**Discussion Questions**

Q15.1 Two waves travel on the same string. Is it possible for them to have (a) different frequencies; (b) different wavelengths; (c) different speeds; (d) different amplitudes; (e) the same frequency but different wavelengths? Explain your reasoning.

Q15.2 Under a tension $F$, it takes 2.00 s for a pulse to travel the length of a taut wire. What tension is required (in terms of $F$) for the pulse to take 6.00 s instead?

Q15.3 What kinds of energy are associated with waves on a stretched string? How could you detect such energy experimentally?

Q15.4 The amplitude of a wave decreases gradually as the wave travels down a long, stretched string. What happens to the energy of the wave when this happens?

Q15.5 For the wave motions discussed in this chapter, does the speed of propagation depend on the amplitude? What makes you say this?

Q15.6 The speed of ocean waves depends on the depth of the water, the deeper the water, the faster the wave travels. Use this to explain why ocean waves crest and “break” as they near the shore.

Q15.7 Is it possible to have a longitudinal wave on a stretched string? Why or why not? Is it possible to have a transverse wave on a steel rod? Again, why or why not? If your answer is yes in either case, explain how you would create such a wave.

Q15.8 An echo is sound reflected from a distant object, such as a wall or a cliff. Explain how you can determine how far away the object is by timing the echo.

Q15.9 Why do you see lightning before you hear the thunder? A familiar rule of thumb is to start counting slowly, once per second, when you see the lightning; when you hear the thunder, divide the number you have reached by 3 to obtain your distance from the lightning in kilometers (or divide by 5 to obtain your distance in miles). Why does this work, or does it?

Q15.10 For transverse waves on a string, is the wave speed the same as the speed of any part of the string? Explain the difference between these two speeds. Which one is constant?

Q15.11 Children make toy telephones by sticking each end of a long string through a hole in the bottom of a paper cup and knotting it so it will not pull out. When the spring is pulled taut, sound can be transmitted from one cup to the other. How does this work? Why is the transmitted sound louder than the sound traveling through air for the same distance?

Q15.12 The four strings on a violin have different thicknesses, but are all under approximately the same tension. Do waves travel faster on the thick strings or the thin strings? Why? How does the fundamental vibration frequency compare for the thick versus the thin strings?

Q15.13 A sinusoidal wave can be described by a cosine function, which is negative just as often as positive. So why isn’t the average power delivered by this wave zero?

Q15.14 Two strings of different mass per unit length $\mu_1$ and $\mu_2$ are tied together and stretched with a tension $F$. A wave travels...
along the string and passes the discontinuity in $\mu$. Which of the following wave properties will be the same on both sides of the discontinuity, and which ones will change? speed of the wave; frequency; wavelength. Explain the physical reasoning behind each of your answers.

Q15.15 A long rope with mass $m$ is suspended from the ceiling and hangs vertically. A wave pulse is produced at the lower end of the rope, and the pulse travels up the rope. Does the speed of the wave pulse change as it moves up the rope, and if so, does it increase or decrease?

Q15.16 In a transverse wave on a string, the motion of the string is perpendicular to the length of the string. How, then, is it possible for energy to move along the length of the string?

Q15.17 Both wave intensity and gravitation obey inverse-square laws. Do they do so for the same reason? Discuss the reason for each of these inverse-square laws as well as you can.

Q15.18 Energy can be transferred along a string by wave motion. However, in a standing wave on a string, no energy can ever be transferred past a node. Why not?

Q15.19 Can a standing wave be produced on a string by superposing two waves traveling in opposite directions with the same frequency but different amplitudes? Why or why not? Can a standing wave be produced by superposing two waves traveling in opposite directions with different frequencies but the same amplitude? Why or why not?

Q15.20 If you stretch a rubber band and pluck it, you hear a (somewhat) musical tone. How does the frequency of this tone change as you stretch the rubber band further? (Try it!) Does this agree with Eq. (15.35) for a string fixed at both ends? Explain.

Q15.21 A musical interval of an octave corresponds to a factor of $2$ in frequency. By what factor must the tension in a guitar or violin string be increased to raise its pitch one octave? To raise it two octaves? Explain your reasoning. Is there any danger in attempting these changes in pitch?

Q15.22 By touching a string lightly at its center while bowing, a violinist can produce a note exactly one octave above the note to which the string is tuned—that is, a note with exactly twice the frequency. Why is this possible?

Q15.23 As we discussed in Section 15.1, water waves are a combination of longitudinal and transverse waves. Defend the following statement: “When water waves hit a vertical wall, the wall is a node of the longitudinal displacement but an antinode of the transverse displacement.”

Q15.24 Violins are short instruments, while cellos and basses are long. In terms of the frequency of the waves they produce, explain why this is so.

Q15.25 What is the purpose of the frets on a guitar? In terms of the frequency of the vibration of the strings, explain their use.

**EXERCISES**

**Section 15.2 Periodic Waves**

15.1 • The speed of sound in air at 20°C is 344 m/s. (a) What is the wavelength of a sound wave with a frequency of 784 Hz, corresponding to the note G₃ on a piano, and how many milliseconds does each vibration take? (b) What is the wavelength of a sound wave one octave higher than the note in part (a)?

15.2 • BIO Audible Sound. Provided the amplitude is sufficiently great, the human ear can respond to longitudinal waves over a range of frequencies from about 20.0 Hz to about 20.0 kHz. (a) If you were to mark the beginning of each complete wave pattern with a red dot for the long-wavelength sound and a blue dot for the short-wavelength sound, how far apart would the red dots be, and how far apart would the blue dots be? (b) In reality would adjacent dots in each set be far enough apart for you to easily measure their separation with a meter stick? (c) Suppose you repeated part (a) in water, where sound travels at 1480 m/s. How far apart would the dots be in each set? Could you readily measure their separation with a meter stick?

