}^3 \]

43. **Strategy** Use the macroscopic form of the ideal gas law, Eq. (13-16).

**Solution**

\[ PV = nRT, \quad \text{so} \quad V = \frac{nRT}{P} = \frac{(1.00 \, \text{mol})[8.314 \, \text{J/(mol·K)}](273.15 \, \text{K} + 0.0 \, \text{K})}{(1.00 \, \text{atm})(1.013 \times 10^5 \, \text{Pa/atm})} = 0.0224 \, \text{m}^3. \]

The result is verified.

44. **Strategy** Replace each quantity with its SI units.

**Solution** \( PV \) has SI units \( \text{Pa} \cdot \text{m}^3 = (\text{N}/\text{m}^2) \cdot \text{m}^3 = \text{N} \cdot \text{m} = \text{J} \).

45. **Strategy** The volume and moles of the gas are constant. Use Gay-Lussac’s law.

**Solution** Find the pressure at the higher temperature.

\[ P \propto T, \quad \text{so} \quad P_f = \frac{T_f}{T_i}P_i = \frac{70.0 \, \text{K} + 273.15 \, \text{K}}{20.0 \, \text{K} + 273.15 \, \text{K}} (115 \, \text{kPa}) = 135 \, \text{kPa}. \]

46. **Strategy** The pressure does not change. Use the microscopic form of the ideal gas law, Eq. (13-13).

**Solution** Find the fraction of air molecules that must be pushed outside.

Initially: \( PV = N_i k T_i \)

Finally: \( PV = N_f k T_f \)

The fraction pushed out is \( f = \frac{N_i - N_f}{N_i} = 1 - \frac{N_f}{N_i} = 1 - \frac{T_i}{T_f} = 1 - \frac{16.0 \, \text{K} + 273.15 \, \text{K}}{20.0 \, \text{K} + 273.15 \, \text{K}} = 0.0136 \)

47. **Strategy** Use Eq. (13-10) and the microscopic form of the ideal gas law, Eq. (13-13).

**Solution** Compute the mass density for each temperature.

\[ \rho = \frac{mN}{V} = \frac{mP}{kT} \quad \text{since} \quad PV = NkT. \]

(a) \[ \rho = \frac{(29 \, \text{u})(1.6605 \times 10^{-27} \, \text{kg/u})(1.0 \, \text{atm})(1.013 \times 10^5 \, \text{Pa/atm})}{(1.38 \times 10^{-23} \, \text{J/K})(273.15 \, \text{K} - 10 \, \text{K})} = 1.3 \, \text{kg/m}^3 \]

(b) \[ \rho = \frac{(29 \, \text{u})(1.6605 \times 10^{-27} \, \text{kg/u})(1.0 \, \text{atm})(1.013 \times 10^5 \, \text{Pa/atm})}{(1.38 \times 10^{-23} \, \text{J/K})(273.15 \, \text{K} + 30 \, \text{K})} = 1.2 \, \text{kg/m}^3 \]
48. **Strategy** The volume and moles of the gas are constant. Use Gay-Lussac’s law.

**Solution** Divide the final by the initial pressure.

\[ P \propto T, \quad \frac{P_f}{P_i} = \frac{T_f}{T_i} = \frac{273.15 \text{ K} + 100.0 \text{ K}}{273.15 \text{ K} - 33 \text{ K}} = 1.55 \]

49. **Strategy** The number of moles of the gas is constant. Use the ideal gas law.

**Solution** Find the volume of the hydrogen.

\[ \frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}, \quad V_f = \frac{P_f V_f}{P_i T_i} = \frac{(1.00 \times 10^5 \text{ N/m}^2)(5.0 \text{ m}^3)(273.15 \text{ K} - 13 \text{ K})}{(0.33 \times 10^3 \text{ N/m}^2)(273.15 \text{ K} + 27 \text{ K})} = 1.3 \times 10^3 \text{ m}^3. \]

50. **Strategy** The number of moles of the gas is constant. Use the ideal gas law.

**Solution** Find the new pressure of the ideal gas.

\[ \frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}, \quad P_f = \frac{P_i V_i T_f}{V_i T_i} = \frac{(1.0 \times 10^5 \text{ Pa})(1.2 \text{ m}^3)(273.15 \text{ K} + 227 \text{ K})}{(0.60 \text{ m}^3)(273.15 \text{ K} + 27 \text{ K})} = 3.3 \times 10^5 \text{ Pa} \]

51. **Strategy** The temperature and the number of moles of the gas are constant. Use Boyle’s law and Eq. (9-3).

**Solution** Find the factor by which the diver’s lungs expand.

\[ P \propto \frac{1}{V}, \quad \frac{V_f}{V_i} = \frac{P_i}{P_f} = 1 + \frac{\rho g d}{P_i} = 1 + \frac{(1.03 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(5.0 \text{ m})}{1.013 \times 10^5 \text{ Pa}} = 1.50. \]

52. **Strategy** Use the microscopic form of the ideal gas law, Eq. (13-13).

**Solution** Find the absolute pressure of intergalactic space.

\[ P = \frac{N k T}{V} = \frac{1 \text{ atom}}{\text{cm}^3} (10^6 \text{ cm}^3/\text{m}^3)(1.38 \times 10^{-23} \text{ J/K})(3 \text{ K}) = 4 \times 10^{-17} \text{ Pa} \]

53. **Strategy** Use the microscopic form of the ideal gas law, Eq. (13-13).

**Solution** Find the number of air molecules released.

\[ N = \frac{P V}{k T}, \quad \text{and } V, k, \text{ and } T \text{ are constant, so} \]

\[ \Delta N = \frac{V \Delta P}{k T} = \frac{(1.0 \text{ m}^3)(15.0 \text{ atm} - 20.0 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})}{(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})} = -1.3 \times 10^{26}. \]

1.3 \times 10^{26} \text{ air molecules were released.}

54. **Strategy** Find the number of moles of molecular oxygen by dividing the total mass by the molar mass. Use the macroscopic form of the ideal gas law, Eq. (13-16).

**Solution** Compute the volume occupied by the gas.

\[ V = \frac{nRT}{P} = \frac{(0.532 \text{ kg})(10^3 \text{ g/kg}) \times [8.314 \text{ J/(mol·K)}](0.0 \text{ K} + 273.15 \text{ K})}{2(15.9994 \text{ g/mol}) \times 1.0 \times 10^5 \text{ Pa}} = 0.38 \text{ m}^3 \]

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55. **Strategy** The number of moles of the gas is constant and \( V = \frac{1}{6} \pi d^3 \). Use the ideal gas law and Eq. (9-3).

