Agricultural Economics & Policy

Risk and the Agricultural Firm

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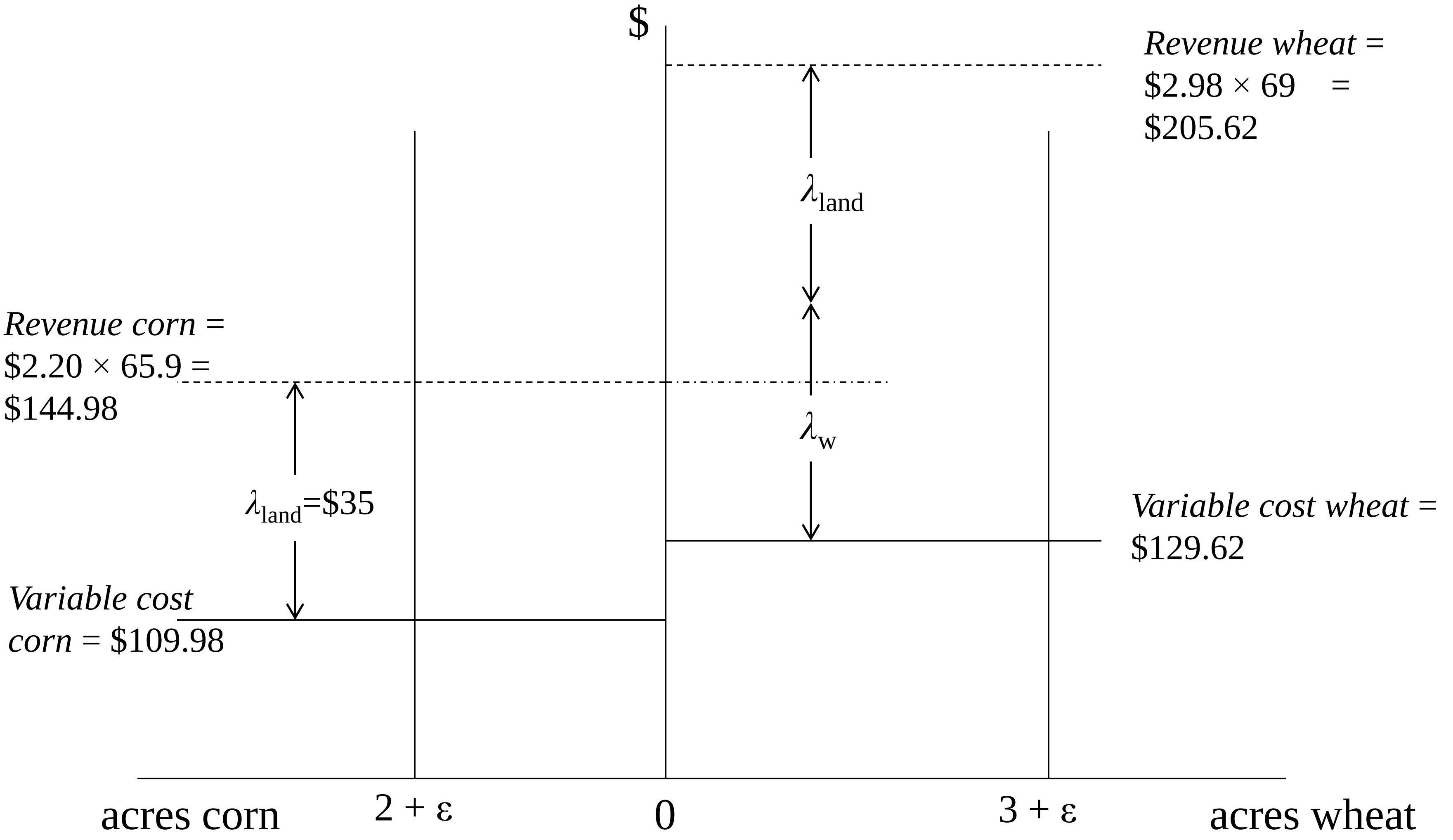
Calibration

Two Crop Example

Item	WHEAT	CORN
Crop prices ($/bu)	$2.98	$2.20
Variable cost ($/acre)	$129.62	$109.98
Average yield (bu/acre)	69.0 bu	65.9 bu
Gross margin ($/acre)	$76.00	$35.00
Observed allocation (acres) (Total acres = 5)	3 ac	2 ac

PMP Calibration: Two-crop Example



Mathematical Representation of Problem

Max $(\$2.98 \times 69 - \$129.62)\, W + (\$2.20 \times 65.9 - \$109.98)\, C$

s.t. (1) $W + C \leq 5$

(2) $W \leq 3.01$

(3) $C \leq 2.01$

$W, C \geq 0$

Recall the gross margins:

Wheat = \$76/ac

Corn = \$35/ac

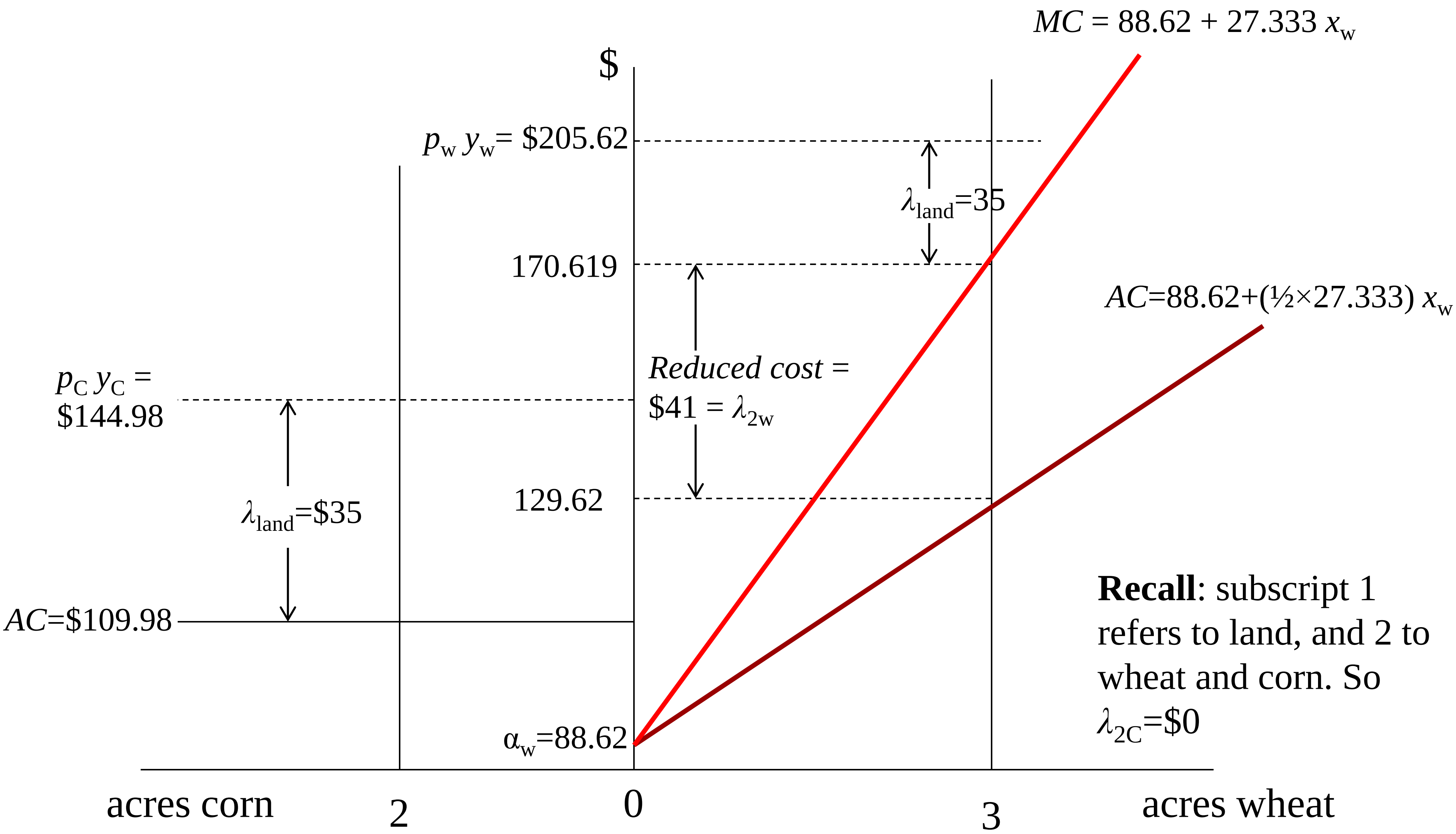
Solving using R gives:

$W = 3.01$, $C = 1.99$;

$\lambda_{land} = 35$, $\lambda_2 = [\lambda_w, \lambda_c] = [41\ 0]$

NOTE: If you do not have the ε=0.01 in constraints (2) and (3), then constraint (1) would be redundant!

