ECON 549
Lab #1
February 26, 2009

Positive Mathematical Programming: Crop Allocation Model

Statement of Problem

Three crops are planted in two regions of the United States, California (CA) and the rest of the US (RUS). The objective is to construct a model that enables us to conduct policy related to the allocation of cropland among these crops in the three regions. For example, we might implement policy that affects yields (tax/subsidy on fertilizer, regulations on chemical use) or output prices (tax on food exports; subsidies on domestic clothing manufacturing that causes cotton prices to rise relative to other crop prices). We are interested in determining the affects of such policy. Of course, our example is contrived and very simplistic, but the idea is to calibrate a model that can then be used to investigate the impacts of such policies. This modeling approach is used in Europe, for example, to predict potential impacts of changes in the Common Agricultural Policy, environmental policy, or liberalization of trade in agricultural commodities.

Required information for the model is given in the following tables:

Region/Crop	Price per unit output	Regional average yields
California		
Cotton	292.4	2.20
Wheat	2.98	85.0
Rice	7.09	70.1
Rest of US		
Cotton	292.4	1.51
Wheat	2.98	69.0
Rice	7.09	48.1

	Resource Variable Costs		
Region/Crop	Land	Water	Capital
California			
Cotton	264.3	25.6	0.1
Wheat	112.0	25.6	0.1
Rice	196.9	25.6	0.1
Rest of US			
Cotton	111.8	28.4	0.1
Wheat	175.0	28.4	0.1
Rice	158.2	28.4	0.1

Region/Crop	Base Resource Use			Regional Resource Constraints		
	Land	Water	Capital	Land	Water	Capital
California				2.65	8.69	2,000.0
Cotton	1.49	4.47	610			
Wheat	0.62	1.14	350			
Rice	0.54	3.08	520			
Rest of US				14.99	28.33	7,000.0
Cotton	5.75	5.23	490			
Wheat	6.50	6.89	280			
Rice	2.74	7.95	480			

Following Howitt, we calibrate the PMP model on yield as opposed to costs. Thus, it is necessary to determine a numeraire input, which in this example is land. Hence, we divide base resource use by land to obtain the regional Leontief coefficients as follows:

Region/Crop	Adjusted base resource use as proportion of land (XX)		
	Land	Water	Capital
California			
Cotton	1.00	3.000	409.396
Wheat	1.00	1.839	564.516
Rice	1.00	5.704	962.963
Rest of US			
Cotton	1.00	0.910	85.217
Wheat	1.00	1.060	43.077
Rice	1.00	2.901	175.182

Region/Crop	Adjusted linear cost	Adjusted net return
California		
Cotton	382.040	261.240
Wheat	215.523	37.777
Rice	439.211	57.798
Rest of US		
Cotton	146.153	295.371
Wheat	209.412	-3.792
Rice	258.120	82.909

Linear Programming Model for Calibration Purposes

$$\text{Maximize} \quad Z = \sum_{g=1}^{2}\sum_{i=1}^{3}\left(p_{g,i}y_{g,i} - C_{g,i}x_{g,i}\right)$$

subject to:

$$\sum_{i=1}^{3} R_{g,i,j}x_{g,i} \le \overline{R}_{g,j}, \ \forall g(=\text{CA, RUS}), j(=\text{land, water, capital})$$

$$x_{g,i} \le XX_{g,i,LAND} \times 1.001, \ \forall g(=\text{CA, RUS}), i(=\text{cotton, wheat, rice})$$

$$x_{g,i} \ge XX_{g,i,LAND} \times 0.999, \ \forall g,i \quad \text{IF} \ \left(p_{g,i}y_{g,i} - C_{g,i}x_{g,i}\right) < 0$$

where p_{gi} refers to price in region *g* of product *i*, *y* refers to (average) yield, and *C* refers to (average) cost, all on a per acre basis. The decision variable is number of acres of each crop to plant in each region, x_{gi}. $\overline{R}_{g,j}$ is the amount of resource *j* available in region *g*, while $R_{g,i,j}$ is the amount of resource *j* needed per acre in region *g* to produce crop *i*. Finally, $XX_{g,i,land}$ is the adjusted base land resource required in region *g* to produce crop *i* (as noted in one of the tables above).

