ECON 403: Crop Allocation Problem

Given the following information for a 1500 ac farm, construct a linear programming model that determines how much of each crop to plant.

| | Observed | Average | | Average |
|--------|----------|---------|---------|----------|
| | Acreage | Yield | Price | costs |
| Crop | (ac) | (bu/ac) | (\$/bu) | (\$/ac) |
| Wheat | 500 | 42 | \$7.50 | \$192.0 |
| Barley | 200 | 70 | \$4.25 | \$169.5 |
| Canola | 450 | 38 | \$11.50 | \$229.0 |
| Peas | 250 | 45 | \$6.75 | \$163.8 |
| Oats | 100 | 110 | \$2.75 | \$152.50 |

Solve the following simple model using R:

Maximize
$$GM = \sum_{k=1}^{n} (p_k x_k y_k - c_k x_k)$$

Subject to
$$\sum_{k=1}^{n} x_k \le 1500$$
$$x_k \ge 0$$

Now include the following constraint and solve the problem again:

$$x_k \le x_k^{obs} + 0.01, \forall k$$

For these constraints find the associated shadow prices, λ_k , and use this information to modify the objective function above assuming a quadratic cost function: $c_k = a x_k + b x_k^2$. Then:

$$b_k = 2 \times \lambda_k / x_k^{obs}$$
 and $a_k = c_k - \frac{1}{2} \times b_k \times x_k^{obs}$

Use the cost function in place of $c_k x_k$ in the objective function, so the revised objective is:

Maximize
$$GM = \sum_{k=1}^{n} (p_k x_k y_k - a_k x_k - b_k x_k^2)$$

Solve the revised problem using quadprod in R.