**Solution** Find the diameter of the bubble when it reaches the surface.

\[
\frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}, \quad \text{so} \quad V_f = \frac{1}{6} \pi d_f^3 = \frac{P_f T_i}{P_i T_f} V_i = \left( \frac{P_f + \rho g h}{P_i T_f} \right) \left( \frac{1}{6} \pi d_i^3 \right)
\]

Solve for \( d_f \).

\[
d_f = d_i \sqrt{\frac{(P_f + \rho g h) T_f}{P_i T_i}}\]

\[
= (1.00 \text{ mm}) \sqrt{\frac{(1.0 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm}) + (1.0 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(80.0 \text{ m})[273.15 \text{ K} + 18 \text{ K}]}{(1.0 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(273.15 \text{ K} + 4 \text{ K})}
\]

\[
= 2.1 \text{ mm}
\]

56. **Strategy** The number of moles of the gas is constant. Use the ideal gas law and Eq. (9-3).

**Solution** Find the volume of the bubble just before it breaks the surface of the water.

\[
\frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}, \quad \text{so} \quad V_f = \frac{(P_f + \rho g d) V_i}{P_i} = \left( \frac{1 + \frac{\rho g d}{P_i}}{P_i} \right) \frac{V_i T_f}{T_i} = \left( \frac{1 + \frac{(1.0 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(20.0 \text{ m})}{1.013 \times 10^5 \text{ Pa}}}{1.013 \times 10^5 \text{ Pa}} \right) \frac{(1.00 \text{ cm}^3)(273.15 \text{ K} + 25.0 \text{ K})}{273.15 \text{ K} + 10.0 \text{ K}}
\]

\[
= 3.09 \text{ cm}^3
\]

57. **Strategy** The number of moles of air is constant. Assume that the temperature is constant.

\[
V_f = \frac{\Delta V}{\Delta t} \quad \text{where} \quad \frac{\Delta V}{\Delta t} = 0.500 \text{ L/s} = 5.00 \times 10^2 \text{ cm}^3/\text{s} \quad \text{and} \quad P_f = P_i + \rho g d. \quad \text{Use Boyle’s law.}
\]

**Solution** Find how long the tank of air will last for each depth.

(a) \[
\frac{V_f}{V_i} = \frac{\Delta V}{\Delta t} = \frac{P_{\text{tank}}}{P_t}, \quad \text{so} \quad \Delta t = \frac{P_{\text{tank}} V_i}{\Delta V (P_\text{atm} + \rho g d)}
\]

\[
= \frac{(1.0 \times 10^7 \text{ Pa})(0.010 \text{ m}^3)}{(5.00 \times 10^2 \text{ cm}^3/\text{s})(10^{-6} \text{ m}^3/\text{cm}^3)(60 \text{ s/min})[1.013 \times 10^5 \text{ Pa} + (1.0 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(2.0 \text{ m})]}
\]

\[
= 28 \text{ min}
\]

(b) \[
\Delta t = \frac{(1.0 \times 10^7 \text{ Pa})(0.010 \text{ m}^3)}{(5.00 \times 10^2 \text{ cm}^3/\text{s})(10^{-6} \text{ m}^3/\text{cm}^3)(60 \text{ s/min})[1.013 \times 10^5 \text{ Pa} + (1.0 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(20.0 \text{ m})]}
\]

\[
= 11 \text{ min}
\]
58. (a) **Strategy** The temperature and the number of moles of the gas are constant. Use Boyle’s law.

**Solution** Find the volume of the oxygen at atmospheric pressure.

\[ P = \frac{1}{V}, \text{ so } V_T = \frac{P_T V_T}{P_t} = \frac{(2200 \text{ lb/in}^2)(0.60 \text{ ft}^3)}{14.70 \text{ lb/in}^2} = 90 \text{ ft}^3. \]

(b) **Strategy** Divide the volume of the oxygen at atmospheric pressure by the volume flow rate.

**Solution** Find how long the cylinder of oxygen will last.

\[ \Delta t = \frac{V}{\Delta V} = \frac{9.0 \times 10^3 \text{ ft}^3}{8 \text{ L/min}} = \frac{1 \text{ L}}{0.0353 \text{ ft}^3} = \frac{1 \text{ h}}{60 \text{ min}} = 5.3 \text{ h}. \]

59. (a) **Strategy** The pressure and number of moles is constant. Use the ideal gas law.

**Solution** Show that \( \Delta V/V_0 = \beta \Delta T \), where \( \beta = 1/T_0 \).

\[ \frac{V}{V_0} = \frac{T}{T_0}, \quad \frac{V_0}{V_0} - 1 = \frac{T}{T_0} - 1, \quad \frac{V - V_0}{V_0} = \frac{T - T_0}{T_0}, \quad \frac{\Delta V}{V_0} = \left( \frac{1}{T_0} \right) \Delta T = \beta \Delta T \]

(b) **Strategy** Use the result of part (a).

**Solution** Compute the coefficient of volume expansion for an ideal gas. Refer to Table 13.2.

\[ \beta_{\text{ideal}} = \frac{1}{T_0} = \frac{1}{273.15 \text{ K} + 20 \text{ K}} = 3410 \times 10^{-6} \text{ K}^{-1} \]

From Table 13.2, \( \beta_{\text{air}} = 3340 \times 10^{-6} \text{ K}^{-1} \) and the range for liquids is \( 182 \times 10^{-6} \text{ K}^{-1} \) for mercury to \( 1240 \times 10^{-6} \text{ K}^{-1} \) for benzene (excepting the negative value for water). So, \( \beta_{\text{ideal}} = \beta_{\text{air}} \) and \( \beta_{\text{ideal}} \) is 3 to 19 times larger than the values for liquids.

60. **Strategy** Use Eq. (13-20).

**Solution** Find the temperature of the ideal gas.

\[ \langle K_T \rangle = \frac{3}{2} kT, \text{ so } T = \frac{2(\langle K_T \rangle)}{3k} = \frac{2(3.20 \times 10^{-20} \text{ J})}{3(1.38 \times 10^{-23} \text{ J/K})} = 1550 \text{ K}. \]

61. **Strategy** The total translational kinetic energy of the gas molecules is equal to the number of molecules times the average translational kinetic energy per molecule. Use Eqs. (13-13) and (13-20).