15.3 • Tsunami! On December 26, 2004, a great earthquake occurred off the coast of Sumatra and triggered immense waves (tsunami) that killed some 200,000 people. Satellites observing these waves from space measured 800 km from one wave crest to the next and a period between waves of 1.0 hour. What was the speed of these waves in m/s and in km/h? Does your answer help you understand why the waves caused such devastation?

15.4 • BIO Ultrasound Imaging. Sound having frequencies above the range of human hearing (about 20,000 Hz) is called ultrasound. Waves above this frequency can be used to penetrate the body and to produce images by reflecting from surfaces. In a typical ultrasound scan, the waves travel through body tissue with a speed of 1500 m/s. For a good, detailed image, the wavelength should be no more than 1.0 mm. What frequency sound is required for a good scan?

15.5 • BIO (a) Audible wavelengths. The range of audible frequencies is from about 20 Hz to 20,000 Hz. What is the range of the wavelengths of audible sound in air? (b) Visible light. The range of visible light extends from 400 nm to 700 nm. What is the range of visible frequencies of light? (c) Brain surgery. Surgeons can remove brain tumors by using a cavitron ultrasonic surgical aspirator, which produces sound waves of frequency 23 kHz. What is the wavelength of these waves in air? (d) Sound in the body. What would be the wavelength of the sound in part (c) in bodily fluids in which the speed of sound is 1480 m/s but the frequency is unchanged?

15.6 • A fisherman notices that his boat is moving up and down periodically, owing to waves on the surface of the water. It takes 2.5 s for the boat to travel from its highest point to its lowest, a total distance of 0.62 m. The fisherman sees that the wave crests are spaced 6.0 m apart. (a) How fast are the waves traveling? (b) What is the amplitude of each wave? (c) If the total vertical distance traveled by the boat were 0.30 m but the other data remained the same, how would the answers to parts (a) and (b) be affected?

**Section 15.3 Mathematical Description of a Wave**

15.7 • Transverse waves on a string have wave speed 8.00 m/s, amplitude 0.0700 m, and wavelength 0.320 m. The waves travel in the $-x$-direction, and at $t = 0$ the $x = 0$ end of the string has its maximum upward displacement. (a) Find the frequency, period, and wavelength of these waves. (b) Write a wave function describing the wave. (c) Find the transverse displacement of a particle at $x = 0.360$ m at time $t = 0.150$ s. (d) How much time must elapse from the instant in part (c) until the particle at $x = 0.360$ m next has maximum upward displacement?

15.8 • A certain transverse wave is described by

$$y(x, t) = (6.50 \text{ mm}) \cos 2\pi \left( \frac{t}{28.0 \text{ cm}} - \frac{x}{0.0360 \text{ s}} \right)$$

Determine the wave’s (a) amplitude; (b) wavelength; (c) frequency; (d) speed of propagation; (e) direction of propagation.

15.9 • CALC Which of the following wave functions satisfies the wave equation, Eq. (15.12)? (a) $y(x, t) = A \cos(kx + \omega t)$; (b) $y(x, t) = A \sin(kx + \omega t)$; (c) $y(x, t) = A(\cos kx + \cos \omega t)$.

(d) For the wave of part (b), write the equations for the transverse velocity and transverse acceleration of a particle at point $x$. 

**Exercises**
15.10 • A water wave traveling in a straight line on a lake is described by the equation
\[ y(x, t) = (3.75 \text{ cm}) \cos(0.450 \text{ cm}^{-1} x + 5.40 \text{ s}^{-1} t) \]
where \( y \) is the displacement perpendicular to the undisturbed surface of the lake. (a) How much time does it take for one complete wave pattern to pass past a fisherman in a boat at anchor, and what horizontal distance does the wave crest travel in that time? (b) What are the wave number and the number of waves per second that pass the fisherman? (c) How fast does a wave crest travel past the fisherman, and what is the maximum speed of his cork floater as the wave causes it to bob up and down?

15.11 • A sinusoidal wave is propagating along a stretched string that lies along the \( x \)-axis. The displacement of the string as a function of time is graphed in Fig. E15.11 for particles at \( x = 0 \) and at \( x = 0.0900 \text{ m} \). (a) What is the amplitude of the wave? (b) What is the period of the wave? (c) You are told that the two points \( x = 0 \) and \( x = 0.0900 \text{ m} \) are within one wavelength of each other. If the wave is moving in the +\( x \)-direction, determine the wavelength and the wave speed. (d) If instead the wave is moving in the −\( x \)-direction, determine the wavelength and the wave speed. (e) Would it be possible to determine definitively the wavelength in parts (c) and (d) if you were not told that the two points were within one wavelength of each other? Why or why not?

15.12 • CALC Speed of Propagation vs. Particle Speed. (a) Show that Eq. (15.3) may be written as
\[ y(x, t) = A \cos \left( \frac{2\pi}{\lambda} (x - vt) \right) \]
(b) Use \( y(x, t) \) to find an expression for the transverse velocity \( v_y \) of a particle in the string on which the wave travels. (c) Find the maximum speed of a particle of the string. Under what circumstances is this equal to the propagation speed \( v \)? Less than \( v \)? Greater than \( v \)?

15.13 • A transverse wave on a string has amplitude 0.300 cm, wavelength 12.0 cm, and speed 6.00 cm/s. It is represented by \( y(x, t) \) as given in Exercise 15.12. (a) At time \( t = 0 \), compute \( y \) at 1.5-cm intervals of \( x \) (that is, at \( x = 0, x = 1.5 \text{ cm}, x = 3.0 \text{ cm}, \) and so on) from \( x = 0 \) to \( x = 12.0 \text{ cm} \). Graph the results. This is the shape of the string at time \( t = 0 \). (b) Repeat the calculations for the same values of \( x \) at times \( t = 0.400 \text{ s} \) and \( t = 0.800 \text{ s} \). Graph the shape of the string at these instants. In what direction is the wave traveling?