PMP Calibrated Model



Recall: subscript 1 refers to land, and 2 to wheat and corn. So $\lambda_{2C}=\$0$

Notice the model is calibrated for one PMP activity but one LP activity is left, and the constraint on wheat still prevents an optimal

Calibrated model

Maximize $[(\$2.98 \times 69)\, W + (\$2.20 \times 65.9)\, C$
$- (88.62 + \tfrac{1}{2} \times 27.333 W) W - 109.98\, C]$

s.t. $W + C \leq 5$
$W, C \geq 0$

Utility Functions and Modeling Agricultural Risk

Risk attitudes: Modeling and managing risk using utility functions

- Kenneth Arrow observed that:

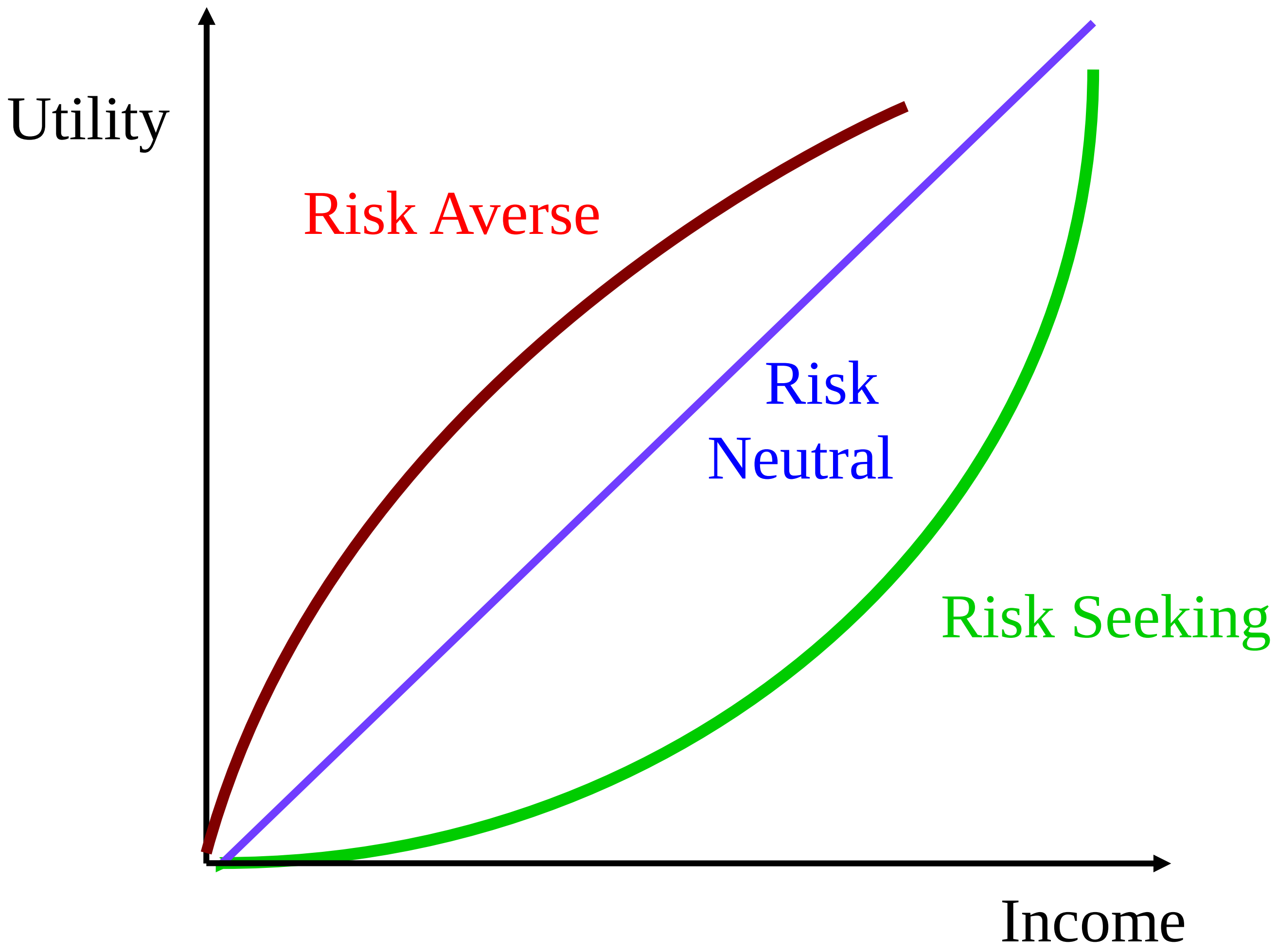
  (1) individuals display an aversion to risks

  (2) risk aversion explains many observed phenomena
- Measures used by economists:
  - expected return (ER)
  - expected utility (EU)

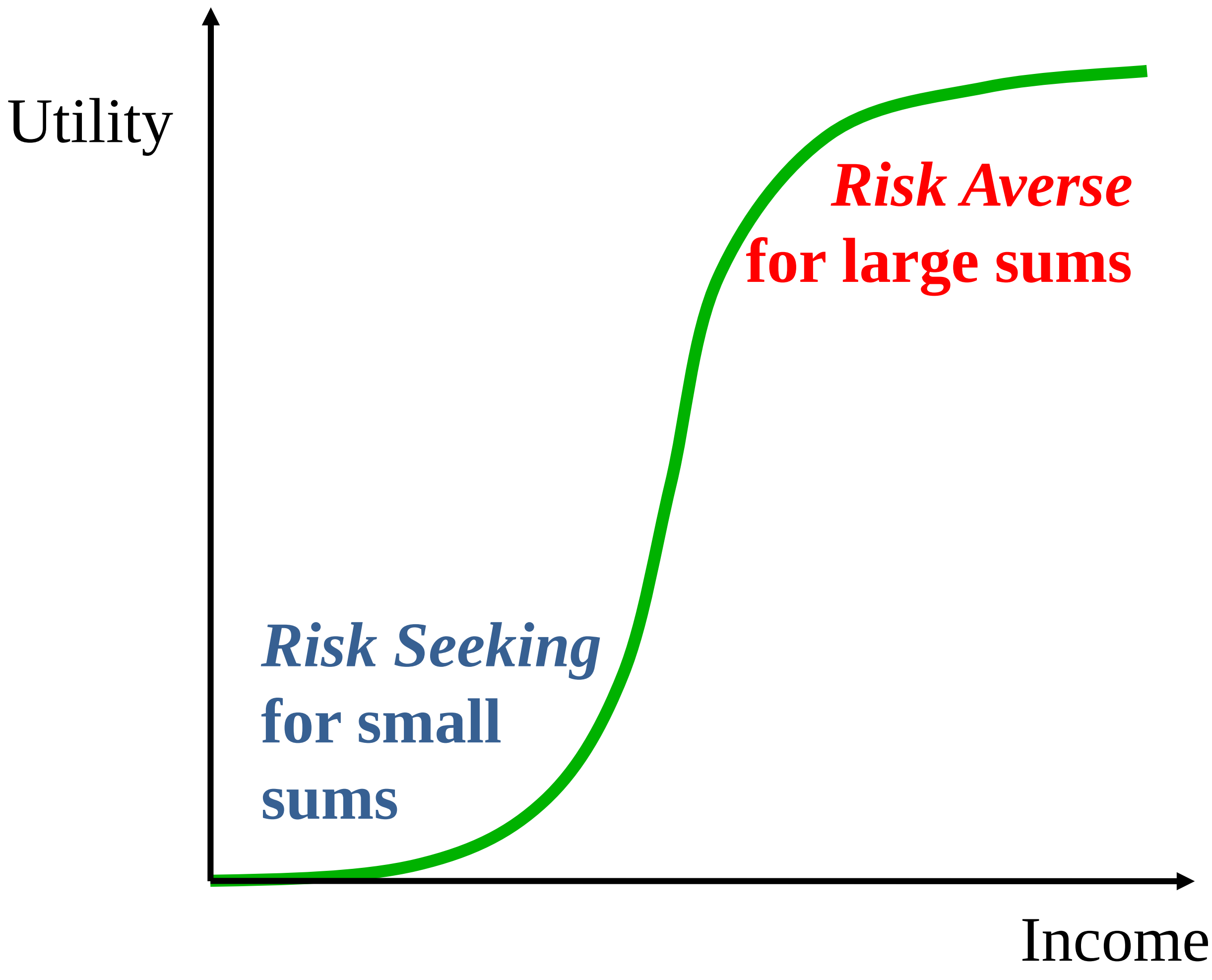
Marginal utility of income/wealth

- Linear marginal utility for income
  - *Risk Neutral*
- Decreasing marginal utility for income
  - *Risk Averse*
- Increasing marginal utility for income
  - *Risk Seeking*

Graphical Marginal Utility



Complex Marginal Utility



Risk Attitude

We begin with formal definitions related to risk attitudes.