**Solution** Find the total translational kinetic energy of the gas molecules.

\[ K_{\text{total}} = N \langle K_T \rangle = N \cdot \frac{3}{2} kT = \frac{3}{2} PV = \frac{3}{2} (1.013 \times 10^5 \text{ Pa})(0.00100 \text{ m}^3) = 152 \text{ J} \]
62. **Strategy**  The total translational kinetic energy of the gas molecules is equal to the number of molecules times the average translational kinetic energy per molecule. Use Eqs. (13-13) and (13-20).

**Solution**  Divide the total translational kinetic energy by the volume for each pressure.

\[
\frac{K_{\text{total}}}{V} = \frac{N\langle K_{\text{tr}}\rangle}{V} = \frac{N\left(\frac{3}{2}kT\right)}{V} = \frac{3}{2} \frac{P}{(1.00 \text{ atm})(1.013\times10^5 \text{ Pa/atm})} = 1.52 \times 10^5 \text{ J/m}^3
\]

\[
\frac{K_{\text{total}}}{V} = \frac{3}{2} \frac{300.0 \text{ atm})(1.013\times10^5 \text{ Pa/atm})}{V} = 4.559 \times 10^7 \text{ J/m}^3
\]

63. **Strategy and Solution**  \(\langle v_\text{rms}^2 \rangle = \frac{1}{3} \langle v^2 \rangle = \frac{1}{3} v_{\text{rms}}^2\), and from Eq. (13–17), \(P = \frac{Nm}{V} \langle v_\text{rms}^2 \rangle\). So,

\[
P = \frac{Nm}{V} \left(\frac{1}{3} v_{\text{rms}}^2\right) = \frac{1}{3} \left(\frac{Nm}{V}\right) v_{\text{rms}}^2 = \frac{1}{3} \rho v_{\text{rms}}^2
\]

64. **Strategy**  The speed of sound at 0.0°C and 1.00 atm is 331 m/s. Refer to Fig 13.13.

**Solution**  According to Fig. 13.13, approximately **50%** of the \(O_2\) molecules are moving faster than the speed of sound.

65. **Strategy**  The total internal kinetic energy of the ideal gas is equal to the number of molecules times the average kinetic energy per molecule. Use Eq. (13-20).

**Solution**  Find the total internal kinetic energy of the ideal gas.

\[
K_{\text{total}} = N\langle K_{\text{tr}}\rangle = N\left(\frac{3}{2}kT\right) = \frac{3}{2} nRT = \frac{3}{2} (1.0 \text{ mol})[8.314 \text{ J/(mol·K)}](273.15 \text{ K} + 0.0 \text{ K}) = 3.4 \text{ kJ}
\]

66. **Strategy**  Use the ideal gas law and Eq. (13-22).

**Solution**  Find the temperature of the nitrogen gas.

\[
P = nRT, \text{ so } T = \frac{PV}{nR}.
\]

Compute the rms speed of the nitrogen molecules.

\[
v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kPV}{mnR}} = \sqrt{\frac{3(1.381\times10^{-23} \text{ J/K})(1.6 \text{ atm})(1.013\times10^5 \text{ Pa/atm})(0.25 \text{ m})}{2(14.0 \text{ u})(1.6605\times10^{-27} \text{ kg/u})(2.0 \text{ mol})(8.314 \text{ J/(mol·K)})}} = 370 \text{ m/s}
\]

67. **Strategy**  Use Eq. (13-22).

**Solution**  Find the ratio of the rms speeds.

\[
\frac{v_{\text{rms, Ar}}}{v_{\text{rms, Ne}}} = \sqrt{\frac{m_{\text{Ne}}}{m_{\text{Ar}}}} = \sqrt{\frac{12}{14}} = \frac{1}{\sqrt{2}}
\]

68. **Strategy**  Use Eq. (13-22).

**Solution**  Find the rms speed of the particle.

\[
v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23} \text{ J/K})(273.15 \text{ K} + 27 \text{ K})}{1.38\times10^{-17} \text{ kg}}} = 3.00 \text{ cm/s}
\]
69. **Strategy** Use Eq. (13-22).

**Solution** Find the rms speeds of the molecules.

(a) \( \frac{3kT}{m} = \frac{3(1.38 \times 10^{-23} \text{ J/K})(273.15 \text{ K} + 0 \text{ K})}{2(14.00674 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 493 \text{ m/s} \)

(b) \( \frac{3(1.38 \times 10^{-23} \text{ J/K})(273.15 \text{ K} + 0 \text{ K})}{2(15.99994 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 461 \text{ m/s} \)

(c) \( \frac{3(1.38 \times 10^{-23} \text{ J/K})(273.15 \text{ K} + 0 \text{ K})}{[12.011 \text{ u} + 2(15.99994 \text{ u})](1.66 \times 10^{-27} \text{ kg/u})} = 393 \text{ m/s} \)

70. **Strategy** Use Eq. (13-22).

**Solution** Find the rms speeds of the atom and molecules.

He: \( \frac{3kT}{m} = \frac{3(1.38 \times 10^{-23} \text{ J/K})(273.15 \text{ K} + 25 \text{ K})}{(4.00260 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 1360 \text{ m/s} \)

\( \frac{3(1.38 \times 10^{-23} \text{ J/K})(160 \text{ K})}{2(1.00794 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 515 \text{ m/s} \)

\( \frac{3(1.38 \times 10^{-23} \text{ J/K})(160 \text{ K})}{2(1.00794 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 1920 \text{ m/s} \)

\( \frac{3(1.38 \times 10^{-23} \text{ J/K})(160 \text{ K})}{2(15.99994 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 482 \text{ m/s} \)

71. **Strategy** Compare the rms speeds to the escape speed. Use Eq. (13-22).

**Solution** Find the rms speeds for atomic and molecular hydrogen.

\( \frac{3kT}{m} = \frac{3(1.38 \times 10^{-23} \text{ J/K})(160 \text{ K})}{(1.00794 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 2.0 \text{ km/s} \)

\( \frac{3(1.38 \times 10^{-23} \text{ J/K})(160 \text{ K})}{2(1.00794 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})} = 1.4 \text{ km/s} \)

\( 60 \text{ km/s} > 2.0 \text{ km/s} > 1.4 \text{ km/s}; \) so yes, the astronaut should expect to find lots of hydrogen there.

72. **Strategy** Use Eqs. (13-13) and (13-22).

**Solution** Find the rms speed for an ideal gas.

\( \frac{3kT}{m} = \frac{3k}{m} \left( \frac{PV}{Nk} \right) = \frac{3PV}{Nm} \)

Form a proportion with the initial and final rms speeds and solve for the pressure \( P \).