15.14 • A wave on a string is described by \( y(x, t) = A \cos(kx - \omega t) \). (a) Graph \( y, v_y, \) and \( a_y \) as functions of \( x \) for \( t = 0 \). (b) Consider the following points on the string: (i) \( x = 0 \); (ii) \( x = \pi/4k \); (iii) \( x = \pi/2k \); (iv) \( x = 3\pi/4k \); (v) \( x = \pi/k \); (vi) \( x = 5\pi/4k \); (vii) \( x = 3\pi/2k \); (viii) \( x = 7\pi/4k \). For a particle at each of these points at \( t = 0 \), describe in words whether the particle is moving and in what direction, and whether the particle is speeding up, slowing down, or instantaneously not accelerating.

Section 15.4 Speed of a Transverse Wave
15.15 • One end of a horizontal rope is attached to a prong of an electrically driven tuning fork that vibrates the rope transversely at 120 Hz. The other end passes over a pulley and supports a 1.50-kg mass. The linear mass density of the rope is 0.0550 kg/m. (a) What is the speed of a transverse wave on the rope? (b) What is the wavelength? (c) How would your answers to parts (a) and (b) change if the mass were increased to 3.00 kg?

15.16 • With what tension must a rope with length 2.50 m and mass 0.120 kg be stretched for transverse waves of frequency 40.0 Hz to have a wavelength of 0.750 m?

15.17 • The upper end of a 3.80-m-long steel wire is fastened to the ceiling, and a 54.0-kg object is suspended from the lower end of the wire. You observe that it takes a transverse pulse 0.0492 s to travel from the bottom to the top of the wire. What is the mass of the wire?

15.18 • A 1.50-m string of weight 0.0125 N is tied to the ceiling at its upper end, and the lower end supports a weight \( W \). Neglect the very small variation in tension along the length of the string that is produced by the weight of the string. When you pluck the string slightly, the waves traveling up the string obey the equation
\[ y(x, t) = (8.50 \text{ mm}) \cos(172 \text{ m}^{-1} x - 4830 \text{ s}^{-1} t) \]
Assume that the tension of the string is constant and equal to \( W \). (a) How much time does it take a pulse to travel the full length of the string? (b) What is the weight \( W \)? (c) How many wavelengths are on the string at any instant of time? (d) What is the equation for waves traveling down the string?

15.19 • A thin, 75.0-cm wire has a mass of 16.5 g. One end is tied to a nail, and the other end is attached to a screw that can be adjusted to vary the tension in the wire. (a) To what tension (in newtons) must you adjust the screw so that a transverse wave of wavelength 3.33 cm makes 875 vibrations per second? (b) How fast would this wave travel?

15.20 • Weighty Rope. If in Example 15.3 (Section 15.4) we do not neglect the weight of the rope, what is the wave speed (a) at the bottom of the rope; (b) at the middle of the rope; (c) at the top of the rope?

15.21 • A simple harmonic oscillator at the point \( x = 0 \) generates a wave on a rope. The oscillator operates at a frequency of 40.0 Hz and with an amplitude of 3.00 cm. The rope has a linear mass density of 50.0 g/m and is stretched with a tension of 5.00 N. (a) Determine the speed of the wave. (b) Find the wavelength. (c) Write the wave function \( y(x, t) \) for the wave. Assume that the oscillator has its maximum upward displacement at time \( t = 0 \). (d) Find the maximum transverse acceleration of points on the rope. (e) In the discussion of transverse waves in this chapter, the force of gravity was ignored. Is that a reasonable approximation for this wave? Explain.

Section 15.5 Energy in Wave Motion
15.22 • A piano wire with mass 3.00 g and length 80.0 cm is stretched with a tension of 25.0 N. A wave with frequency 120.0 Hz and amplitude 1.6 mm travels along the wire. (a) Calculate the average power carried by the wave. (b) What happens to the average power if the wave amplitude is halved?

15.23 • A horizontal wire is stretched with a tension of 94.0 N, and the speed of transverse waves for the wire is 492 m/s. What must the amplitude of a traveling wave of frequency 69.0 Hz be in order for the average power carried by the wave to be 0.365 W?

15.24 • A light wire is tightly stretched with tension \( T \). Transverse traveling waves of amplitude \( A \) and wavelength \( \lambda_1 \) carry average power \( P_{av,1} = 0.400 \text{ W} \). If the wavelength of the waves is doubled, so \( \lambda_2 = 2\lambda_1 \), while the tension \( T \) and amplitude \( A \) are not altered, what then is the average power \( P_{av,2} \) carried by the waves?

15.25 • A jet plane at takeoff can produce sound of intensity 10.0 W/m² at 30.0 m away. But you prefer the tranquil sound of
normal conversation, which is 1.0 \mu W/m^2. Assume that the plane behaves like a point source of sound. (a) What is the closest distance you should live from the airport runway to preserve your peace of mind? (b) What intensity from the jet does your friend experience if she lives twice as far from the runway as you do? (c) What power of sound does the jet produce at takeoff?

**15.26 • Threshold of Pain.** You are investigating the report of a UFO landing in an isolated portion of New Mexico, and you encounter a strange object that is radiating sound waves uniformly in all directions. Assume that the sound comes from a point source and that you can ignore reflections. You are slowly walking toward the source. When you are 7.5 m from it, you measure its intensity in all directions. Assume that the sound comes from a point source and that the intensity is at a distance of 1.0 W/m^2. An intensity of 1.0 W/m^2 is often used as the “threshold of pain.” How much closer to the source can you move before the sound intensity reaches this threshold?

