Delay Analysis and Routing for Two-Dimensional VANETs Using Carry-and-Forward Mechanism

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APPENDIX A
THE PROOF OF THEOREM 4.4

Proof. To calculate \( \Pr\{Y_{wn}\} \), we need to consider three cases. First, no vehicles in the cluster will turn north directly, and the probability of this case is denoted by \( \Pr\{X_{en}\} \). Denote \( P_w^n \) the probability that a vehicle turns north from west at the intersection. We have \( \Pr\{X_{en}\} = (1 - P_w^n)^{|c_s|} \), where \(|c_s|\) is the total number of vehicles in the cluster.

Second, the cluster cannot find a vehicle from east to north to carry the data, the corresponding probability is denoted by \( \Pr\{X_{cn}\} \). In this case, \( \Pr\{X_{cn}\} \) equals the probability that the vehicles in the cluster cannot meet a vehicle from east to north before they cross the intersection, i.e., \( \Pr\{X_{cn}\} = 1 - \int_{0}^{\frac{\Delta t}{v}} \lambda_{en} e^{-\lambda_{en} t} dt = e^{-\lambda_{en}(c_s+R')/v} \), where \((c_s+R')/v\) is the effective communication time for the whole cluster to cross the intersection and this can be viewed as the time that the message is waiting at the intersection for the next-hop carrier. Third, the cluster cannot find a vehicle from south to north. The corresponding probability is denoted by \( \Pr\{X_{sn}\} \). Similarly, we have \( \Pr\{X_{sn}\} = 1 - \int_{0}^{\frac{\Delta t}{v}} \lambda_{sn} e^{-\lambda_{sn} t} dt = e^{-\lambda_{sn}(c_s+R')/v} \).

Note that the event \( Y_{wn} \) happens when one of the above three cases does not happen. Hence,

\[
\Pr\{Y_{wn}\} = 1 - \Pr\{X_{en}\} \times \Pr\{X_{cn}\} \times \Pr\{X_{sn}\} = 1 - (1 - P_w^n)^{|c_s|} e^{-(\lambda_{en} + \lambda_{sn})(c_s+R')/v}. 
\]

We thus have completed the proof. \( \square \)

APPENDIX B
THE PROOF OF THEOREM 5.2

Proof. First, we consider the expectation of \( Z_{wn} \). Assume that there is a cluster carrying the message and traveling from west to an intersection, and its cluster size is \( c_s \). Note that the vehicle arrival processes are Poisson processes, which means that the inter-vehicle distances between any two neighboring vehicles are i.i.d. with the same average value which equals \( \frac{c_s}{v[|c_s|-1]} \).

Define \( \Delta t = \frac{c_s}{v[|c_s|-1]} \). We divide the possible values of \( Z_{wn} \) into several intervals, i.e., \([0, \frac{R'}{v}) \), \( \left( \frac{R'}{v}, \frac{R'}{v} + \Delta t \right) \), \( \frac{R'}{v} + \Delta t \), \( \cdots \), \( \left( \frac{R'}{v} + (|c_s| - 2)\Delta t, \frac{R'}{v} + (|c_s| - 1)\Delta t \right) \), \( \frac{R'}{v} + (|c_s| - 1)\Delta t \), \( \cdots \), \( \left( \frac{R'}{v} + |c_s|\Delta t, \frac{R'}{v} + (|c_s| + 1)\Delta t \right) \). Then, we consider the following five cases.

Case 1: if \( Z_{wn} \in [0, \frac{R'}{v}) \), which means that the data carrier in the cluster can find a vehicle traveling from west or south to north. In this case, since the aggregation of two Poisson process is still a Poisson process, we have the PDF of the time that this case occurs as \( f(t) = \lambda_{es}^n e^{-\lambda_{es}^n t}, t \in [0, \frac{R'}{v}) \) and the probability of this case is

\[
\Pr\{Z_{wn} \in \left[0, \frac{R'}{v} \right)\} = 1 - e^{-\lambda_{es}^n R'/v}. 
\]

Case 2: if \( Z_{wn} = \frac{R'}{v} \), which means that the first vehicle in the cluster cannot find a vehicle traveling from west or south to north to carry the data and it carries the data to the north directly. In this case, we have

\[
\Pr\{Z_{wn} = \frac{R'}{v}\} = P_w^n e^{-\lambda_{es}^n R'/v}. 
\]

Case 3: if \( Z_{wn} \in \left( \frac{R'}{v}, \frac{R'}{v} + (\ell + 1)\Delta t \right) \) for \( \ell = 0, 1, \ldots, |c_s| - 2 \), it means that the \( (\ell + 2) \)-th vehicle in the cluster will carry the data and not go north either) and find an appropriate vehicle in the intersection for the next-hop data carrier. In this case, we have

\[
f(t) = (1 - P_w^n)^{\ell+1} \lambda_{es}^n e^{-\lambda_{es}^n t}, t \in \left( \frac{R'}{v} + \ell \Delta t, \frac{R'}{v} + (\ell + 1)\Delta t \right). 
\]