\( \frac{v_0}{0.90v_0} = \frac{1}{0.90} = \sqrt{\frac{3PV}{Nm}} = \sqrt{\frac{P_0}{P}} \)

so \( P = 0.90^2 P_0 = 0.90^2 (2.0 \text{ atm}) = 1.6 \text{ atm} \).
73. **Strategy** Use Eq. (13-20).

**Solution** Find the temperature of the ideal gas.

\[ \langle K_{tr} \rangle = \frac{3}{2} kT, \text{ so } T = \frac{2\langle K_{tr} \rangle}{3k} = \frac{2(4.60 \times 10^{-20} \text{ J})}{3(1.38 \times 10^{-23} \text{ J/K})} = 2220 \text{ K}. \]

74. **Strategy and Solution** The average translational kinetic energy of a molecule in an ideal gas is \( \langle K_{tr} \rangle = \frac{1}{2} m(v^2) \).

The rms speed is \( v_{rms} = \sqrt{(v^2)} \), so \( \frac{1}{2} m v_{rms}^2 = \langle K_{tr} \rangle \). From Eq. (13-19), we know that \( \langle K_{tr} \rangle = \frac{3PV}{2N} \), and using the ideal gas law, \( PV = NkT \), we have \( \frac{1}{2} m v_{rms}^2 = \frac{3}{2} \left( \frac{PV}{N} \right) = \frac{3}{2} kT \), so \( v_{rms} = \sqrt{\frac{3kT}{m}} \).

75. **Strategy and Solution** The average translational energy of a molecule in an ideal gas is \( \langle K_{tr} \rangle = \frac{1}{2} m(v^2) \). The rms speed is \( v_{rms} = \sqrt{(v^2)} \), so \( \frac{1}{2} m v_{rms}^2 = \langle K_{tr} \rangle \). From Eq. (13-19), we know that \( \langle K_{tr} \rangle = \frac{3PV}{2N} \), and using the ideal gas law, \( PV = nRT \), we have \( \frac{1}{2} m v_{rms}^2 = \frac{3}{2} \left( \frac{PV}{N} \right) = \frac{3}{2} \left( \frac{nRT}{N} \right) \), so \( v_{rms} = \sqrt{\frac{3}{m} \left( \frac{n}{N} \right) RT} = \sqrt{\frac{3RT}{M}} \).

76. **Strategy** Form a proportion with the two reaction rates and solve for the activation energy. Use Eq. (13-24).

**Solution** Find the activation energy.

\[ \frac{1.878}{1} = e^{\frac{E_a}{k \left( \frac{1}{T_1} - \frac{1}{T_2} \right)}} \]

\[ \ln 1.878 = \frac{E_a}{k} \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \]

\[ E_a = \frac{k \ln 1.878}{\frac{1}{T_1} - \frac{1}{T_2}} = \frac{(1.38 \times 10^{-23} \text{ J/K}) \ln 1.878}{\frac{1}{273.15 \text{ K} + 5.0 \text{ K}} - \frac{1}{273.15 \text{ K} + 10.0 \text{ K}}} = 1.4 \times 10^{-19} \text{ J} \]
77. **Strategy** Form a proportion with the two reaction rates and solve for the temperature increase. Use Eq. (13-24).

**Solution** Find the temperature increase.

\[
\frac{1.035}{1} = \frac{e^{\frac{k}{kT_2}}}{e^{\frac{k}{kT_1}}} = e^{\frac{k}{k}(\frac{1}{T_1} - \frac{1}{T_2})}
\]

\[
\ln 1.035 = \frac{E_a}{k} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)
\]

\[
\frac{k \ln 1.035}{E_a} = \frac{1}{T_1} - \frac{1}{T_2} = \frac{1}{k \ln 1.035} E_a
\]

\[
T_2 = \left( \frac{1}{T_1} \frac{k \ln 1.035}{E_a} \right)^{-1}
\]

\[
\Delta T = \left[ \frac{1}{273.15 K + 10.00 K} - \frac{(1.38 \times 10^{-23} \text{ J/K}) \ln 1.035}{2.81 \times 10^{-19} \text{ J}} \right]^{-1} - (273.15 \text{ K} + 10.00 \text{ K}) = 0.14^\circ \text{C}
\]

78. **Strategy** Form a proportion with the two reaction rates and solve for the activation energy. Use Eq. (13-24).

**Solution** Find the activation energy implied by the rule.

\[
\frac{2}{1} = \frac{e^{\frac{k}{kT_2}}}{e^{\frac{k}{kT_1}}} = e^{\frac{k}{k}(\frac{1}{T_1} - \frac{1}{T_2})}
\]

\[
\ln 2 = \frac{E_a}{k} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)
\]

\[
E_a = \frac{k \ln 2}{\left( \frac{1}{T_1} - \frac{1}{T_2} \right)} = \frac{(1.38 \times 10^{-23} \text{ J/K}) \ln 2}{273.15 \text{ K} + 90.0 \text{ K} - 273.15 \text{ K} + 100.0 \text{ K}} = 1.3 \times 10^{-19} \text{ J}
\]

79. **Strategy** Use Eqs. (13-13) and (13-25).

**Solution** Estimate the mean free path for a nitrogen molecule is each situation.

(a) \[
\Lambda = \frac{1}{\sqrt{2\pi d^2}} \frac{N}{\tau} = \frac{1}{\sqrt{2\pi d^2}} \frac{\frac{P}{kT}}{\sqrt{2\pi d^2}} = \frac{kT}{\sqrt{2\pi d^2} P} = \frac{(1.38 \times 10^{-23} \text{ J/K})(290 \text{ K})}{\sqrt{2\pi}(3 \times 10^{-10} \text{ m})^2(1.0 \times 10^5 \text{ Pa})} = 100 \text{ nm}
\]

(b) \[
\Lambda = \frac{(1.38 \times 10^{-23} \text{ J/K})(230 \text{ K})}{\sqrt{2\pi}(3 \times 10^{-10} \text{ m})^2(5.0 \times 10^4 \text{ Pa})} = 200 \text{ nm}
\]

(c) \[
\Lambda = \frac{(1.38 \times 10^{-23})(230 \text{ K})}{\sqrt{2\pi}(3 \times 10^{-10} \text{ m})^2(1 \times 10^3 \text{ Pa})} = 8 \text{ \mu m}
\]
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80. **Strategy** Use Eq. (13-26).

**Solution** Find the time for the perfume molecule to diffuse 5.00 m in one direction.

\[ x_{\text{rms}} = \sqrt{2D} \text{, so } t = \frac{x_{\text{rms}}^2}{2D} = \frac{(5.00 \text{ m})^2}{2(1.00 \times 10^{-5} \text{ m}^2/\text{s})} = 1.25 \times 10^6 \text{ s}. \]

81. **Strategy** Use Eq. (13-26).