**15.27 • Energy Output.** By measurement you determine that sound waves are spreading out equally in all directions from a point source and that the intensity is 0.026 W/m^2 at a distance of 4.3 m from the source. (a) What is the intensity at a distance of 3.1 m from the source? (b) How much sound energy does the source emit in one hour if its power output remains constant?

**15.28 • A fellow student with a mathematical bent tells you that the wave function of a traveling wave on a thin rope is \( y(x, t) = 2.30 \text{ mm} \cos \left( \frac{6.98 \text{ rad/m}}{x + (742 \text{ rad/s})t} \right) \). Being more practical, you measure the rope to have a length of 1.35 m and a mass of 0.00338 kg. You are then asked to determine the following: (a) amplitude; (b) frequency; (c) wavelength; (d) wave speed; (e) direction the wave is traveling; (f) tension in the rope; (g) average power transmitted by the wave.

**15.29 • At a distance of 7.00 \times 10^{12} \text{ m} from a star, the intensity of the radiation from the star is 15.4 \text{ W/m}^2. Assuming that the star radiates uniformly in all directions, what is the total power output of the star?

### Section 15.6 Wave Interference, Boundary Conditions, and Superposition

**15.30 • Reflection.** A wave pulse on a string has the dimensions shown in Fig. E15.30 at \( t = 0 \). The wave speed is 40 cm/s. (a) If point \( O \) is a fixed end, draw the total wave on the string at \( t = 15 \text{ ms}, 20 \text{ ms}, 25 \text{ ms}, 30 \text{ ms}, 35 \text{ ms}, 40 \text{ ms}, \) and \( 45 \text{ ms} \). (b) Repeat part (a) for the case in which point \( O \) is a free end.

**15.31 • Reflection.** A wave pulse on a string has the dimensions shown in Fig. E15.31 at \( t = 0 \). The wave speed is 5.0 m/s. (a) If point \( O \) is a fixed end, draw the total wave on the string at \( t = 1.0 \text{ ms}, 2.0 \text{ ms}, 3.0 \text{ ms}, 4.0 \text{ ms}, 5.0 \text{ ms}, 6.0 \text{ ms}, \) and \( 7.0 \text{ ms} \). (b) Repeat part (a) for the case in which point \( O \) is a free end.

**15.32 • Interference of Triangular Pulses.** Two triangular wave pulses are traveling toward each other on a stretched string as shown in Fig. E15.32. Each pulse is identical to the other and travels at 2.00 cm/s. The leading edges of the pulses are 1.00 cm apart at \( t = 0 \). Sketch the shape of the string at \( t = 0.250 \text{ s}, t = 0.500 \text{ s}, t = 0.750 \text{ s}, t = 1.000 \text{ s}, \) and \( t = 1.250 \text{ s} \).

**15.33 • Suppose that the left-traveling pulse in Exercise 15.32 is below the level of the unstretched string instead of above it. Make the same sketches that you did in that exercise.

**15.34 • Two pulses are moving in opposite directions at 1.0 cm/s on a taut string, as shown in Fig. E15.34. Each square is 1.0 cm. Sketch the shape of the string at the end of (a) 6.0 s; (b) 7.0 s; (c) 8.0 s.

**15.35 • Interference of Rectangular Pulses.** Figure E15.35 shows two rectangular wave pulses on a stretched string traveling toward each other. Each pulse is traveling with a speed of 1.00 mm/s and has the height and width shown in the figure. If the leading edges of the pulses are 8.00 mm apart at \( t = 0 \), sketch the shape of the string at \( t = 4.00 \text{ s}, t = 6.00 \text{ s}, \) and \( t = 10.0 \text{ s} \).
the wave equation, Eq. (15.12), for $v = \omega/k$. (b) Explain why the
relationship $v = \omega/k$ for traveling waves also applies to standing
waves.

15.39 • CALC Let $y_1(x, t) = A \cos(k_1x - \omega_1t)$ and $y_2(x, t) =
A \cos(k_2x - \omega_2t)$ be two solutions to the wave equation, Eq.
(15.12), for the same $\omega$. Show that $y(x, t) = y_1(x, t) + y_2(x, t)$ is
also a solution to the wave equation.

15.40 • A 1.50-m-long rope is stretched between two supports
with a tension that makes the speed of transverse waves 48.0 m/s. What are
the wavelength and frequency of (a) the fundamental; (b) the second overtone; (c) the fourth harmonic?

15.41 • A wire with mass 40.0 g is stretched so that its ends are
tied down at points 80.0 cm apart. The wire vibrates in its fundamental
mode with frequency 60.0 Hz and with an amplitude at the
antinodes of 0.300 cm. (a) What is the speed of propagation of
transverse waves in the wire? (b) Compute the tension in the wire.
(c) Find the maximum transverse velocity and acceleration of
particles in the wire.

15.42 • A piano tuner stretches a steel piano wire with a tension
of 800 N. The steel wire is 0.400 m long and has a mass of 3.00 g.
(a) What is the frequency of its fundamental mode of vibration?
(b) What is the number of the highest harmonic that could be heard
by a person who is capable of hearing frequencies up to 10,000 Hz.

15.43 • CALC A thin, taut string tied at both ends and oscillating in
its third harmonic has its shape described by the equation $y(x, t) =
(5.60 \text{ cm}) \sin\left(\frac{(0.0340 \text{ rad/cm})x}{\sin\left(\frac{50.0 \text{ rad/s}}{s}\right)}\right)$, where the ori-
gin is at the left end of the string, the $x$-axis is along the string,
and the $y$-axis is perpendicular to the string. (a) Draw a sketch that
shows the standing-wave pattern. (b) Find the amplitude of
the two traveling waves that make up this standing wave. (c) What is
the length of the string? (d) Find the wavelength, frequency,
period, and speed of the traveling waves. (e) Find the maximum
transverse speed of a point on the string. (f) What would be the
equation $y(x, t)$ for this string if it were vibrating in its eighth harmonic?