Certainty Equivalence (CE)

Let » denote 'is preferred to'.

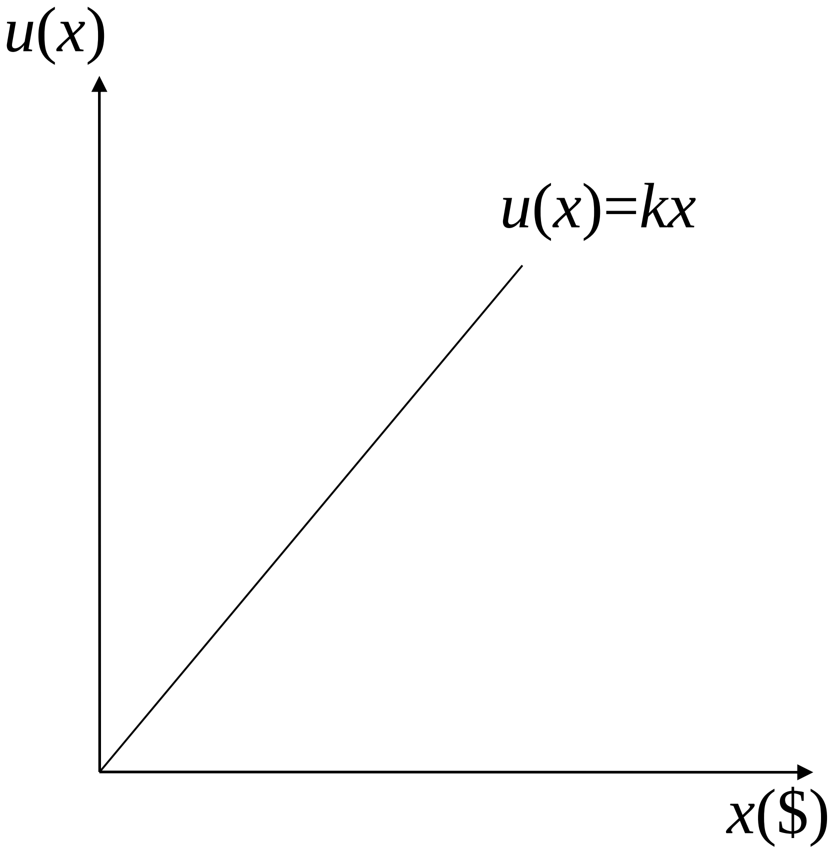
If $\mathbf{A_1} \gg \mathbf{A_2}$ and $\mathbf{A_2} \gg \mathbf{A_3}$, then there exists **p** such that the decision maker (DM) is indifferent to receiving $\mathbf{A_2}$ with certainty and the lottery:

$$\mathbf{A_2 \sim pA_1 + (l - p)A_3}$$

(where ~ denotes indifference)

$\mathbf{A_2}$ is the **CE** of $\mathbf{pA_1 + (l - p)A_3}$

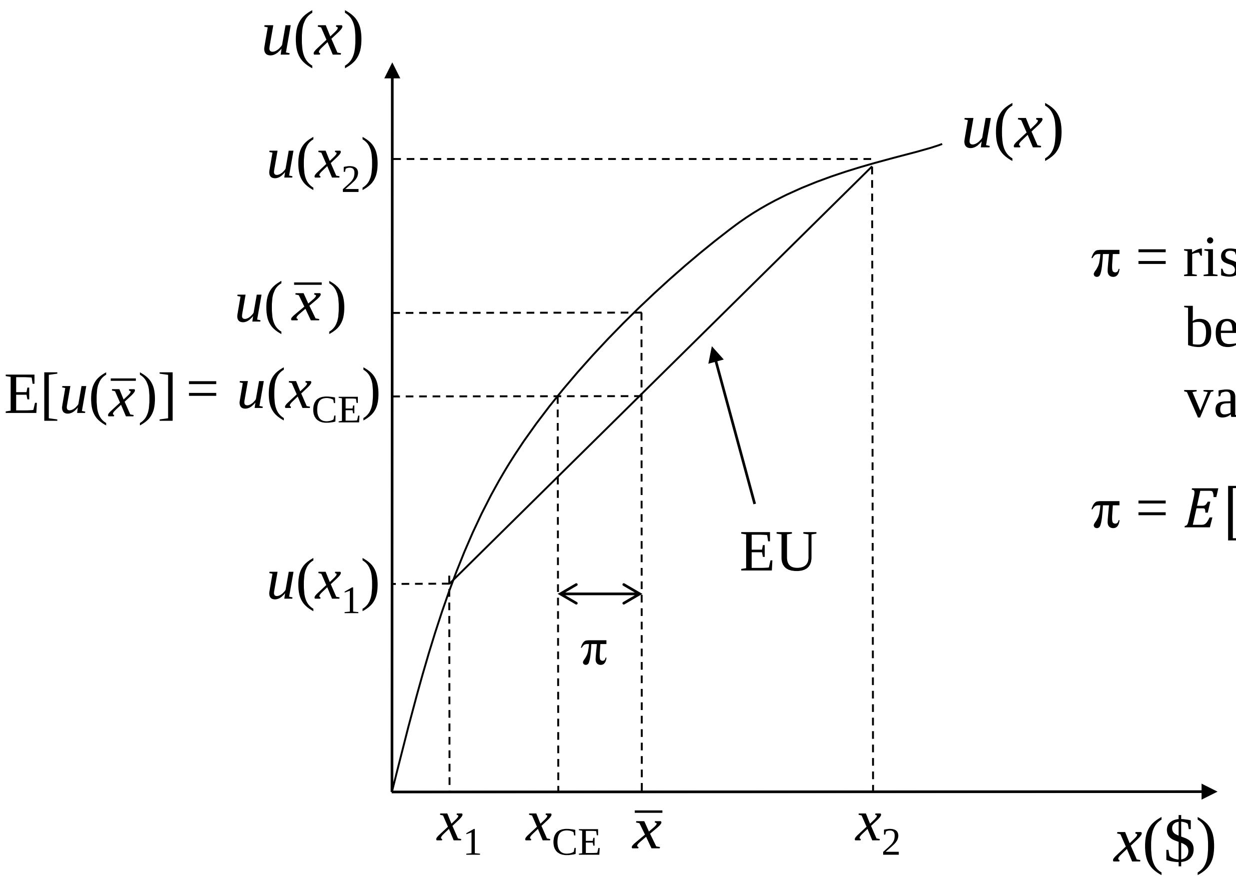
Risk neutral utility function



Straight line (quasi-concave, quasi-convex) utility function indicates risk neutral decision maker (DM).

$$u'(x) = k > 0,\ u''(x) = 0$$

Risk-averse utility function



π = risk premium = difference between expected monetary value and CE

$\pi = E[u(\bar{x})] - u(x_{CE})$

Strictly concave utility function indicates risk aversion.

$u'(x) > 0$, $u''(x) < 0$ and $\bar{x} = ½ (x_1 + x_2)$

$E[u(x)] = p\, u(x_1) + (1–p)\, u(x_2) = CE$

With risk aversion:

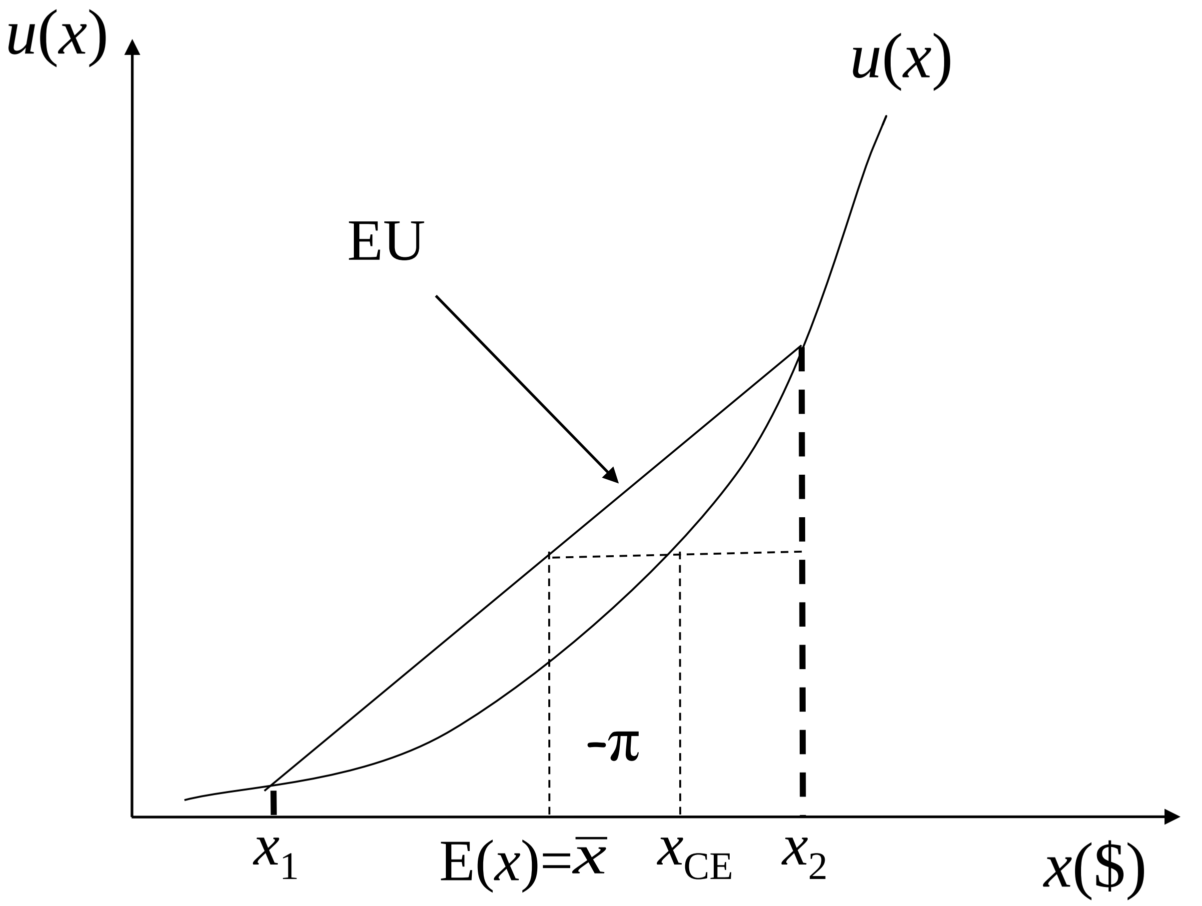
Utility increases with wealth but marginal utility (MU) falls, which implies the farmer prefers a certain return to an equal but uncertain one.

$$u'(x) > 0,\ u''(x) < 0$$

Risk taker has a strictly convex utility function:

$$u'(x) > 0,\ u''(x) > 0$$

Risk taker utility function



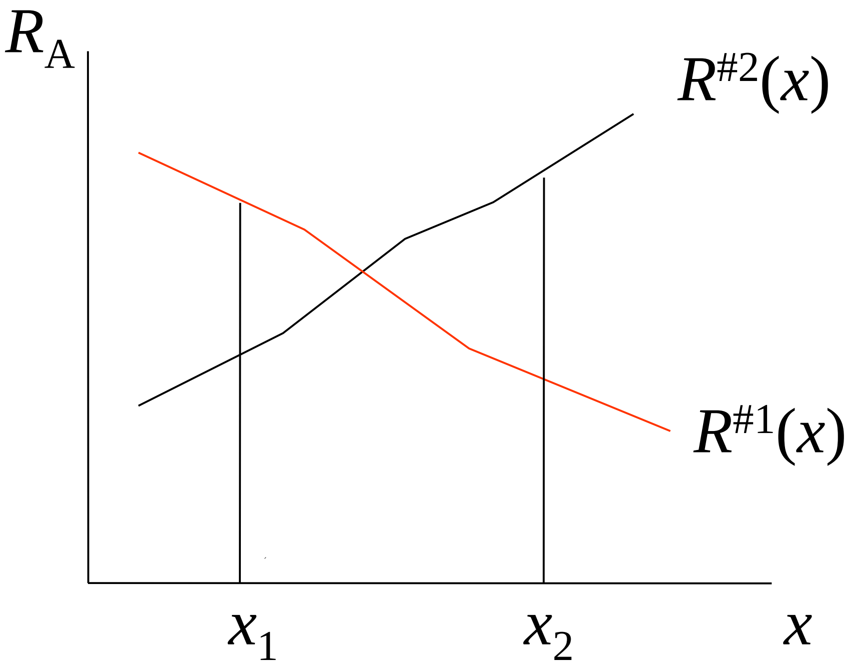
Extreme risk lovers concentrate on upside risk and tend to be less concerned about downside risk. Risk premium is negative (–π) → DM is willing to pay to take on risk

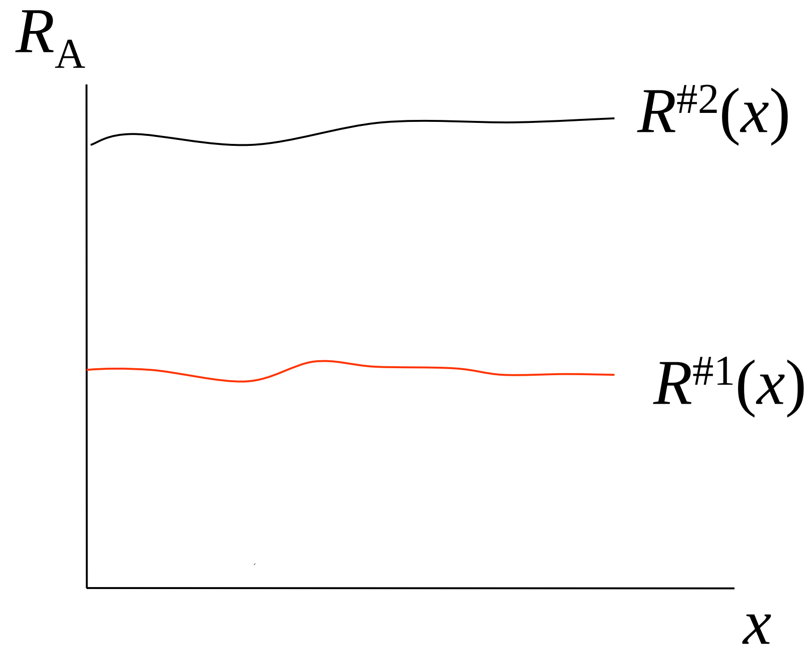
Measures of Risk Aversion

Absolute : $$R_A(x) = -\frac{u''(x)}{u'(x)}$$

Relative : $$R_R(x) = -\frac{xu''(x)}{u'(x)}$$

- Unaffected by transformations in *u*
- Positive values imply risk aversion – the larger the value, the greater the risk aversion
- Negative values imply risk taking

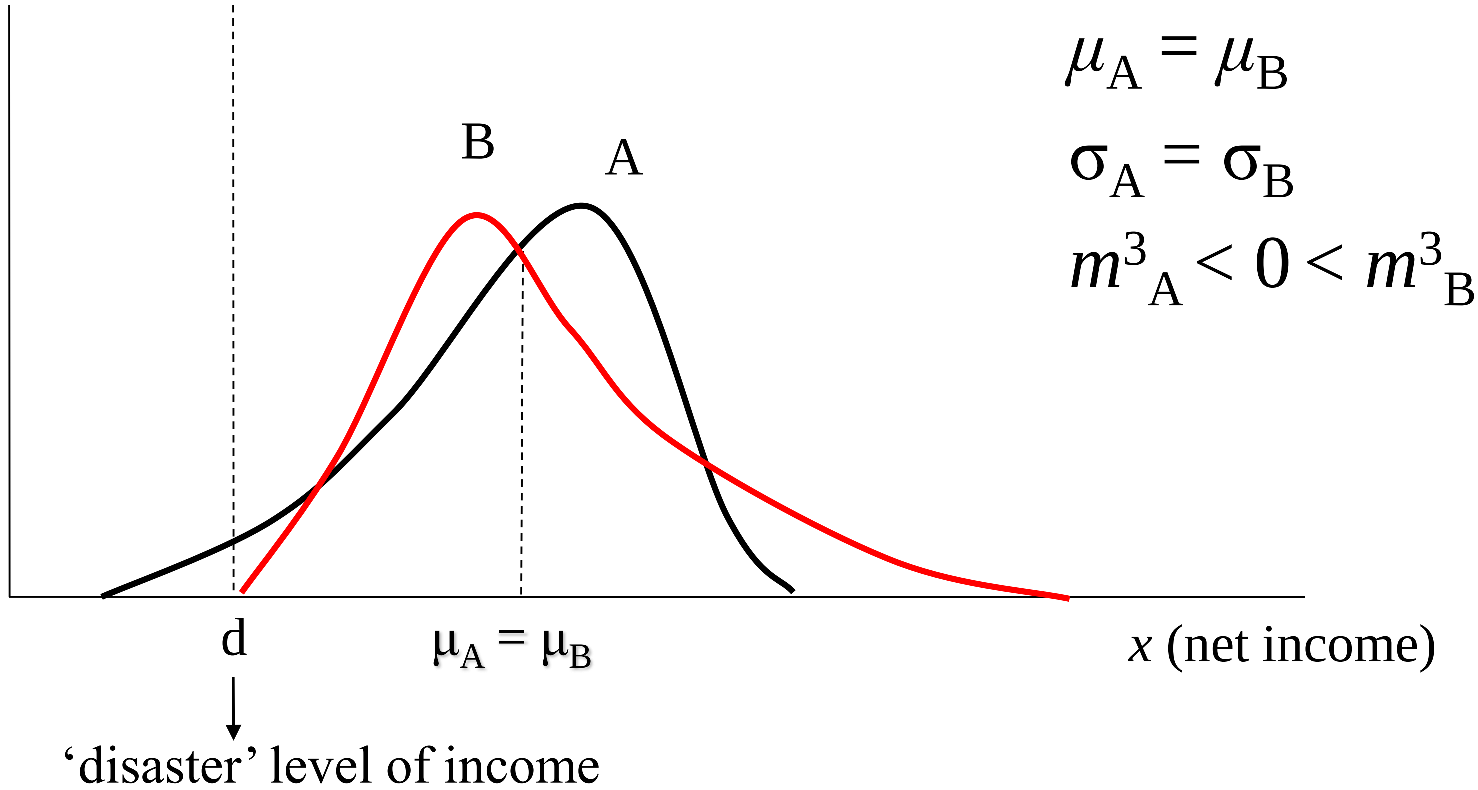




One person has greater risk aversion than the other, but only ‘in the small’ as it depends on points x_1 and x_2

person #2 is everywhere more risk averse than #1 – ‘risk aversion in the large’