**Solution** Estimate the time it takes a sucrose molecule to move 5.00 mm in one direction.

\[ x_{\text{rms}} = \sqrt{2D} \text{, so } t = \frac{x_{\text{rms}}^2}{2D} = \frac{(5.00 \times 10^{-3} \text{ m})^2}{2(5.0 \times 10^{-10} \text{ m}^2/\text{s})} = 2.5 \times 10^4 \text{ s}. \]

82. **Strategy** Use Eq. (13-26). Use a proportion to find the time.

**Solution** Find the time for a perfume molecule to diffuse 6.0 m in one direction.

\[ x_{\text{rms}} = \sqrt{2D} \text{, so } t = \frac{x_{\text{rms}}^2}{2D} \text{ and } t = \frac{x_{\text{rms}}^2}{x_{\text{rms}}^2} t = \frac{6.0 \text{ m}}{(6.0 \text{ m})^2} (20 \text{ s}) = 80 \text{ s}. \]

83. **Strategy** Use Eq. (10-10) for the change in pressure and Eq. (13-7) for the fractional change in volume of the gasoline.

**Solution** The gasoline expands outward against the can, so the negative sign in Eq. (10-10) is inappropriate.

\[ \Delta P = P_2 - P_1 = B \frac{\Delta V}{V_0} = B \beta \Delta T, \text{ so} \]

\[ P_2 = P_1 + B \frac{\Delta V}{V_0} = 1.0 \text{ atm} + (1.00 \times 10^9 \text{ N/m}^2)(950 \times 10^{-6} \text{ K}^{-1})(15.0 \text{ K}) \left( \frac{1 \text{ atm}}{1.013 \times 10^5 \text{ Pa}} \right) = 140 \text{ atm}. \]

84. **Strategy** Use Eqs. (10-4) and (13-4).

**Solution** Find the stress.

\[ \text{stress} = Y \frac{\Delta L}{L_0} = Y \alpha \Delta T = (2.0 \times 10^{11} \text{ N/m}^2)(12 \times 10^{-6} \text{ K}^{-1})(20.0 \text{ K}) = 4.8 \times 10^7 \text{ N/m}^2. \]

85. **Strategy** Find the temperature at which the radius of the steel sphere and the internal radius of the brass ring are the same. Use Eq. (13-5).

**Solution** Set the final lengths of the radii equal and solve for the final temperature.

\[ L_{b0} + L_{b0} \alpha_b \Delta T = L_{s0} + L_{s0} \alpha_s \Delta T \]

\[ L_{b0} \alpha_b (T_f - T_i) = L_{s0} - L_{b0} + L_{s0} \alpha_s (T_f - T_i) \]

\[ T_f (L_{b0} \alpha_b - L_{s0} \alpha_s) = L_{s0} - L_{b0} + T_i (L_{b0} \alpha_b - L_{s0} \alpha_s) \]

\[ T_f = \frac{L_{s0} - L_{b0} + T_i}{L_{b0} \alpha_b - L_{s0} \alpha_s} \]

\[ T_f = \frac{1.0010 \text{ cm} - 1.0000 \text{ cm}}{(1.0000 \text{ cm})(19 \times 10^{-6} \text{ K}^{-1}) - (1.0010 \text{ cm})(12 \times 10^{-6} \text{ K}^{-1})} + 22.0^\circ \text{C} = 165^\circ \text{C} \]
86. (a) **Strategy** Sum the molar masses of the elements that make up the molecule.

**Solution** Find the molar mass of oleic acid.

\[
18(12.011 \text{ g/mol}) + 34(1.00794 \text{ g/mol}) + 2(15.9994 \text{ g/mol}) = 282.47 \text{ g/mol}
\]

(b) **Strategy** Divide the mass by the molar mass.

**Solution** Find the number of moles of oleic acid in one drop.

\[
\frac{2.3 \times 10^{-5} \text{ g}}{282.47 \text{ g/mol}} = 8.1 \times 10^{-8} \text{ mol}
\]

(c) **Strategy** The volume is equal to the area times the height of the spread-out drop.

**Solution** Find \(d\).

\[
V = Ah = A(7d), \text{ so } d = \frac{V}{7A} = \frac{2.6 \times 10^{-5} \text{ cm}^3}{7(70.0 \text{ cm}^2)} = 5.3 \times 10^{-8} \text{ cm}
\]

(d) **Strategy** Divide the total area by the area of one drop.

**Solution** Find the number of oleic acid molecules in one drop.

\[
\frac{\text{total area}}{\text{area of one molecule}} = \frac{70.0 \text{ cm}^2}{(5.3 \times 10^{-8} \text{ cm})^2} = 2.5 \times 10^{16} \text{ molecules}
\]

(e) **Strategy** Divide the number of molecules by the number of moles.

**Solution** Estimate Avogadro’s number.

\[
\frac{2.5 \times 10^{16}}{8.1 \times 10^{-8} \text{ mol}} = 3.1 \times 10^{23} \text{ mol}^{-1}
\]

87. **Strategy** The relative number of atoms of an element contained within the molecule is equal to the molecular mass of the molecule times the percentage of the molecular mass of the molecule that is the mass of the atoms of element divided by the molecular mass of the element.

**Solution** Find the chemical formula.

\[
\begin{align*}
(63 \text{ u})(0.016) &= 1.0 \text{ H}; \quad (63 \text{ u})(0.222) = 1.0 \text{ N}; \quad (63 \text{ u})(0.762) = 3.0 \text{ O} \\
1.00794 \text{ u} & \quad 14.00674 \text{ u} & \quad 15.9994 \text{ u}
\end{align*}
\]

The chemical formula is \([\text{HNO}_3]\).
88. **Strategy** Plot the pressure on the vertical axis and the temperature on the horizontal axis. Estimate the value of absolute zero by drawing a best-fit line through the data points and finding the temperature at which the line intersects the $T$-axis (where the pressure is zero).

**Solution** The graph is shown.

According to the graph, absolute zero is approximately $-270^\circ C$.

89. (a) **Strategy** Use Eq. (13-20).

**Solution** Compute the average kinetic energy of the air molecules.

$$\langle K_r \rangle = \frac{3}{2} kT = \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K}) \left[ \frac{5}{9} (98.6 - 32.0) \text{ K} + 273.15 \text{ K} \right] = 6.42 \times 10^{-21} \text{ J}$$

(b) **Strategy** Since the average kinetic energy is directly proportional to the absolute temperature, find the percent change in the absolute temperature.