Solution to calibration LP:

$Z = 2347.283529$

	Acres Planted		Shadow values on upper calibration constraints (λ_{2i})		Shadow values on lower calibration constraints (μ_{2i})	
	CA	RUS	CA	RUS	CA	RUS
Cotton	1.491	5.756	217.448	295.371	—	—
Wheat	0.619	6.435	0.000	0.000	—	-3.792
Rice	0.540	2.743	0.000	82.909	—	—

	Shadow values on resource constraints (λ_{2j})	
	CA	RUS
Land	28.253	0.000
Water	5.180	0.000
Capital	0.000	0.000

Notice that some crops have negative nominal revenue, but are nonetheless observed in practice. These are *rotational crops* and differ from *marginal crops*. Rotational crops might be used to prevent disease when some other marginal crop is planted year after year. To take into account the negative revenues, multiply yield by some factor, say 1.2. Then

$$Y_{g,i} = y_{g,i} \times 1.2$$

This is the maximum yield and we could get this information from actual data on crop production in the region (a table of yields) or using prior information on elasticities of supply. Now, we have to adjust the opportunity costs of the marginal crops:

$$A_g = \lambda_{1(g,land)} \times 0.2$$

Then, the opportunity cost of land, OC, is:

$$OC_g = \lambda_{1(g,land)} - A_g = 0.8\ \lambda_{1(g,land)}$$

Define a flag function:

k=1 if $XX_{g,i,land} \neq 0$ and $\lambda_{2(g,i)}$=0 and $\mu_{g,i}$=0 (marginal crops),
k=0 otherwise

Now specify a yield function for marginal crops in each region using a 20% reduction in opportunity costs:

$$y^M_{g,i} = A_g/(p_{g,i} \times x^*_{g,i}) \text{ and } y^C_{g,i} = y_{gi} + (y^M_{g,i} \times x^*_{g,i}) \qquad \text{if } k=1$$

where y^M is the yield for marginal crops, y^C is the maximum calibrated yield, y is average regional yield (from observation and provided in first table above), $x^*_{g,i}$ are the optimal allocations of activities as determined from the above LP.

Now calculate the yield functions for normal PMP crops:

$$y^M_{g,i} = (\lambda_{2(g,i)} + A_g) / (p_{g,i} \times x^*_{g,i}) \text{ and } y^C_{g,i} = y_{gi} + (y^M_{g,i} \times x^*_{g,i}) \qquad \text{if } k=0$$

Now calculate the yield function for rotational crops where net revenue was negative.

$$y^C_{g,i} = Y_{gi} \text{ and } y^M_{g,i} = (Y_{gi} - y_{gi}) / x^*_{g,i} \quad \text{if } \mu_{g,i} < 0$$

The cost adjustment for rotational crops:

$$A^R_{g,i} = C_{g,i} \qquad \text{if } \mu_{g,i} \leq 0$$
$$A^R_{g,i} = A^R_{g,i} - [(y^C_{g,i} - y_{gi}) \times p_{g,i}] - (\mu_{g,i} + A_g) \quad \text{if } \mu_{g,i} < 0$$

Policy changes can be entered by, for example, changing prices or quantity available of a resource. Thus, a 15% reduction of wheat prices in all regions would be $p_{g,wheat} = 0.85\ p_{g,wheat}$ and a 10% reduction in water availability in California as $R_{CA,water} = 0.9\ R_{CA,water}$.

Results for current model:

	Fixed Maximum Yield ($Y_{g,i}$)	
	CA	RUS
Cotton	2.640	1.812
Wheat	102.0	82.800
Rice	84.120	57.720

Adjustment to marginal crop duals (A_g): $A_{CA} = 5.651$

Opportunity cost of land (OC_g): $OC_{CA} = 22.602$

	Adjusted cost for rotationals ($A^R_{g,i}$)	
	CA	RUS
Cotton	382.040	146.153
Wheat	215.523	164.496
Rice	439.211	258.120

k=1 for California wheat and rice.