15.44 • The wave function of a standing wave is $y(x, t) =
4.44 \text{ mm} \sin\left(\frac{32.5 \text{ rad/m}}{m}x\right) \sin\left(\frac{754 \text{ rad/s}}{s}\right)$. For the two travel-
ing waves that make up this standing wave, find the (a) amplitude;
(b) wavelength; (c) frequency; (d) wave speed; (e) wave functions.
(f) From the information given, can you determine which harmonic
this is? Explain.

15.45 • CALC Consider again the rope and traveling wave of Exercise
15.28. Assume that the ends of the rope are held fixed and that this
traveling wave and the reflected wave are traveling in the opposite
direction. (a) What is the wave function $y(x, t)$ for the standing
wave that is produced? (b) In which harmonic is the standing wave
oscillating? (c) What is the frequency of the fundamental
oscillation?

15.46 • One string of a certain musical instrument is 75.0 cm
long and has a mass of 8.75 g. It is being played in a room where
the speed of sound is 344 m/s. (a) To what tension must you adjust
the string so that, when vibrating in its second overtone, it pro-
duces sound of wavelength 0.765 m? (Assume that the breaking
stress of the wire is very large and isn’t exceeded.) (b) What fre-
quency sound does this string produce in its fundamental mode of
vibration?

15.47 • The portion of the string of a certain musical instrument
between the bridge and upper end of the finger board (that part
of the string that is free to vibrate) is 60.0 cm long, and this length
of the string has mass 2.00 g. The string sounds an $A_4$ note (440 Hz)
when played. (a) Where must the player put a finger (what distance
$x$ from the bridge) to play a $D_5$ note (587 Hz)? (See Fig. E15.47.)

For both the $A_4$ and $D_5$ notes, the string vibrates in its funda-
mental mode. (b) Without retuning, is it possible to play a $G_4$
(note (392 Hz) on this string? Why or why not?

15.48 • (a) A horizontal string tied at both ends is vibrating in its
fundamental mode. The trav-
eling waves have speed $v$, fre-
quency $f$, amplitude $A$, and
wavelength $\lambda$. Calculate the max-
imum transverse velocity and maximum transverse acceleration
of points located at (i) $x = \lambda/2$, (ii) $x = \lambda/4$, and (iii) $x = \lambda/8$ from
the left-hand end of the string. (b) At each of the points in part (a),
what is the amplitude of the motion? (c) At each of the points in
part (a), how much time does it take the string to go from its largest
upward displacement to its largest downward displacement?

15.49 • Guitar String. One of the 63.5-cm-long strings of an
ordinary guitar is tuned to produce the note $B_3$ (frequency 245 Hz)
when vibrating in its fundamental mode. (a) Find the speed of
transverse waves on this string. (b) If the tension in this string is
increased by 1.0%, what will be the new fundamental frequency of
the string? (c) If the speed of sound in the surrounding air is
344 m/s, find the frequency and wavelength of the sound wave
produced in the air by the vibration of the $B_3$ string. How do these
compare to the frequency and wavelength of the standing wave on
the string?

15.50 • Waves on a Stick. A flexible stick 2.0 m long is not
fixed in any way and is free to vibrate. Make clear drawings of this
stick vibrating in its first three harmonics, and then use your draw-
ings to find the wavelengths of each of these harmonics. (Hint: Should the ends be nodes or antinodes?)

PROBLEMS

15.51 • CALC A transverse sine wave with an amplitude of 2.50 mm
and a wavelength of 1.80 m travels from left to right along a long,
horizontal, stretched string with a speed of 36.0 m/s. Take the origin
at the left end of the undisturbed string. At time $t = 0$ the left
end of the string has its maximum upward displacement. (a) What
are the frequency, angular frequency, and wave number of the
wave? (b) What is the function $y(x, t)$ that describes the wave?
(c) What is $y(t)$ for a particle at the left end of the string? (d) What
is $y(x)$ for a particle 1.35 m to the right of the origin? (e) What
is the maximum magnitude of transverse velocity of any particle
of the string? (f) Find the transverse displacement and the transverse
velocity of a particle 1.35 m to the right of the origin at time
$t = 0.0625$ s.

15.52 • A transverse wave on a rope is given by

$$y(x, t) = \left(0.750 \text{ cm}\right) \cos\left(\pi\left(0.400 \text{ cm}^{-1}\right)x + (250 \text{ s}^{-1})t\right)$$

(a) Find the amplitude, period, frequency, wavelength, and speed
of propagation. (b) Sketch the shape of the rope at these values of
$x$: 0, 0.0005 s, 0.0010 s. (c) Is the wave traveling in the $+x$ or
$-x$-direction? (d) The mass per unit length of the rope is
0.0500 kg/m. Find the tension. (e) Find the average power of this
wave.

15.53 • Three pieces of string, each of length $L$, are joined
together end to end, to make a combined string of length $3L$. The
first piece of string has mass per unit length $\mu_1$, the second piece...
has mass per unit length \( \mu_2 = 4 \mu_1 \), and the third piece has mass per unit length \( \mu_3 = \mu_1/4 \). (a) If the combined string is under tension \( F \), how much time does it take a transverse wave to travel the entire length \( 3L \)? Give your answer in terms of \( L, F, \) and \( \mu_1 \). (b) Does your answer to part (a) depend on the order in which the three pieces are joined together? Explain.

15.54 • CP A 1750-N irregular beam is hanging horizontally by its ends from the ceiling by two vertical wires (A and B), each 1.25 m long and weighing 0.360 N. The center of gravity of this beam is one-third of the way along the beam from the end where wire A is attached. If you pluck both strings at the same time at the beam, what is the time delay between the arrival of the two pulses at the ceiling? Which pulse arrives first? (Neglect the effect of the weight of the wires on the tension in the wires.)