**Solution** Find the percentage by which the kinetic energy of the molecules increased.

$$\frac{\Delta K}{K_0} \times 100\% = \frac{\Delta T}{T_0} \times 100\% = \frac{\frac{5}{9} (100.0 - 98.6) \text{ K}}{\frac{5}{9} (98.6 - 32.0) \text{ K} + 273.15 \text{ K}} \times 100\% = 0.25\%$$

90. **Strategy** Use the ideal gas law with the number of moles constant to find $V_a$, the volume of air at standard temperature and pressure.

**Solution** Find the number of breaths taken per day.

$$\frac{P_a V_a}{T_a} = \frac{P_{L} V_{L}}{T_{L}}$$

so $V_a = \frac{P_{L} T_a V_{L}}{P_{L} T_{L}}$.

$$V_a = \frac{(450 \text{ mm Hg})(133.3 \text{ Pa/mm Hg})(0 \text{ K} + 273.15 \text{ K})(100 \text{ cm}^3)}{(4000 \text{ breaths})}$$

$$= 4000 \text{ breaths}$$
91. **Strategy** Use the ideal gas law.

**Solution**

(a) Find the number of moles in terms of the pressure.

\[ PV = nRT, \text{ so } n = \frac{PV}{RT}. \]

Find the percent change in the number of moles of air in the cabin.

\[ \frac{n_f - n_i}{n_i} \times 100\% = \frac{P_fV_f}{RT} - \frac{P_iV_i}{RT} \times 100\% = \frac{P_f - P_i}{P_i} \times 100\% = \frac{7.62 \times 10^4 \text{ Pa} - 1.01 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \times 100\% = -25\% \]

The number of moles decreases by 25%.

(b) Find the final temperature in terms of the pressure.

\[ \frac{T_f}{T_i} = \frac{P_f}{P_i} \frac{V_i}{nR}, \text{ so } T_f = \frac{P_f}{P_i} T_i = \frac{7.62 \times 10^4 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}} \frac{25.0 \text{ K} + 273.15 \text{ K}}{225 \text{ K}} = 225 \text{ K} = -48^\circ \text{C} \]

92. **Strategy** The extra volume is equal to the difference between the final and initial volumes. Let the initial radius be of a cannonball be \( r_0 \) and the final radius be \( r_0 + \Delta r \). Use Eq. (13-4).

**Solution** Find the extra volume required.

\[ V_{\text{extra}} = 500 \left( \frac{4}{3} \pi \right) [(r_0 + \Delta r)^3 - r_0^3] = 500 \left( \frac{4}{3} \pi \right) r_0^3 \left[ 1 + \frac{\Delta r}{r_0} \right]^3 - 1 = 500 \left( \frac{4}{3} \pi \right) r_0^3 [(1 + \alpha \Delta T)^3 - 1] \]

\[ = 500 \left( \frac{4}{3} \pi \right) (0.05 \text{ m})^3 \left[ 1 + (12 \times 10^{-6} \text{ K}^{-1})(8 \text{ K})^3 - 1 \right] = 80 \text{ cm}^3 \]

93. **Strategy and Solution** The average of the test scores is \( \frac{83 + 62 + 81 + 77 + 68 + 92 + 88 + 83 + 72 + 75}{10} = 78.1 \) The rms value is \( \sqrt{\frac{83^2 + 62^2 + 81^2 + 77^2 + 68^2 + 92^2 + 88^2 + 83^2 + 72^2 + 75^2}{10}} = 78.6 \). The most probable value is 83, since it appears twice as often as any other score.

94. **Strategy** Use the ideal gas law. Let \( L \) be the length of the cylinder and \( d \) be the distance the piston must be pushed down before air will flow into the tire.

**Solution** The volume of the pump just as air begins to flow into the tire is \( V_f = A(L - d) \).

\[ PV = nRT, \text{ so } V = \frac{nRT}{P}. \]

Find \( d \).

\[ \frac{V_f}{V_i} = \frac{nRT}{P_i} = \frac{P_f}{P_i}, \text{ so } V_f = A(L - d) = AL - Ad = \frac{P_f}{P_i} V_i = \frac{P_f}{P_i} AL, \text{ or} \]

\[ d = L \left( 1 - \frac{P_i}{P_f} \right) = (18.0 \text{ in}) \left( 1 - \frac{14.70}{14.70 + 40.0} \right) = 13.2 \text{ inches}. \]
95. (a) **Strategy** The slope of a graph is rise over run.

**Solution** Find the slope of pressure versus temperature.

\[
\frac{\Delta P}{\Delta T} = \frac{8.00 \text{ mm}}{20.0^\circ \text{C}} = 0.400 \text{ mm Hg/}^\circ \text{C}
\]

(b) **Strategy** Use the ideal gas law and the result of part (a).

**Solution** Find the number of moles of gas present.

\[
\frac{(\Delta P)V}{R \Delta T} = \frac{0.500 \text{ L} \times (0.400 \text{ mm Hg/}^\circ \text{C}) \times 10^{-3} \text{ m}^3 \times 1 \text{ L}}{8.314 \text{ J/(mol} \cdot \text{K})}(1.333 \times 10^2 \text{ Pa/mm Hg}) = 3.21 \times 10^{-3} \text{ mol}
\]

96. **Strategy** Use the ideal gas law.

**Solution**

(a) The top pushes on the gas with a force of magnitude \( F = mg \). So, the total pressure of the gas at equilibrium is \( P_{\text{total}} = \frac{mg}{A} + P_{\text{atm}} \).

Find the volume of the gas.

\[
P V = n R T, \quad V = \frac{n R T}{P} = \frac{n R T}{\frac{mg}{A} + P_{\text{atm}}} = \frac{(2.25 \times 10^{-3} \text{ mol}) \times (8.314 \text{ J/(mol} \cdot \text{K}) \times (23.0 \text{ K} + 273.15 \text{ K})}{5.40 \text{ kg} \times (9.80 \text{ m/s}^2) \times 70.0 \times 10^{-4} \text{ m}^2} + 1.013 \times 10^5 \text{ Pa} = 50.9 \text{ cm}^3.
\]

(b) When the gas is again in equilibrium, the new pressure is the same as the old.

\[
\frac{V_f}{V_i} = \frac{n R T_f}{P} = \frac{223.0 + 273.15}{23.0 + 273.15} = 1.675
\]

The volume of the gas increases by a factor of 1.675.