Moments of a probability distribution

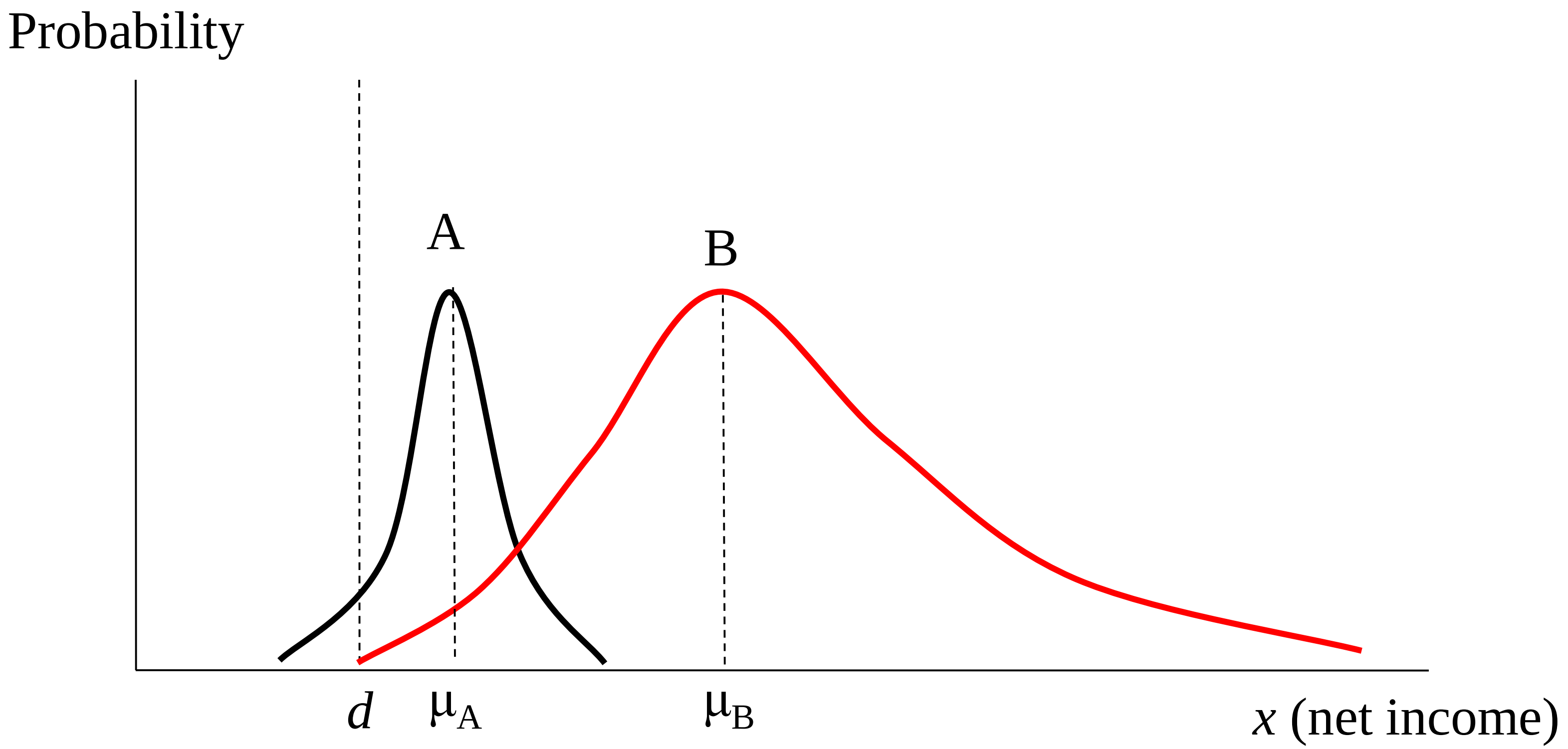


Variance and mean rank A and B equally

Skewness ranks B over A ($m^3{}_A < m^3{}_B \rightarrow$ B is preferred)

Consider 'chance of loss' as a risk constraint: Prob($x \leq$ d) $\leq \alpha$

Kurtosis is the 4th moment; there also exist higher moments, although this depends on functional form of the probability density function.



Risk constraint: $P(x \leq d) \leq \alpha$ (d is disaster level)

In diagram: $\mu_A < \mu_B$ and $\sigma_A < \sigma_B$

Chance of loss ranks A as more risky than B, while variance ranks B as more risky than A

<u>Power utility function</u>: Decreasing absolute risk aversion but constant relative risk aversion

$$U(x) = \frac{x^{1-\lambda}}{1-\lambda}$$

Then

$$U'(x) = x^{-\lambda} \text{ and } U''(x) = -\lambda x^{-\lambda-1}$$

Consequently,

$$R_A(x) = -\frac{U''(x)}{U'(x)} = -\frac{-\lambda x^{-\lambda-1}}{x^{-\lambda}} = \frac{\lambda}{x}$$

$$R_R(x) = -x\frac{U''(x)}{U'(x)} = -x\frac{-\lambda x^{-\lambda-1}}{x^{-\lambda}} = \lambda$$

<u>Exponential utility function</u>: Constant absolute risk aversion but increasing relative risk aversion

$$U(x) = a - be^{-\lambda x},\ b, \lambda > 0$$

Then

$$U'(x) = b\lambda e^{-\lambda x} \text{ and } U''(x) = -b\lambda^2 e^{-\lambda x}$$

Consequently,

$$R_A(x) = -\frac{U''(x)}{U'(x)} = -\frac{-b\lambda^2 e^{-\lambda x}}{b\lambda e^{-\lambda x}} = \lambda$$

$$R_R(x) = -x\frac{U''(x)}{U'(x)} = -x\frac{-b\lambda^2 e^{-\lambda x}}{b\lambda e^{-\lambda x}} = \lambda x$$

<u>Quadratic utility function</u>: Increasing absolute and relative risk aversion

$$U(x) = x - \frac{1}{2}\lambda x^2,\ \lambda > 0 \quad \text{Then}$$

$$U'(x) = 1 - \lambda x \text{ and } U''(x) = -\lambda$$

Consequently,

$$R_A(x) = -\frac{U''(x)}{U'(x)} = \frac{\lambda}{1-\lambda x}$$

$$R_R(x) = -x\frac{U''(x)}{U'(x)} = -x\frac{-\lambda}{1-\lambda x} = \frac{\lambda x}{1-\lambda x}$$

$$\frac{dR_A(x)}{dx} = \frac{\lambda^2}{(1-\lambda x)^2} > 0; \quad \frac{dR_R(x)}{dx} = \frac{\lambda}{1-\lambda x}\left[1 + \frac{\lambda x}{1-\lambda x}\right] > 0$$

Modeling risk attitude

- There are various ways to model risk attitudes
- Economists have come up with conditions that decision makers (DMs) should meet if their decisions are to be considered ‘rational’
  - Expected utility maximization (EUM) is considered a benchmark in this regard, although many decision criteria fail to meet its requirements
  - We adopt EUM as a benchmark for comparison purposes

DECISION RULES

Maximization of (1) expected (net) return (ER) or (2) expected utility (EU)

Question: Does a decision rule violate the expected utility maximization (EUM) hypothesis?