	Marginal yield parameter ($y^M_{g,i}$)	
	CA	RUS
Cotton	0.512	0.176
Wheat	3.063	2.145
Rice	1.477	4.264

	Calibrated maximum yield ($y^C_{g,i}$)	
	CA	RUS
Cotton	2.963	2.520
Wheat	86.896	82.800
Rice	70.897	59.794

PMP Model

Now the PMP model solution for the base year becomes:

$$\text{Maximize} \quad Z = Z1 + Z2$$
$$\text{subject to}:$$
$$Z1 = \sum_{g=1}^{2}\sum_{i=1}^{3}\left[p_{g,i}(y_{g,i}^{C} - y_{g,i}^{M})z_{g,i} - A_g^R\right]z_{g,i}$$
$$Z2 = \sum_{g=1}^{2}\sum_{i=1}^{3} x_{g,i}^{*}\left[(y_{g,i}^{C} - y_{g,i})p_{g,i} - (\mu_{g,i} + A_g)\right], \quad \text{if } \mu_{g,i} < 0$$
$$\sum_{i=1}^{3} R_{g,,i,,j} z_{g,i} \le \overline{R}_{g,j}, \ \forall g (= \text{CA}, \text{RUS}), j (= \text{land, water, capital})$$

where z_{gi} refers to allocation of acreage to each crop in each region (activities or decisions), and $x^{*}_{g,i}$ refers to the optimal allocations of land to crops found in the previous calibration constrained LP.

Solution to the PMP model:

Z = 2347.2835 (Solved with CONOPT3)

	Calibrated maximum yield ($y^{C}_{g,i}$)	
	CA	RUS
Cotton	2.963	2.520
Wheat	86.896	82.800
Rice	70.897	59.794

	Marginal yield = $y^{C}_{g,i} - 2\, y^{M}_{g,i}\, z^{*}_{g,i}$	
	CA	RUS
Cotton	1.437	0.500
Wheat	83.104	55.200
Rice	69.303	36.406

	Acres planted, LP Model ($x^{*}_{g,i}$)		Acres planted, nonlinear PMP Model ($z^{*}_{g,i}$)	
	CA	RUS	CA	RUS
Cotton	1.491	5.756	1.491	5.756
Wheat	0.619	6.435	0.619	6.435
Rice	0.540	2.743	0.540	2.743

	Percent difference in land allocation = $(z^{*}_{g,i} - XX_{g,i,land}) \times 100/ XX_{g,i,land}$	
	CA	RUS
Cotton	0.100	0.100
Wheat	–0.168	–1.000
Rice	–0.083	0.100

Shadow prices in the LP from resource constraints
$\lambda_{1(CA,\ land)} = 28.253$
$\lambda_{1(CA,\ water)} = 5.180$

Shadow prices in the PMP (nonlinear) model from resource constraints
$\lambda_{1(CA,\ land)} = 22.602$
$\lambda_{1(CA,\ water)} = 5.180$