97. **Strategy** Assume that each air molecule is at the center of a sphere (with volume \( V/N \)) of diameter \( d \). Then the average distance between air molecules is approximately \( d \). Use the microscopic form of the ideal gas law, Eq. (13-13).

**Solution** Estimate the average distance between air molecules.

\[
P V = P \left( \frac{N}{6} \frac{1}{\pi d^3} \right) = N k T, \quad \text{so } d = \left( \frac{6 k T}{\pi P} \right)^{1/3} = \left[ \frac{6(1.38 \times 10^{-23} \text{ J/K})(273.15 \text{ K} + 0.0 \text{ K})}{\pi(1.00 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})} \right]^{1/3} = 4 \text{ nm}
\]
98. **(a) Strategy** Find the distance between N\textsubscript{2} molecules and multiply by the scale factor
\[
\frac{0.0375 \text{ m}}{0.30 \times 10^{-9} \text{ m}} = 1.25 \times 10^8
\]
to get the distance between ping pong balls. Let each N\textsubscript{2} molecule be at the center of a sphere with diameter \(d\). Then the average distance between N\textsubscript{2} molecules is approximately \(d\). The volume of each sphere is \(V/N\) with \(V = 0.0224 \text{ m}^3\) for \(N = 6.02 \times 10^{23}\) molecules, \(P = 1.00 \text{ atm}\), and \(T = 0.0^\circ \text{C}\).

**Solution** Find the average distance between the ping-pong balls.
\[
\frac{V}{N} = \frac{1}{6} \pi d^3, \text{ so } d = \sqrt[3]{\frac{6V}{\pi N}}.
\]
Therefore, the distance between ping-pong balls is \( (1.25 \times 10^8) \sqrt[3]{\frac{6(0.0224 \text{ m}^3)}{\pi (6.02 \times 10^{23})}} = 52 \text{ cm} \).

**(b) Strategy** Find the mean free path for an N\textsubscript{2} molecule and multiply by the scale factor to find the average distance between ping pong ball collisions. Use Eq. (13-25) and the results of part (a).

**Solution** Find the mean free path.
\[
\Lambda = \frac{kT}{\sqrt{2\pi d^2 N/V}} = \frac{kT}{\sqrt{2\pi d^2 P}}
\]
So, the average distance between ping pong ball collisions is
\[
\frac{(1.25 \times 10^8)(1.38 \times 10^{-23} \text{ J/K})(273.15 \text{ K})}{(0.30 \times 10^{-9} \text{ m})^2(1.00 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})} = 12 \text{ m}.
\]

99. **(a) Strategy** Use Eq. (13-10) to find the number of air molecules per m\(^3\); multiply by 0.21 to find the number of O\textsubscript{2} molecules.

**Solution** Find the number of oxygen molecules per cubic meter.
\[
\frac{N_{\text{O}_2}}{V} = \frac{0.21 \rho_{\text{air}}}{m_{\text{air}}} = \frac{0.21(1.20 \times 10^3 \text{ g/m}^3)(6.022 \times 10^{23} \text{ mol}^{-1})}{0.78(2)(14.00674 \text{ g/mol}) + 0.21(2)(15.9994 \text{ g/mol}) + 0.01(39.948 \text{ g/mol})}
\]
\[
= 5.2 \times 10^{24} \text{ m}^{-3}
\]

**(b) Strategy** The ratio of the surface volume of air to the volume of the air at a depth of 100.0 m multiplied by 21% gives the percentage of O\textsubscript{2} molecules. Use the ideal gas law with \(N\), \(k\), and \(T\) constant and Eq. (9-3).

**Solution** Find the appropriate percentage of oxygen molecules in the tank.
\[
\frac{V_2}{V_1} = \frac{21%}{21%} = \frac{P_{\text{atm}}}{P_{\text{atm}} + \rho gh} = 1.9%
\]

100. **Strategy** Use Eq. (13-22). Form a proportion.

**Solution** Show that, in two gases at the same temperature, the rms speeds are inversely proportional to the square root of the molecular masses.
\[
(v_{\text{rms}})_1 = \frac{3kT}{m_1} \text{ and } (v_{\text{rms}})_2 = \frac{3kT}{m_2}, \text{ so } (v_{\text{rms}})_1 = \frac{\sqrt{3kT/m_1}}{\sqrt{m_2}} = \frac{\sqrt{m_2}}{\sqrt{m_1}}.
\]

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101. **Strategy** Use Eq. (13-4).

**Solution** Find the approximate temperature for the SR-71 while it is in flight.

\[ \frac{\Delta L}{L_0} = \alpha \Delta T, \text{ so } T_f = T_i + \frac{\Delta L}{\alpha L_0} = 20^\circ C + \frac{0.20 \text{ m}}{(10.1 \times 10^{-6} \text{ K}^{-1})(32.70 \text{ m})} = 630^\circ C. \]

102. **Strategy** Use Eq. (13-26).

**Solution** Find the time it takes for a water molecule to diffuse out through the pore.

\[ x_{\text{rms}} = \sqrt{2D_t}, \text{ so } t = \frac{x_{\text{rms}}^2}{2D} = \frac{(2.5 \times 10^{-5} \text{ m})^2}{2(2.4 \times 10^{-5} \text{ m}^2/\text{s})} = 1.3 \times 10^{-5} \text{ s}. \]

103. **Strategy** Use the microscopic form of the ideal gas law, Eq. (13-13).

**Solution** Find the number of air molecules \( N \).

\[ NkT = PV, \text{ so } N = \frac{PV}{kT} = \frac{(1.00 \times 10^5 \text{ Pa})(0.125 \times 10^{-3} \text{ m})^3}{(1.38 \times 10^{-23} \text{ J/K})(310 \text{ K})} = 1.9 \times 10^{14} \text{ molecules}. \]

104. (a) **Strategy** If \( V = 10.0 \text{ L} \) and \( N \) is the number of \( \text{N}_2 \) molecules, \( V/N \) is approximately the volume of a sphere centered on one molecule with diameter \( d \). \( d \) is approximately the nearest-neighbor distance.

**Solution** Estimate the nearest-neighbor distance.

\[ \frac{1}{6} \pi d^3 = \frac{V}{N}, \text{ so } d = \left( \frac{6V}{\pi N} \right)^{1/3} = \left[ \frac{6(10.0 \text{ L})(10^{-3} \text{ m}^3/\text{L})}{(12 \text{ g})(6.022 \times 10^{23} \text{ mol}^{-1})} \right]^{1/3} = 4.2 \text{ nm}. \]

(b) **Strategy and Solution** Since the nearest neighbor distance is significantly larger than the diameter of an \( \text{N}_2 \) molecule (4.2 nm/0.3 nm = 14), the gas is dilute.