As we show in the next slides, expected revenue maximization satisfies the EUM hypothesis:

→ For a linear utility function, EU leads to the same outcome as ER

ER Maximization:

$$\underset{j}{\text{Max}}\left[E(R_j)\right] = \underset{j}{\text{Max}}\sum_{i=1}^{n} p_{ij}x_{ij},\ j = 1,....,k \text{ (actions)}$$

$$i = 1,.....,n \text{ (outcomes)}$$

p_{ij} = probability event i occurs and action j is taken

x_{ij} = payoff if event i occurs and action j is taken

DM generally maximizes expected utility rather than expected net return

EU maximization:

$$max\,E\left[u(x)\right] = \max_{j} \sum_{i} p_{ij} u(x_{ij})$$

p_{ij} = probability of event *i* occurring and you plant crop *j*

Event *i* might refer to a certain level of GDDs, precipitation, pests, weeds, low price at harvest, et cetera.

Mean-Variance (EV) analysis

Background to mean-variance analysis:

A Taylor series expansion about mean μ_j gives:

$$E[u_j] = f(\mu_j, \sigma^2{}_j, m^3{}_j, m^4{}_j, \ldots),$$

where $\sigma^2{}_j$ is the variance, $m^3{}_j$ is skewness, $m^4{}_j$ is kurtosis and there exist higher moments of the probability distribution

EUM works only if expected utility has two moments – only if (1) the utility function is quadratic, or (2) net returns are normally distributed. With only two moments:

$$E(u_j) = f(\mu_j, \sigma^2_j)$$

In contrast, if utility is linear there is only one moment:

$$E(u_j) = f(\mu_j)$$

Variance or standard deviation simply measures dispersion of net returns and is defined as:

$$V(x) = \sigma^2 = \sum_i \left[x_i - E(x_i)\right]^2 p(x_i)$$

Problem with *V*(*x*) is that deviations above the mean are penalized the same as those below the mean.

Consider again the exponential utility function and normally distributed net returns. Does this satisfy EUM hypothesis?

$$u(x) = a - b\, e^{-\lambda x} \qquad b, \lambda > 0 \qquad \text{(exponential)}$$

If x ~ Normal, then max E[$u(x)$] is equivalent to maximizing

$$E[u(x)] = (1/\lambda) \exp[\lambda(\mu + ½\, \lambda\sigma^2)]$$

In essence, we can write the expected utility function as:

$$E[u(x)] = a - be^{-\lambda E(x) + \frac{1}{2}\lambda^2 V(x)}$$

$$E[u(x)] = E(x) - ½\, \lambda\, V(x) \qquad \text{by transformation}$$

Since there are only two moments in this expression, EV applies when maximizing an exponential utility function and assuming normality of x.

A normal distribution is fully described by the first two moments: E(x) and V(x).

Recall, for exponential utility function:

$R_A = \lambda$ is the degree of risk aversion.

EV Decision Rule:

$$\text{Max E}[u(x)] = \text{E}(x) - \tfrac{1}{2}\lambda \text{V}(x), \; \lambda \text{ given.}$$

It provides an ordering of alternatives consistent with the EUM hypothesis.

If λ is unknown, the EV criterion can be used to order risky choices into efficient and inefficient sets.

Two versions of EV Model:

- Freund
- Markowitz

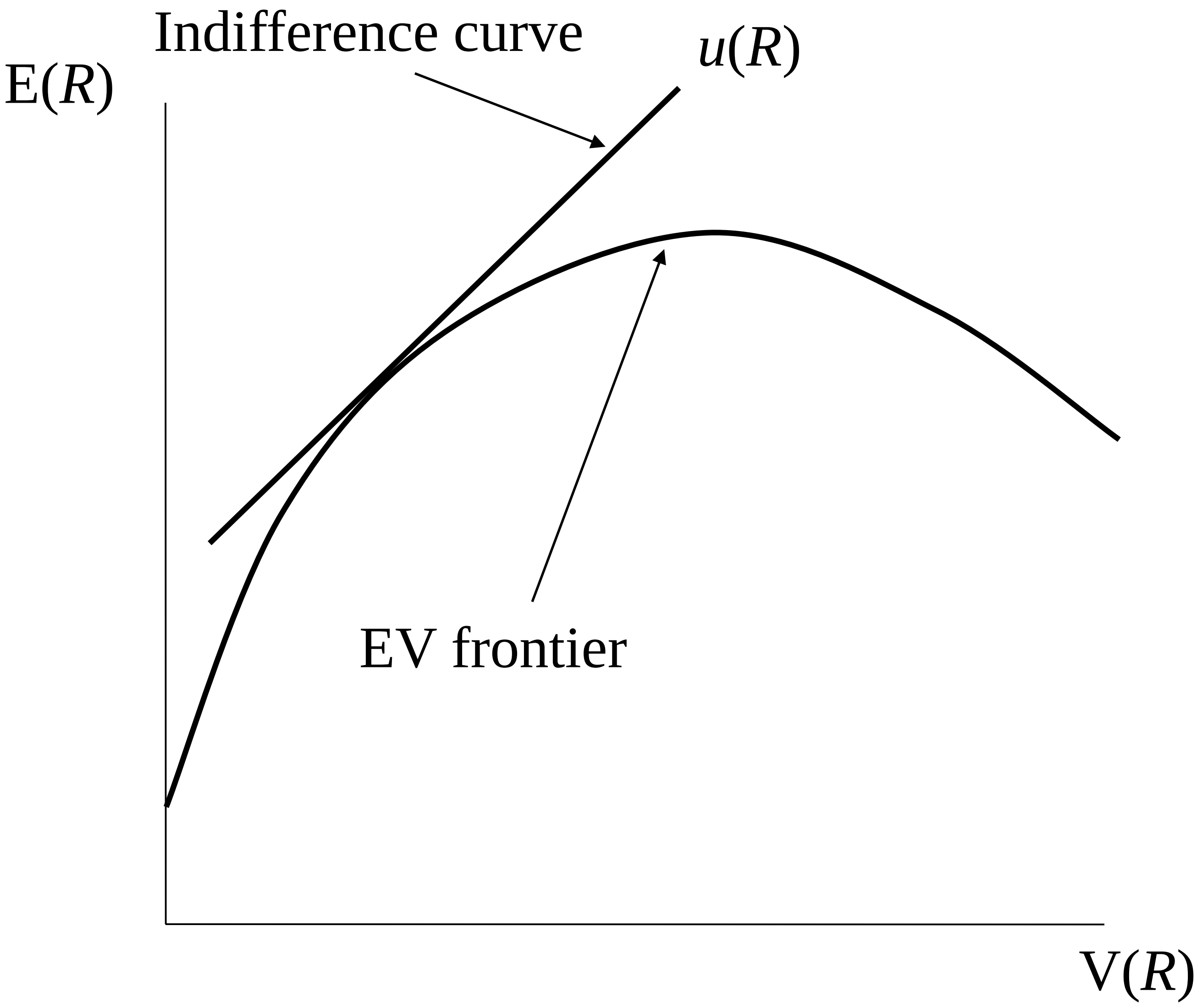
Freund Approach

Max	$E(R) - \frac{1}{2} \lambda V(R)$	
s.t.	$\boldsymbol{AX} \leq \boldsymbol{b}$	(technical constraints)
	$\boldsymbol{x} \geq 0$	(non-negativity)