15.55 • CALC Ant Joy Ride. You place your pet ant Klyde (mass \( m \)) on top of a horizontal, stretched rope, where he holds on tightly. The rope has mass \( M \) and length \( L \) and is under tension \( F \). You start a sinusoidal transverse wave of wavelength \( \lambda \) and amplitude \( A \) propagating along the rope. The motion of the rope is in a vertical plane. Klyde’s mass is so small that his presence has no effect on the propagation of the wave. (a) What is Klyde’s top speed as he oscillates up and down? (b) Klyde enjoys the ride and begins for more. You decide to double his top speed by changing the tension while keeping the wavelength and amplitude the same. Should the tension be increased or decreased, and by what factor?

15.56 • Weightless Ant. An ant with mass \( m \) is standing peacefully on top of a horizontal, stretched rope. The rope has mass per unit length \( \mu \) and is under tension \( F \). Without warning, Cousin Throckmorton starts a sinusoidal transverse wave of wavelength \( \lambda \) propagating along the rope. The motion of the rope is in a vertical plane. What minimum wave amplitude will make the ant become momentarily weightless? Assume that \( m \) is so small that the presence of the ant has no effect on the propagation of the wave.

15.57 • CP When a transverse sinusoidal wave is present on a string, the particles of the string undergo SHM. This is the same motion as that of a mass \( m \) attached to an ideal spring of force constant \( k \), for which the angular frequency of oscillation was found in Chapter 14 to be \( \omega = \sqrt{k/m} \). Consider a string with tension \( F \) and mass per unit length \( \mu \), along which is propagating a sinusoidal wave with amplitude \( A \) and wavelength \( \lambda \). (a) Find the “force constant” \( k \) of the restoring force that acts on a short segment of the string of length \( \Delta x \) (where \( \Delta x \ll \lambda \)). (b) How does the “force constant” calculated in part (b) depend on \( F, \mu, A, \) and \( \lambda \)? Explain the physical reasons this should be so.

15.58 • Music. You are designing a two-string instrument with metal strings 35.0 cm long, as shown in Fig. P15.58. Both strings are under the same tension. String \( S_1 \) has a mass of 8.00 g and produces the note middle C (frequency 262 Hz) in its fundamental mode. (a) What should be the tension in the string? (b) What should be the mass of string \( S_2 \) so that it will produce A-sharp (frequency 466 Hz) as its fundamental? (c) To extend the range of your instrument, you include a fret located just under the strings but not normally touching them. How far from the upper end should you put this fret so that when you press \( S_1 \) tightly against it, this string will produce C-sharp (frequency 277 Hz) in its fundamental? That is, what is \( x \) in the figure?

(d) If you press \( S_2 \) against the fret, what frequency of sound will it produce in its fundamental?

15.59 • CP The lower end of a uniform bar of mass 45.0 kg is attached to a wall by a frictionless hinge. The bar is held by a horizontal wire attached at its upper end so that the bar makes an angle of 30.0° with the wall. The wire has length 0.330 m and mass 0.0920 kg. What is the frequency of the fundamental standing wave for transverse waves on the wire?

15.60 • CP You are exploring a newly discovered planet. The radius of the planet is 7.20 \times 10^7 \text{ m}. You suspend a lead weight from the lower end of a light string that is 4.00 m long and has mass 0.0280 kg. You measure that it takes 0.0600 \text{ s} for a transverse pulse to travel from the lower end to the upper end of the string. On earth, for the same string and lead weight, it takes 0.0390 \text{ s} for a transverse pulse to travel the length of the string. The weight of the string is small enough that its effect on the tension in the string can be neglected. Assuming that the mass of the planet is distributed with spherical symmetry, what is its mass?

15.61 • For a string stretched between two supports, two successive standing-wave frequencies are 525 Hz and 630 Hz. There are other standing-wave frequencies lower than 525 Hz and higher than 630 Hz. If the speed of transverse waves on the string is 384 m/s, what is the length of the string? Assume that the mass of the wire is small enough for its effect on the tension in the wire to be neglected.

15.62 • CP A 5.00-m, 0.732-kg wire is used to support two uniform 235-N posts of equal length (Fig. P15.62). Assume that the wire is essentially horizontal and that the speed of sound is 344 m/s. A strong wind is blowing, causing the wire to vibrate in its 5th overtone. What are the frequency and wavelength of the sound this wire produces?

Figure P15.62

15.63 • CP A 1.80-m-long uniform bar that weighs 536 N is suspended in a horizontal position by two vertical wires that are attached to the ceiling. One wire is aluminum and the other is copper. The aluminum wire is attached to the left-hand end of the bar, and the copper wire is attached 0.40 m to the left of the right-hand end. Each wire has length 0.600 m and a circular cross section with radius 0.280 mm. What is the fundamental frequency of transverse standing waves for each wire?

15.64 • A continuous succession of sinusoidal wave pulses are produced at one end of a very long string and travel along the length of the string. The wave has frequency 70.0 Hz, amplitude 5.00 mm, and wavelength 0.600 m. (a) How long does it take the wave to travel a distance of 8.00 m along the length of the string? (b) How long does it take a point on the string to travel a distance of 8.00 m, once the wave train has reached the point and set it into motion? (c) In parts (a) and (b), how does the time change if the amplitude is doubled?

15.65 • CALC Waves of Arbitrary Shape. (a) Explain why any wave described by a function of the form \( y(x, t) = f(x - vt) \) moves in the \( +x \)-direction with speed \( v \). (b) Show that \( y(x, t) = f(x - vt) \) satisfies the wave equation, no matter what the functional form of \( f \). To do this, write \( y(x, t) = f(u) \), where
\( u = x - vt \). Then, to take partial derivatives of \( y(x, t) \), use the chain rule:

\[
\frac{\partial y(x, t)}{\partial t} = \frac{df(u)}{du} \frac{\partial u}{\partial t} = \frac{df(u)}{du} (v)
\]

\[
\frac{\partial^2 y(x, t)}{\partial x \partial t} = \frac{df(u)}{du} \frac{\partial u}{\partial x} \frac{\partial u}{\partial t} = \frac{df(u)}{du} (v)
\]

(c) A wave pulse is described by the function \( y(x, t) = D e^{-(Bx - Ct)^2} \) where \( B, C, \) and \( D \) are all positive constants. What is the speed of this wave?