Case 4: if \( Z_{wn} = \frac{R'}{v} + l\Delta t \) for \( l = 1, 2, \ldots, |c_s| - 1 \), it means that the \( (l + 1) \)-th vehicle in the cluster will carry and forward the message to north, and the corresponding probability is

\[
\Pr\{Z_{wn} = \frac{R'}{v} + l\Delta t\} = P_w^n (1 - P_w^n)^l e^{-\lambda_{es}^n (\frac{R'}{v} + l\Delta t)}. 
\]

Case 5: if \( Z_{wn} \in (\frac{R'}{v} + |c_s|, \infty) \), it means that the last vehicle in the cluster will carry the data to east or south to find the vehicle traveling back to this intersection. The probability of this case is given by

\[
\Pr\{Z_{wn} \in \left(\frac{R'}{v} + |c_s|, \infty\right)\} = 1 - \Pr[Y_{wn}], 
\]
Combing the above five cases and using the definition of expectation, one infers that
\[
\mathbf{E}\{Z_{wn}\} = \int_0^{\frac{R'}{v}} t\lambda^e_t e^{-\lambda^e_t t} dt + \frac{R'}{v} P^e_w e^{-\lambda^e_{n+1} T} + (1 - P^e_w) \int_0^{\frac{R'}{v} + \Delta t} t\lambda^e_t e^{-\lambda^e_t t} dt \\
+ \left(\frac{R'}{v} + \Delta t\right) P^e_w (1 - P^e_w) e^{-\lambda^e_{n+1} \left(\frac{R'}{v} + \Delta t\right)} \\
+ (1 - P^e_w)^2 \int_0^{\frac{R'}{v} + 2\Delta t} t\lambda^e_t e^{-\lambda^e_t t} dt \\
\ldots \]
\[
+ \left(1 - P^e_w\right)^{|c_s| - 1} \int_0^{\frac{R'}{v} + \Delta t} t\lambda^e_t e^{-\lambda^e_t t} dt \\
+ \left(1 - P^e_w\right)^{|c_s| - 1} e^{-\lambda^e_{n+1} \frac{R'}{v}} \sum_{l=0}^{\frac{|c_s| - 1}{2}}\left(1 - P^e_w\right)^l \left(\frac{R'}{v} + \ell \Delta t\right) e^{-\lambda^e_t \ell \Delta t} \\
+ \sum_{l=1}^{\frac{|c_s| - 1}{2}}\left(1 - P^e_w\right)^l e^{-\lambda^e_{n+1} \left(R + (l-1)\Delta t\right)} \left(\frac{R'}{v} + (l - 1) \Delta t\right) \\
\times (1 - e^{-\lambda^e_{n+1} \Delta t}) + A^e_{n+1} (\Delta t)] \\
+ (1 - \text{Pr}\{Y_{wn}\}) \left[\frac{R'}{v} + c_s + \frac{P^w_e}{2\lambda_e (1 - P^w_e)} + \frac{P^s_w}{2\lambda_s (1 - P^w_e)}\right] \\
+ (1 - \text{Pr}\{Y_{wn}\}) \frac{P^w_e}{1 - P^w_e} \mathbf{E}\{Z_{en}\} \\
+ (1 - \text{Pr}\{Y_{wn}\}) \frac{P^s_w}{1 - P^w_e} \mathbf{E}\{Z_{sn}\} \\
= C_{wn} + B^e_{wn} \mathbf{E}\{Z_{en}\} + B^s_{wn} \mathbf{E}\{Z_{sn}\},
\]
where \( C_{wn} \) is the constant part, \( B^e_{wn} = (1 - \text{Pr}\{Y_{wn}\}) \frac{P^w_e}{1 - P^w_e} \) and \( B^s_{wn} = (1 - \text{Pr}\{Y_{wn}\}) \frac{P^s_w}{1 - P^w_e} \), i.e., we have an equation to relate \( \mathbf{E}\{Z_{wn}\}, \mathbf{E}\{Z_{en}\}, \) and \( \mathbf{E}\{Z_{sn}\} \).

Similarly, considering \( \mathbf{E}\{Z_{en}\} \) and \( \mathbf{E}\{Z_{sn}\} \), we can have the other two equations. Combining these three equations, we can formulate the linear equation set in the following matrix form.
\[
\begin{bmatrix}
1 & -B^e_{wn} & -B^s_{wn} \\
-B^e_{en} & 1 & -B^s_{en} \\
-B^e_{sn} & -B^s_{en} & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{E}\{Z_{wn}\} \\
\mathbf{E}\{Z_{en}\} \\
\mathbf{E}\{Z_{sn}\}
\end{bmatrix}
= \begin{bmatrix}
C_{wn} \\
C_{en} \\
C_{sn}
\end{bmatrix}.
\]

Note that in the above matrix, we have
\[ B^e_{wn} + B^e_{en} = (1 - \text{Pr}\{Y_{wn}\}) \frac{P^w_e}{1 - P^w_e} < 1, \]
for the first row and the same result also holds for the other two rows in the matrix. Thus, for each row in the matrix, the sum of the absolute values of non-diagonal elements is small than the value of the diagonal element. Hence, the matrix in the equation relate \( \mathbf{E}\{Z_{wn}\}, \mathbf{E}\{Z_{en}\}, \) and \( \mathbf{E}\{Z_{sn}\}, \) is of full rank or invertible, and solving the above equation gives the result shown as that in Theorem 5.2. \( \square \)