GAMS Code

```
$TITLE POSITIVE PROGRAMMING QUADRATIC CROP EXAMPLE
* Created by R E HOWITT,  JANUARY 1994
* e-mail   rehowitt.ucdavis.edu

* This example is ECON 549, Lab 2009
* PMP function calculated on yields not crops. The adjustment for marginal
* crop supply functions and rotational crops is based on an arbitrary 20%
* yield variation. Preferably, this would be based on empirical evidence
* regarding yield variability.
* ---------------------------------------------------

$OFFSYMLIST OFFSYMXREF
 OPTION LIMROW = 0
 OPTION LIMCOL = 0
 option iterlim =1500

 SETS  I  PRODUCTION PROCESSES /COT,WHT,RI/
      J  RESOURCES           /LAND,WATER,CAPITAL/
      G   REGIONS           / CAL   CALIFORNIA
                              RUS   REST OF US/

 ALIAS (I,K)
 ALIAS (J,L)

* -------------------------------------------------------------
*   DATA (see Lab Handout)
* -------------------------------------------------------------
 PARAMETER P(I,G)    PRICE PER UNIT OUTPUT
    /COT.CAL       292.4
     COT.RUS       292.4
     WHT.CAL        2.98
     WHT.RUS        2.98
     RI.CAL         7.09
     RI.RUS         7.09/

 PARAMETER Y(I,G)   REGIONAL AVERAGE YIELDS
     /COT.CAL       2.20
      COT.RUS       1.51
      WHT.CAL       85.0
      WHT.RUS       69.0
      RI.CAL        70.1
      RI.RUS        48.1/

 PARAMETER R(J,G)   REGIONAL RESOURCE CONSTRAINTS
     /LAND.CAL       2.65
      LAND.RUS      14.99
      WATER.CAL      8.69
      WATER.RUS     28.33
      CAPITAL.CAL  2000.0
      CAPITAL.RUS  7000.0/
```

```
TABLE C(I,G,J)  RESOURCE VARIABLE COSTS
                  LAND    WATER     CAPITAL
     COT.CAL     264.3       25.6       0.1
     COT.RUS     111.8       28.4       0.1
     WHT.CAL      112.0      25.6       0.1
     WHT.RUS     175.0       28.4       0.1
     RI.CAL       196.9       25.6       0.1
     RI.RUS       158.2       28.4       0.1

TABLE X(I,G,J)  BASE RESOURCE USE
                  LAND    WATER     CAPITAL
     COT.CAL      1.49      4.47        610
     COT.RUS      5.75      5.23        490
     WHT.CAL      0.62      1.14        350
     WHT.RUS      6.50      6.89        280
     RI.CAL       0.54      3.08        520
     RI.RUS       2.74      7.95        480

 PARAMETER
      LEON(I,G,J)          REGIONAL LEONTIEF COEFFICIENTS
      CL(I,G)              LINEAR COST
      NET(I,G)             NET CASH RETURN       ;

 LEON(I,G,J) = (X(I,G,J)/X(I,G,"LAND") );
 CL(I,G) = SUM(J, C(I,G,J)*LEON(I,G,J) ) ;
 NET(I,G) = (Y(I,G) * P(I,G)) - CL(I,G) ;

* ---------------------------------------------------------------
* LP to calculate resource and PMP dual variables
* ---------------------------------------------------------------

VARIABLES     ACRES(I,G)       ACRES  PLANTED
              LINPROF          LP PROFIT

POSITIVE VARIABLE ACRES;

EQUATIONS     RESOURCE(J,G)   CONSTRAINED RESOURCES
              CALIBU(I,G)       UPPER CALIBRATION CONSTRAINTS
              CALIBL(I,G)       LOWER CALIBRATION CONSTRAINTS
              LPROFIT           LP OBJECTIVE FUNCTION;

RESOURCE(J,G)..       SUM(I,LEON(I,G,J)*ACRES(I,G)) =L= R(J,G);
CALIBU(I,G)..            ACRES(I,G) =L= X(I,G,"LAND")* 1.001;
CALIBL(I,G)$(NET(I,G) LT 0)..    ACRES(I,G) =G= X(I,G,"LAND")* 0.99;

LPROFIT..   LINPROF =E= SUM((I,G),((P(I,G)*Y(I,G))-CL(I,G))*ACRES(I,G));

MODEL  CALIBRATE /RESOURCE,CALIBU,CALIBL,LPROFIT/;

SOLVE CALIBRATE USING LP MAXIMIZING LINPROF;

DISPLAY ACRES.L, ACRES.M;
```

```
* ------------------------------------------------------------------
* Calculation of coefficients of PMP quadratic yield function
* ------------------------------------------------------------------

 PARAMETER
     OP(G)        Opportunity cost of land
     CN(I,G)     ADJUSTED costs for rotationals
     ADJ(G)      ADJUSTMENT to marginal crop duals
     FLAG(I,G)   Marginal crops
     FXYLD(I,G)  Fixed maximum yield