**15.66** CP A vertical, 1.20-m length of 18-gauge (diameter of 1.024 mm) copper wire has a 100.0-N ball hanging from it. (a) What is the wavelength of the third harmonic for this wire? (b) A 500.0-N ball now replaces the original ball. What is the change in the wavelength of the third harmonic caused by replacing the light ball with the heavy one? (Hint: See Table 11.1 for Young’s modulus.)

**15.67** (a) Show that Eq. (15.25) can also be written as \( P = \frac{1}{2} F k u A^2 \), where \( k \) is the wave number of the wave. (b) If the tension \( F \) in the string is quadrupled while the amplitude \( A \) is kept the same, how must \( k \) and \( \omega \) each change to keep the average power constant? (Hint: Recall Eq. (15.6).]

**15.68** CALC Equation (15.7) for a sinusoidal wave can be made more general by including a phase angle \( \phi \), where \( 0 \leq \phi \leq 2\pi \) (in radians). Then the wave function \( y(x, t) \) becomes

\[
y(x, t) = A \cos(kx - \omega t + \phi)
\]

(a) Sketch the wave as a function of \( x \) at \( t = 0 \) for \( \phi = 0 \), \( \phi = \pi/4 \), \( \phi = \pi/2 \), \( \phi = 3\pi/4 \), and \( \phi = 3\pi/2 \). (b) Calculate the transverse velocity \( v_y = \frac{\partial y}{\partial t} \). (c) At \( t = 0 \), a particle on the string at \( x = 0 \) has displacement \( y = A/\sqrt{2} \). Is this enough information to determine the value of \( \phi \)? In addition, if you are told that a particle at \( x = 0 \) is moving toward \( y = 0 \) at \( t = 0 \), what is the value of \( \phi \)? (d) Explain in general what you must know about the wave’s behavior at a given instant to determine the value of \( \phi \).

**15.69** A sinusoidal transverse wave travels on a string. The string has length 8.00 m and mass 6.00 g. The wave speed is 30.0 m/s, and the wavelength is 0.200 m. (a) If the wave is to have an average power of 50.0 W, what must be the amplitude of the wave? (b) For this same string, if the amplitude and wavelength are the same as in part (a), what is the average power for the wave if the tension is increased such that the wave speed is doubled?

**15.70** CALC Energy in a Triangular Pulse. A triangular wave pulse on a taut string travels in the positive \( x \)-direction with speed \( v \). The tension in the string is \( F \), and the linear mass density of the string is \( \mu \). At \( t = 0 \), the shape of the pulse is given by

\[
y(x, 0) = \begin{cases} 0 & \text{if } x < -L \\ h(L + x)/L & \text{for } -L < x < 0 \\ h(L - x)/L & \text{for } 0 < x < L \\ 0 & \text{for } x > L 
\end{cases}
\]

(a) Draw the pulse at \( t = 0 \). (b) Determine the wave function \( y(x, t) \) at all times \( t \). (c) Find the instantaneous power in the wave. Show that the power is zero except for \(-L < (x - vt) < L\) and that in this interval the power is constant. Find the value of this constant power.

**15.71** CALC Instantaneous Power in a Wave. (a) Graph \( y(x, t) \) as given by Eq. (15.7) as a function of \( x \) for a given time \( t \) (say, \( t = 0 \)). On the same axes, make a graph of the instantaneous power \( P(x, t) \) as given by Eq. (15.23). (b) Explain the connection between the slope of the graph of \( y(x, t) \) versus \( x \) and the value of \( P(x, t) \). In particular, explain what is happening at points where \( P = 0 \), where there is no instantaneous energy transfer. (c) The quantity \( P(x, t) \) always has the same sign. What does this imply about the direction of energy flow? (d) Consider a wave moving in the \(-x\)-direction, for which \( y(x, t) = A \cos(kx + \omega t) \). Calculate \( P(x, t) \) for this wave, and make a graph of \( y(x, t) \) and \( P(x, t) \) as functions of \( x \) for a given time \( t \) (say, \( t = 0 \)). What differences arise from reversing the direction of the wave?

**15.72** A vibrating string 50.0 cm long is under a tension of 1.00 N. The results from five successive stroboscopic pictures are shown in Fig. P15.72. The strobe rate is set at 5000 flashes per minute, and observations reveal that the maximum displacement occurred at flashes 1 and 5 with no other maxima in between. (a) Find the period, frequency, and wavelength for the traveling waves on this string. (b) In what normal mode (harmonic) is the string vibrating? (c) What is the speed of the traveling waves on the string? (d) How fast is point \( P \) moving when the string is in (i) position 1 and (ii) position 3? (e) What is the mass of this string? (See Section 15.3.)

Figure P15.72

**15.73** Clothesline Nodes. Cousin Throckmorton is once again playing with the clothesline in Example 15.2 (Section 15.3). One end of the clothesline is attached to a vertical post. Throcky holds the other end loosely in his hand, so that the speed of waves on the clothesline is a relatively slow 0.720 m/s. He finds several frequencies at which he can oscillate his end of the clothesline so that a light clothespin 45.0 cm from the post doesn’t move. What are these frequencies?