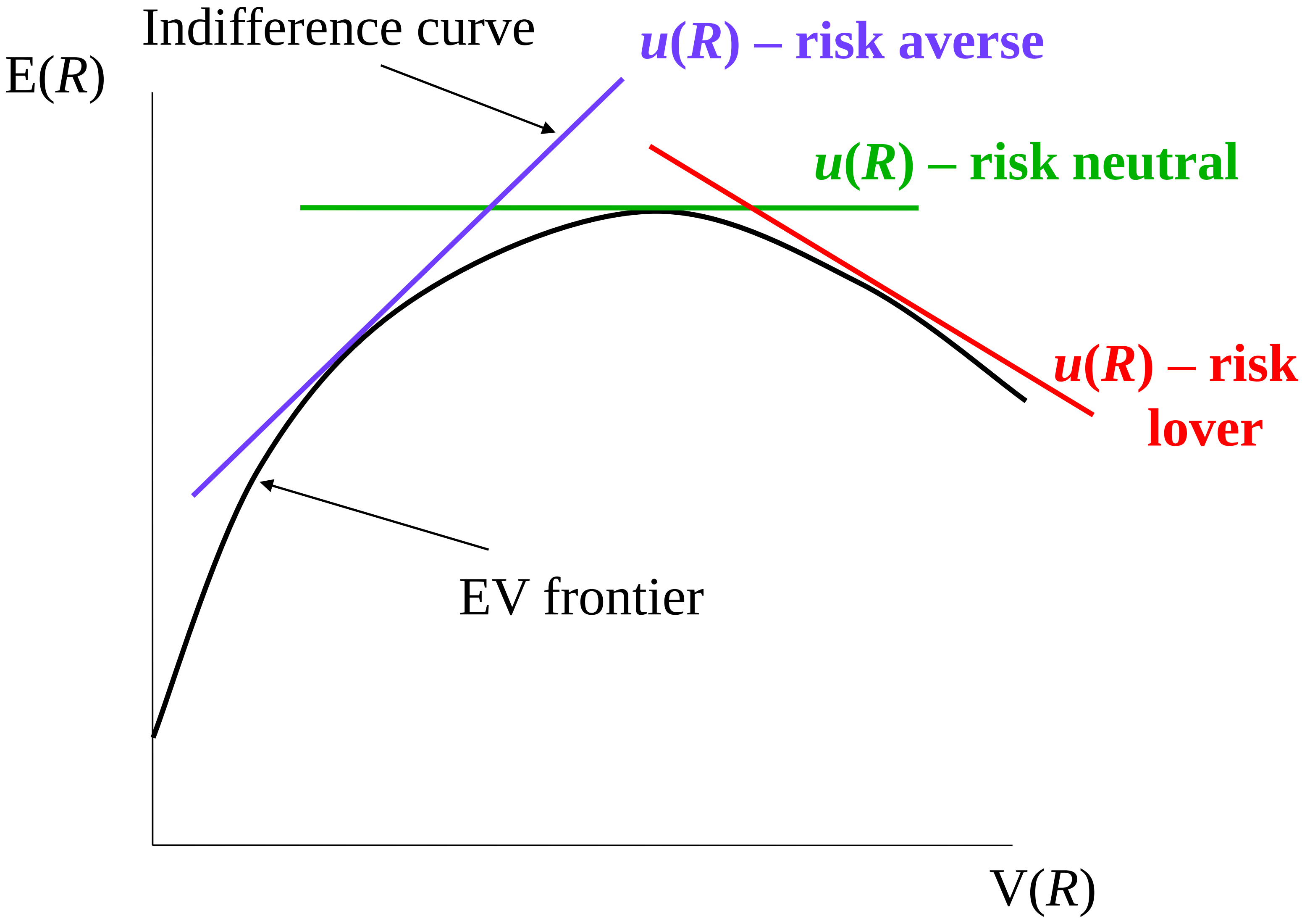
NOTE: R is a function of the decision vector, $\boldsymbol{X}$

λ is an Arrow-Pratt risk aversion coefficient discussed earlier.

If λ is unknown, we could vary λ and solve the program for its various values.



EV is the expected return – variance


Indifference curve
E(*R*)
u(*R*) – risk averse
u(*R*) – risk neutral
u(*R*) – risk lover
EV frontier
V(*R*)

Further points

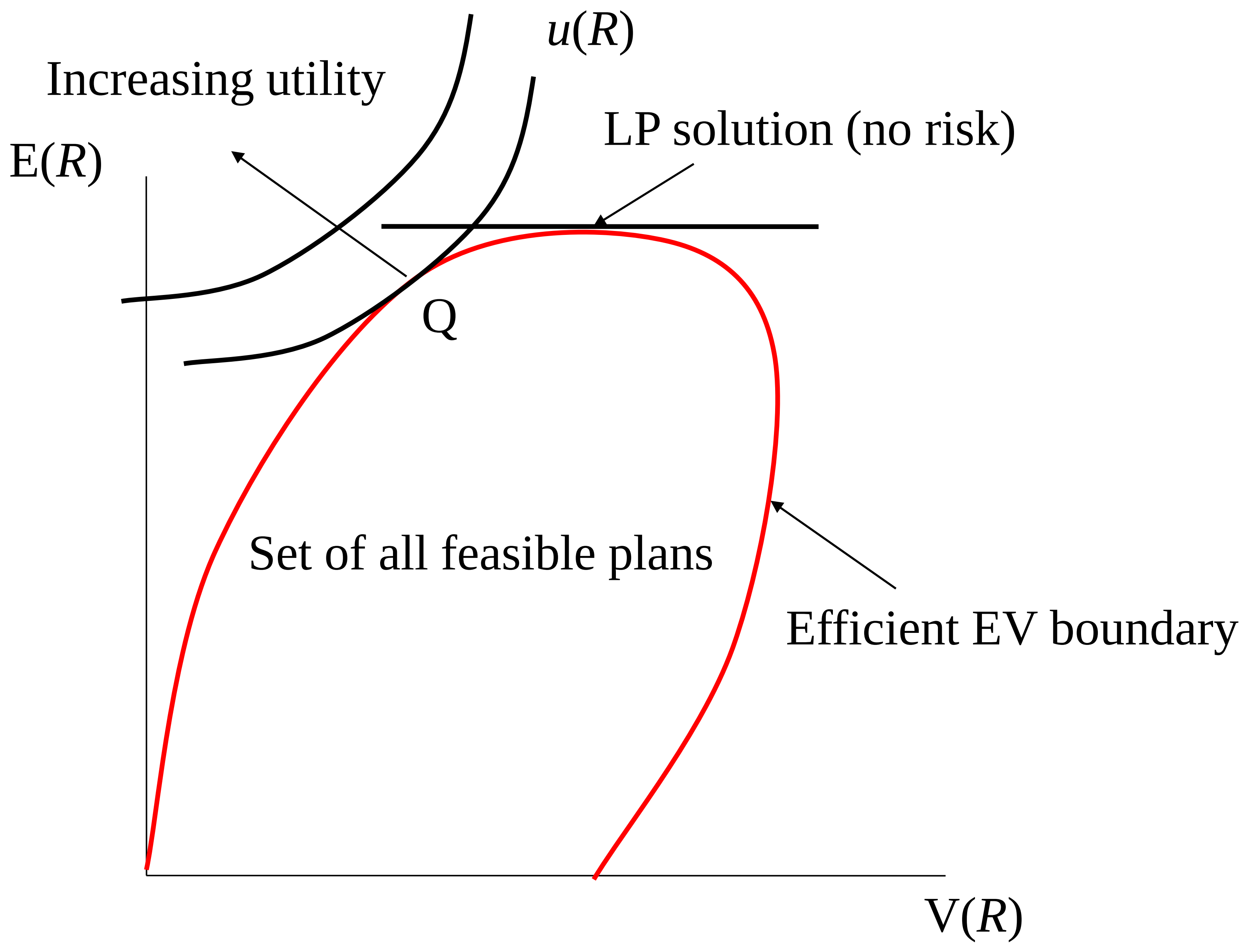
- Quadratic programming has sometimes been referred to as risk programming because EV analysis requires use of QP
- The Freund method of the previous diagram employs an ***elicited*** risk parameter to identify the optimal point on the EV frontier.
- An alternative that does not elicit risk parameters is the Markowitz approach.

Markowitz Approach to EV Analysis

Minimize	$V(R)$
s.t.	$E(R) \geq k$
	$\boldsymbol{A}\,\boldsymbol{X} \leq \boldsymbol{b}$
	$\boldsymbol{X} \geq 0$

where k is varied in some iterative fashion to trace out the set of risk efficient (minimum variance) solutions – EV frontier (see diagram next slide)

Again $\boldsymbol{X}$ is the vector of activity levels or decision variables


u(R)
Increasing utility
LP solution (no risk)
E(R)
Q
Set of all feasible plans
Efficient EV boundary
V(R)

Notes pertaining to previous diagram

- We do not know the DM's utility function or tradeoff between expected returns and variance of returns – Markowitz's approach cannot identify the optimal plan Q
- The LP solution is obtained by maximizing expected return E(*R*) because V(R) requires a quadratic, which would result in a nonlinear objective. So the LP solution can only give the highest expected outcome.

Agricultural BRM in the U.S.