105. **Strategy** Use Eq. (13-22).

**Solution** Find the decrease in the rms speed of the air molecules.

\[ \Delta v_{\text{rms}} = \sqrt{\frac{3kT_f}{m} - \frac{3kT_i}{m}} \]

\[ = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(10.0 \text{ K} + 273.15 \text{ K})}{0.750(2)(14.00674 \text{ u}) + 0.250(2)(15.9994 \text{ u})}(1.66 \times 10^{-27} \text{ kg/u})} - \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(40.0 \text{ K} + 273.15 \text{ K})}{0.750(2)(14.00674 \text{ u}) + 0.250(2)(15.9994 \text{ u})}(1.66 \times 10^{-27} \text{ kg/u})} \]

\[ = -25 \text{ m/s}. \]

The rms speed will have decreased by 25 m/s.
106. **Strategy** Use Eqs. (13-4) and (13-5).

**Solution**

(a) Find $T$ such that $\Delta L = 0.00200$ cm.

\[
\Delta L = \alpha L_0 \Delta T, \ \text{so} \ \ T = \frac{\Delta L}{\alpha L_0} + T_0 = \frac{0.00200 \text{ cm}}{(12 \times 10^{-6} \text{ K}^{-1})(7.00000 \text{ cm})} + 20.0^\circ C = 44^\circ C
\]

(b) Set the diameters ($L_s$ and $L_b$) equal and solve for $T$.

\[
T = T_0 + \frac{L_{b0} - L_0}{L_{s0} \alpha - L_{s0} \alpha_b} = 20.0^\circ C + \frac{7.00200 \text{ cm} - 7.00000 \text{ cm}}{(7.00000 \text{ cm})(12 \times 10^{-6} \text{ K}^{-1}) - (7.00200 \text{ cm})(19 \times 10^{-6} \text{ K}^{-1})} = -21^\circ C
\]

107. **Strategy** The increase in volume of the mercury minus the increase in volume of the glass bulb equals the volume of mercury that moves up the tube. Use Eqs. (13-6) and (13-7).

**Solution** Find the how far the thread of mercury moves $h$.

\[
\Delta V_{Hg} - \Delta V_g = A_{tube} h
\]

\[
V_0 \beta_{Hg} \Delta T - V_0 \beta_g \Delta T = (A_0 + 2 \alpha_g A_0 \Delta T) h
\]

\[
h = \frac{V_0 (\beta_{Hg} - \beta_g) \Delta T}{A_0 (1 + 2 \alpha_g \Delta T)} = \frac{(0.200 \times 10^{-6} \text{ m}^3)[(182 - 9.75) \times 10^{-6} \text{ K}^{-1}][1.00 \text{ K}]}{\frac{1}{4} \pi (0.120 \times 10^{-3} \text{ m})^2 [1 + 2(3.25 \times 10^{-6} \text{ K}^{-1})(1.00 \text{ K})]} = 3.05 \text{ mm}
\]

108. (a) **Strategy** Use Eq. (13-4).

**Solution** Find the temperature to which the band must be heated.

\[
\frac{\Delta L}{L_0} = \alpha \Delta T, \ \text{so} \ \ T = T_0 + \frac{\Delta L}{\alpha L_0} = 20.0^\circ C + \frac{134.460 \text{ cm} - 134.448 \text{ cm}}{(12 \times 10^{-6} \text{ K}^{-1})(134.448 \text{ cm})} = 27.4^\circ C.
\]

(b) **Strategy** Use Eq. (10-4).

**Solution** Find the tension in the band when it cools.

\[
\frac{F}{A} = y \frac{\Delta L}{L_0}, \ \text{so} \ \ F = A Y \frac{\Delta L}{L_0} = (0.0500 \text{ m})(0.00500 \text{ m})(2.0 \times 10^{11} \text{ N/m}^2) \frac{134.460 \text{ cm} - 134.448 \text{ cm}}{134.448 \text{ cm}} = 4.5 \text{ kN}.
\]
109. **Strategy** Use ideal gas law and Hooke’s law.

**Solution** Find the pressure of the gas.

\[ PV = nRT, \text{ so } P_{gas} = \frac{nRT}{V}. \]

The force with which the piston pushes on the spring is equal to

\[ F = (P_{gas} - P_{atm})A_{piston}. \]

Set this equal to \( F = k\Delta x \) to find the spring constant.

\[ k\Delta x = (P_{gas} - P_{atm})A_{piston} = \left( \frac{nRT}{V} - P_{atm} \right)A_{piston}, \text{ so} \]

\[ k = \left[ \frac{(6.50 \times 10^{-2} \text{ mol})(8.314 \text{ J/(mol K)})(20.0 \text{ K} - 273.15 \text{ K})}{(0.120 \text{ m} + 0.0540 \text{ m}) \pi (0.0800/2 \text{ m})^2} - 1.013 \times 10^5 \text{ Pa} \right] \frac{(0.0800/2 \text{ m})^2}{0.0540 \text{ m}} = 7.4 \times 10^3 \text{ N/m}. \]

110. **Strategy** Use Eq. (13-4) and the formula for arc length.

**Solution**

(a) Find the relation for the radius of the strip.

\[ \frac{\Delta L_1}{L_0} = \alpha_1 \Delta T \quad \text{and} \quad \frac{\Delta L_2}{L_0} = \alpha_2 \Delta T, \quad \alpha_2 > \alpha_1. \]

\[ L_0 + \Delta L_1 = R\theta \quad \text{and} \quad L_0 + \Delta L_2 = (R + s)\theta \quad (\text{arc length}). \]

Solve for \( R \).

\[ L_0 + \Delta L_2 = (R + s) \frac{L_0 + \Delta L_1}{R} \]

\[ R(L_0 + L_0\alpha_2 \Delta T) = (R + s)(L_0 + L_0\alpha_1 \Delta T) \]

\[ R(1 + \alpha_2 \Delta T) = R(1 + \alpha_1 \Delta T) + s(1 + \alpha_1 \Delta T) \]

\[ R(\alpha_2 - \alpha_1) \Delta T = s(1 + \alpha_1 \Delta T) \]

\[ R = \frac{s}{(\alpha_2 - \alpha_1) \Delta T} \quad (\alpha_1 \Delta T << 1) \]

(b) Compute the radius.

\[ R = \frac{0.1 \times 10^{-3} \text{ m}}{[(19 - 12) \times 10^{-6} \text{ K}^{-1}](20.0 \text{ K})} = 0.7 \text{ m} \]