**15.74** CALC A guitar string is vibrating in its fundamental mode, with nodes at each end. The length of the segment of the string that is free to vibrate is 0.386 m. The maximum transverse acceleration of a point at the middle of the segment is 8.40 \times 10^3 \text{m/s}^2 and the maximum transverse velocity is 3.80 m/s. (a) What is the amplitude of this standing wave? (b) What is the wave speed for the transverse traveling waves on this string?

**15.75** CALC A string that lies along the \(+x\)-axis has a free end at \( x = 0 \). (a) By using steps similar to those used to derive Eq. (15.28), show that an incident traveling wave \( y_1(x, t) = A \cos(kx + \omega t) \) gives rise to a standing wave \( y(x, t) = 2A \cos \omega t \cos kx \). (b) Show that the standing wave has an antinode at its free end (\( x = 0 \)). (c) Find the maximum displacement, maximum speed, and maximum acceleration of the free end of the string.

**15.76** A string with both ends held fixed is vibrating in its third harmonic. The waves have a speed of 192 m/s and a frequency of 240 Hz. The amplitude of the standing wave at an antinode is 0.400 cm. (a) Calculate the amplitude at points on the string a distance of (i) 40.0 cm; (ii) 20.0 cm; and (iii) 10.0 cm from the left end of the string. (b) At each point in part (a), how much time does it take the string to go from its largest upward displacement to its largest downward displacement? (c) Calculate the maximum
transverse velocity and the maximum transverse acceleration of the string at each of the points in part (a).

15.77 A uniform cylindrical steel wire, 55.0 cm long and 1.14 mm in diameter, is fixed at both ends. To what tension must it be adjusted so that, when vibrating in its first overtone, it produces the note D-sharp of frequency 311 Hz? Assume that it stretches an insignificant amount. (Hint: See Table 12.1.)

15.78 Holding Up Under Stress. A string or rope will break apart if it is placed under too much tensile stress [Eq. (11.8)]. Thicker ropes can withstand more tension without breaking because the thicker the rope, the greater the cross-sectional area and the smaller the stress. One type of steel has density 7800 kg/m³ and will break if the tensile stress exceeds $7.0 \times 10^6$ N/m². You want to make a guitar string from 4.0 g of this type of steel. In use, the guitar string must be able to withstand a tension of 900 N without breaking. Your job is the following: (a) Determine the maximum tension in the string can have. (b) Determine the highest possible fundamental frequency of standing waves on this string, if the entire length of the string is free to vibrate.

15.79 Combining Standing Waves. A guitar string of length $L$ is plucked in such a way that the total wave produced is a sum of the fundamental and the second harmonic. That is, the standing wave is given by

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

where

$$y_1(x, t) = C \sin \omega_1 t \sin \frac{1}{L} \pi x$$

$$y_2(x, t) = C \sin \omega_2 t \sin \frac{2}{L} \pi x$$

with $\omega_1 = \sqrt{k/L}$ and $\omega_2 = \sqrt{k/2L}$. (a) At what values of $x$ are the nodes of $y_1$? (b) At what values of $x$ are the nodes of $y_2$? (c) Graph the total wave at $t = 0$, $t = \frac{1}{4} T$, $t = \frac{1}{2} T$, and $t = \frac{3}{4} T$. (d) Does the sum of the two standing waves $y_1$ and $y_2$ produce a standing wave? Explain.

15.80 CP When a massive aluminum sculpture is hung from a steel wire, the fundamental frequency for transverse standing waves on the wire is 250.0 Hz. The sculpture (but not the wire) is then completely submerged in water. (a) What is the new fundamental frequency? (Hint: See Table 12.1.) (b) Why is it a good approximation to treat the wire as being fixed at both ends?

15.81 CP A large rock that weighs 164.0 N is suspended from the lower end of a thin wire that is 3.00 m long. The density of the rock is 3200 kg/m³. The mass of the wire is small enough that its effect on the tension in the wire can be neglected. The upper end of the wire is held fixed. When the rock is in air, the fundamental frequency for transverse standing waves on the wire is 42.0 Hz. When the rock is totally submerged in a liquid, with the rock just below the surface, the fundamental frequency for the wire is 28.0 Hz. What is the density of the liquid?

15.82 Tuning an Instrument. A musician tunes the C-string of her instrument to a fundamental frequency of 65.4 Hz. The vibrating portion of the string is 0.600 m long and has a mass of 14.4 g. (a) With what tension must the musician stretch it? (b) What percent increase in tension is needed to increase the frequency from 65.4 Hz to 73.4 Hz, corresponding to a rise in pitch from C to D?

15.83 One type of steel has a density of $7.8 \times 10^3$ kg/m³ and a breaking stress of $7.0 \times 10^8$ N/m². A cylindrical guitar string is to be made of 4.00 g of this steel. (a) What are the length and radius of the longest and thinnest string that can be placed under a tension of 900 N without breaking? (b) What is the highest fundamental frequency that this string could have?

### Challenge Problems

15.84 CP CALC A deep-sea diver is suspended beneath the surface of Loch Ness by a 100-m-long cable that is attached to a boat on the surface (Fig. P15.84). The diver and his suit have a total mass of 120 kg and a volume of 0.0800 m³. The cable has a diameter of 2.00 cm and a linear mass density of $\mu = 1.10$ kg/m. The diver thinks he sees something moving in the murky depths and jerks the end of the cable back and forth to send transverse waves up the cable as a signal to his companions in the boat. (a) What is the tension in the cable at its lower end, where it is attached to the diver? Do not forget to include the buoyant force that the water exerts on the moving cable. (b) What is the maximum tension in the cable? (c) Determine the tension in the cable a distance $x$ above the diver. The buoyant force on the cable must be included in your calculation. (d) Show that for all $x$ the total energy per unit length multiplied by the wave speed is zero, and vice versa. (e) Show that the instantaneo power in the wave, given by Eq. (15.22), is equal to the total energy per unit length multiplied by the wave speed $v$. Explain why this result is reasonable.