ECON 403: Crop Allocation Problem

Given the following information for a 1500 ac farm, construct a linear programming model that determines how much of each crop to plant.

Crop	Observed Acreage (ac)	Average Yield (bu/ac)	Price ($/bu)	Average costs ($/ac)
Wheat	500	42	$7.50	$192.0
Barley	200	70	$4.25	$169.5
Canola	450	38	$11.50	$229.0
Peas	250	45	$6.75	$163.8
Oats	100	110	$2.75	$152.50

Solve the following simple model using R:

Maximize $$\text{GM} = \sum_{k=1}^{n}(p_k x_k y_k - c_k x_k)$$

Subject to $$\sum_{k=1}^{n} x_k \leq 1500$$

$$x_k \geq 0$$

Now include the following constraint and solve the problem again:

$$x_k \leq x_k^{obs} + 0.01, \forall k$$

For these constraints find the associated shadow prices, λ_k, and use this information to modify the objective function above assuming a quadratic cost function: $c_k = a\, x_k + b\, x_k^2$. Then:

$$b_k = 2 \times \lambda_k / x_k^{obs} \text{ and } a_k = c_k - \tfrac{1}{2} \times b_k \times x_k^{obs}$$

Use the cost function in place of $c_k\, x_k$ in the objective function, so the revised objective is:

Maximize $$\text{GM} = \sum_{k=1}^{n}(p_k x_k y_k - a_k x_k - b_k x_k^2)$$

Solve the revised problem using quadprod in R.

ANSWERS

1. ORIGNAL LP

To the table we add the net return ($/ac) for each crop:

Crop	Observed Acreage (ac)	Average Yield (bu/ac)	Price ($/bu)	Average costs ($/ac)	Net return ($/ac)
Wheat	500	42	$7.50	$192.0	$123.00
Barley	200	70	$4.25	$169.5	$128.00
Canola	450	38	$11.50	$229.0	$208.00
Peas	250	45	$6.75	$163.8	$139.95
Oats	100	110	$2.75	$152.50	$150.00

Upon solving the LP without any calibration constraints, we get the following solution: **Objective = $312,000**; solutions provided in table below.

2. CALIBRATION LP

When we add calibration constraints, the **objective value = $230,687.60**; solutions for acreage and shadow prices associated with the calibration constraints are as follows: the shadow price on the constraint regarding the total land available is $123.00; the shadow price for each of the crops is given in the last column of the table below.

	LP Model		Calibration Model (LP)	
Crop	Acreage (ac)	Shadow prices	Acreage (ac)	Shadow prices
Wheat	0	42	499.996	0
Barley	0	70	200.001	5.00
Canola	1500	38	450.001	85.00
Peas	0	45	250.001	16.95
Oats	0	110	100.001	27.00

3. CALIBRATION QP

We use the shadow prices in the above table to calculate the values of the intercept and slope terms for each crop:

	QP Model		
Crop	Intercept	Slope	Optimal allocation
Wheat	192.00	0.00	484.56
Barley	164.50	0.05	209.69
Canola	144.00	0.3778	451.28
Peas	146.85	0.1356	253.57
Oats	125.50	0.54	100.90

Notice that the calibrated solution (last column in above table) is close to but not exactly the orig inal allocation, but it is sufficient. The calibrated QP model has **objective value of $230,566.4**

R-Code

```
# Crop allocation problem with calibration
library(ROI)
require(ROI.plugin.lpsolve)
require(ROI.plugin.quadprog)

crops <- c('Wheat', 'Barley', 'Canola', 'Peas', 'Oats')
yield <- c(42, 70, 38, 45, 110)  # crop yield in bu per acre
price <- c(7.50, 4.25, 11.50, 6.75, 2.75)  # $ per bushel
cost <- c(192.0, 169.5, 229.0, 163.8, 152.50)  # $ per acre
obs <- c(500, 200, 450, 250, 100)  # observed acres planted to each crop

# --------------- LINEAR PROBLEM ---------------------------------------------
obj <- price*yield - cost
f.mat <- matrix(c(1,1,1,1,1),1,5)  # General form: const.mat <- matrix(c(20, 12, 1/15, 1/15),
nrow=2, byrow=TRUE)
f.dir <- c('<=')
f.rhs <- 1500
LP1 <- OP( obj, L_constraint(L=f.mat, f.dir, f.rhs), max = TRUE ) # default minimize

sol <- ROI_solve(LP1)

solution(sol, type = c("primal")) # solution for primal problem
solution(sol, type = c("dual"))  # solution for dual problem: minimize by subject to A'y >= c

#### Extract solutions

sol$solution
sol$objval
sol$status
sol$message # detailed message for solution

# --------------- QUADRATIC PROBLEM ---------------------------------------------
# Maximize expected utility: Change the value of GM to find Expected revenue (E) and variance
(V) values
# matrix of covariance: source of nonlinearity
GM <- 312000  # Target revenue
CV <- matrix(c(1.11,  0.79,    1.12,    0.95,    0.58, 0.79,       0.68,    0.71,    0.67,    0.47, 1.12,
        0.71,    1.65,    1.19,    0.72,
          0.95, 0.67,    1.19,    1.28,    0.58, 0.58,       0.47,    0.72,    0.58,    0.55), nrow=5,
ncol=5)
Dmat <- 0.005*CV
dvec <- rep(0,5)
Amat <- rbind(f.mat, diag(1,5,5), obj)
bvec <- c(1500, rep(0,5), GM)
```

```
QP <- OP(Q_objective (Q = Dmat, L = dvec),   # quadratic objective: qudratic part, x'Qx; linear
part: Lx
         L_constraint(L = Amat, dir = c('<=', rep(">=", 6)), rhs = bvec) ) # right hand side, vector b

solQ <- ROI_solve(QP, solver = "quadprog") # choose quadratic solver

solQ$solution
solQ$objval
solQ$status
solQ$message #not much information

# --------------- CALIBRATION CONSTRAINTS ----------------------------------------------
# Add calibration constraints to above model

cal.mat <- rbind(f.mat, diag(1,5))
cal.dir <- rep('<=', 6)
cal.rhs <- c(1500, obs+0.001); dim(cal.rhs) <- c(6,1)

acres <- lp(direction='max', obj, cal.mat, '<=', cal.rhs, compute.sens=TRUE)
GrossReturn <- acres$objval
planting <- acres$solution
lamda <- acres$duals[1:6]
GrossReturn
planting
lamda

# ----- CALCULATE THE INTERCEPT AND SLOPE TERMS ON MARGINAL COST -------
lam <- lamda[2:6]
slope <- 2 * lam/obs
intercept <- cost - 0.5*slope*obs
C <- diag(slope)
C[1,1] <- 0.001
dvec <- price*yield - intercept
LHS <- rbind(rep(1,5), diag(1,5))
right <- c(1500, 0, 0, 0, 0, 0); dim(right) <- c(6,1)

QP <- OP( Q_objective(Q = -C, L = dvec), L_constraint(L = LHS, dir = c("<=", rep(">=", 5)),
                                          rhs = c(1500,0,0,0,0,0)), max = TRUE )
outcome <- ROI_solve(QP, solver = 'quadprog')
outcome$objval
outcome$solution
```