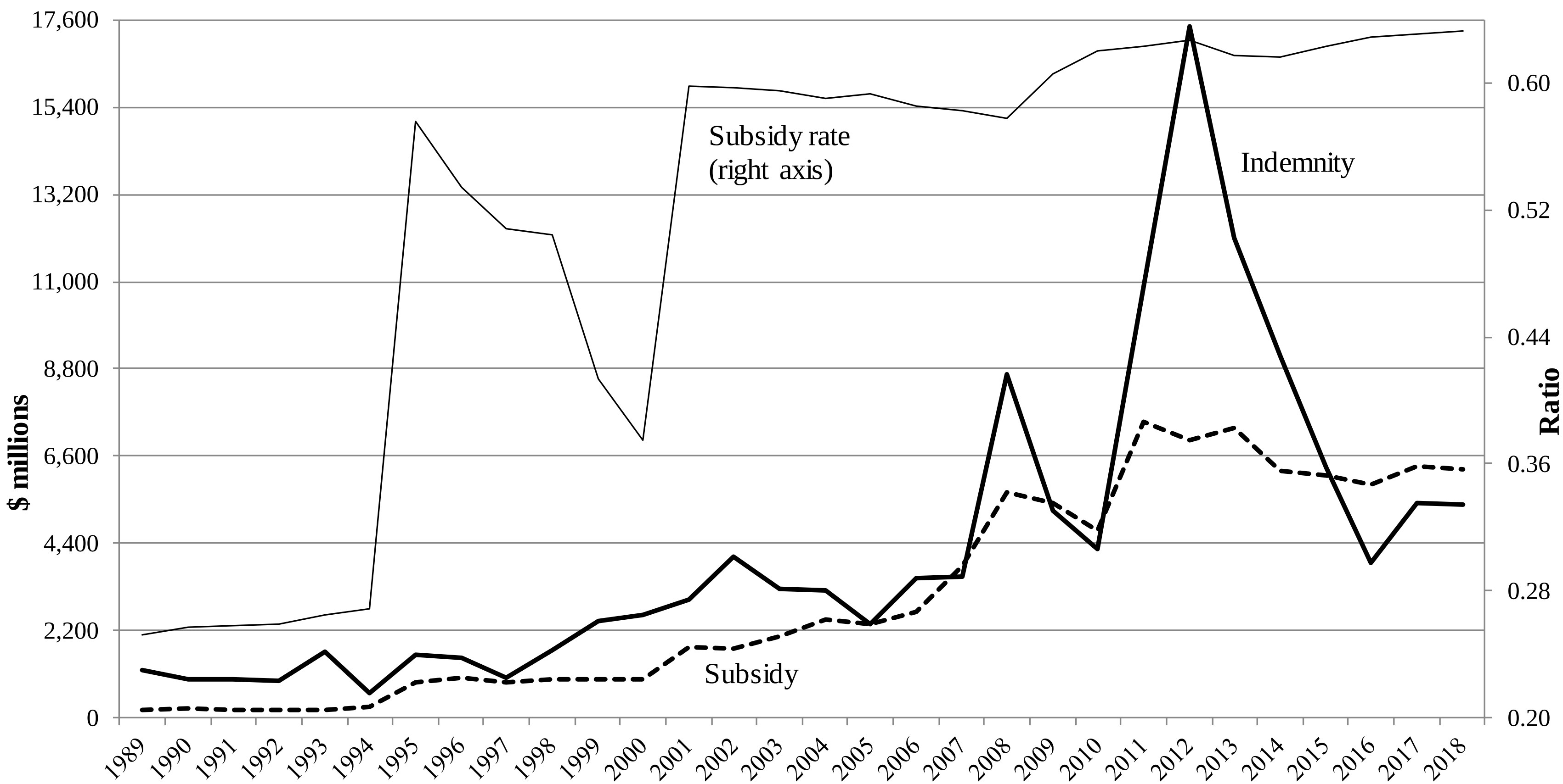
- Deep Loss Protection: Crop Insurance
  - *Federal Crop Insurance Act* (1980)
    - Mandated shift to private delivery
    - Gov't covered O&A costs plus underwriting risks; companies received 33% of premiums to cover O&A
    - 30% subsidy rate on premiums
    - Only 18% uptake
  - *Federal Crop Insurance Reform Act* (1994)
    - Premium subsidy of 40%; mandated expansion of crops covered
    - lowered the A&O payment as a proportion of total premiums to 31%, and eventually to 27%
    - Included a catastrophic loading factor of 13.4% on premiums to ensure underwriting function

Spending by Type of U.S. Crop Program, 1961-2012, Annual Average based on Fiscal Year (FY)

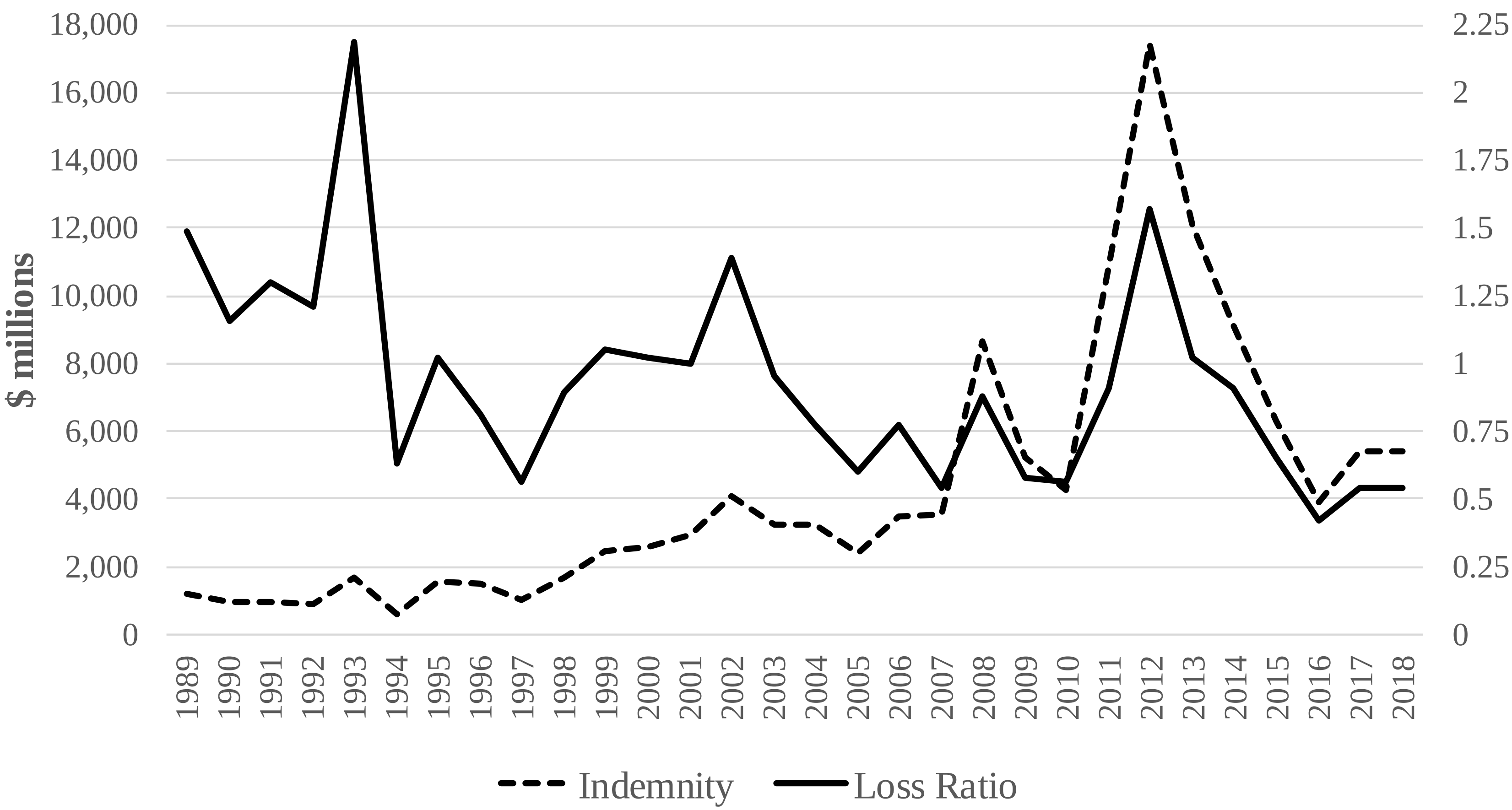
Period (FYs)	Total Payments	Low-price required	Yield or revenue decline required		
			Fixed payment	Disaster	Insurance
1961-1973	$1.7	100%	0%	0%	0%
1974-1995	$7.5	88%	0%	8%	4%
1996-2006	$14.5	49%	35%	7%	9%
2007-2012	$11.6	12%	39%	8%	41%

Low price programs include non-recourse/marketing loans, deficiency payments/ CCP, PIK; fixed payments are programs that use a fixed unit rate multiplied by historical yields; disaster consists primarily of ad hoc disaster relief; and insurance refers to indemnities paid to farmers for losses minus premiums they paid

Crop Insurance Subsidy and Indemnities (left axis) and Subsidy Rate (right axis), Annual, 1989 through 2018



Net indemnities from U.S. Crop Insurance Programs (left axis) and Ratio of Crop Insurance Payments to Total Insurance Premiums (right axis), 1989-2018



Agricultural Risk Protection Act (2000)

- Increased premium subsidy so that it averaged 62%
- increased A&O payments and extended the premium subsidy to take advantage of the Harvest Price Option (HPO), which uses the higher of the spring planting or harvest price.
  - HPO facilitated a massive shift out of yield insurance into revenue insurance
  - Proportion of eligible acres in the U.S. covered by yield insurance fell from 93% in 1996 to only 15% in 2013.

2014 Farm Bill Reference Prices ($US) i.e. target price

Commodity	Reference price	Commodity	Reference price
Wheat	5.50/bu	Soybeans	8.40/bu
Corn	3.70/bu	Other oilseeds	20.15/cwt
Grain sorghum	3.95/bu	Peanuts	535/ton
Barley	4.95/bu	Dry peas	11/cwt
Oats	2.40/bu	Lentils	19.97/cwt
Long grain rice	14/cwt	Small chickpeas	19.04/cwt
Medium grain rice	14/cwt	Large chickpeas	21.54/cwt