     MYLD(I,G)   Marginal yield parameter
     XYLD(I,G)   Calibrated maximum yield
 ;

* NOTE: FXYLD is used to calibrate crops (activities) that have negative
*  nominal net revenues. These are termed rotational crops and differ from
*  marginal crops. FXYIELD can be specified in a table of empirical data on
*  yield variability or rotational crops can be calibrated using prior
*  estimates of elasticities of supply.

   FXYLD(I,G) = Y(I,G) * 1.2 ;
* ----------------------------------------------------------------
* Adjusting opportunity cost of land for marginal yield of marginal crop
* in each region
* ----------------------------------------------------------------
   ADJ(G) = RESOURCE.M("LAND",G) * 0.2 ;
   OP(G) = RESOURCE.M("LAND",G) - ADJ(G) ;

   FLAG(I,G) = 0;
   FLAG(I,G)$((X(I,G,"LAND") NE 0) AND (CALIBU.M(I,G) EQ 0)
            AND (CALIBL.M(I,G) EQ 0) ) = 1 ;

* ----------------------------------------------------------------
*  Specification of yield function for marginal crops in each region
* (20% reduction in opportunity cost)
* ----------------------------------------------------------------

   MYLD(I,G)$FLAG(I,G)
        =  ADJ(G) /( P(I,G) * ACRES.L(I,G)) ;

   XYLD(I,G)$FLAG(I,G)
         = Y(I,G) + (MYLD(I,G) * ACRES.L(I,G)) ;

* ----------------------------------------------------------------
* Calculation yield functions for normal PMP crops
* ----------------------------------------------------------------

   MYLD(I,G)$(FLAG(I,G) EQ 0) = (CALIBU.M(I,G) + ADJ(G)) /( P(I,G) * ACRES.L(I,G)) ;
   XYLD(I,G)$(FLAG(I,G) EQ 0) = Y(I,G) + (MYLD(I,G) * ACRES.L(I,G)) ;

* ----------------------------------------------------------------
* Calculations of yield function for rotational crops (negative net return)
* ----------------------------------------------------------------
```

```
   XYLD(I,G)$(CALIBL.M(I,G) LT 0 ) =  FXYLD(I,G) ;

   MYLD(I,G)$(CALIBL.M(I,G) LT 0 ) = ( XYLD(I,G) - Y(I,G)) / ACRES.L(I,G) ;

* ------------------------------------------------------------
* Cost adjustment for rotational crops
* ------------------------------------------------------------

   CN(I,G)   = CL(I,G) ;
   CN(I,G)$(CALIBL.M(I,G) LT 0) = CN(I,G) - (((XYLD(I,G) - Y(I,G)) * P(I,G))
         - ( CALIBL.M(I,G) + ADJ(G))) ;

* ------------------------------------------------------------
* You can put policy changes here. E.g., 15% reduction of wheat prices in all regions
*  P("WHT",G) = P("WHT",G) * 0.85 ;

*   10% reduction in water available in California
*   R("WATER","CAL") = R("WATER","CAL") * 0.90 ;

* ------------------------------------------------------------
*  PMP Model solution for base year
* ------------------------------------------------------------
 VARIABLES     Z                    NONLINEAR LAND ALLOCATION
               NLPROF               NONLINEAR PROFIT  ;

 POSITIVE VARIABLE Z;

 EQUATIONS     RESOURCEN(J,G)          CONSTRAINED RESOURCES
               NPROFIT                 NLP OBJECTIVE FUNCTION;

 RESOURCEN(J,G)..    SUM(I,LEON(I,G,J)*Z(I,G)) =L= R(J,G);

 NPROFIT.. SUM((I,G), ( P(I,G)*(XYLD(I,G) - MYLD(I,G)* Z(I,G))
    - CN(I,G))*Z(I,G)) - SUM((I,G)$(CALIBL.M(I,G) LT 0),  ACRES.L(I,G)
    * (((XYLD(I,G) - Y(I,G)) * P(I,G)) - (CALIBL.M(I,G) + ADJ(G))) )
         =E= NLPROF;

 MODEL  PRIMAL  /RESOURCEN,NPROFIT/ ;

 SOLVE  PRIMAL   USING NLP MAXIMIZING NLPROF;

* ------------------------------------------------------------
*  Display Results
* ------------------------------------------------------------
 PARAMETER    MARGYLD(I,G)   MARGINAL YIELD
              PERDIF(I,G)        PERCENT DIFFERENCE IN LAND ALLOCATION ;

  PERDIF(I,G) = (Z.L(I,G) - X(I,G,"LAND")) * 100 / X(I,G,"LAND") ;
  MARGYLD(I,G) = XYLD(I,G) -(2* MYLD(I,G)* Z.L(I,G)) ;

DISPLAY  XYLD, MARGYLD, Y, LINPROF.L, NLPROF.L, ACRES.L, Z.L, PERDIF,
               RESOURCE.M, RESOURCEN.M ;
```