Commodity price stabilization:  
the price uncertainty case

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Abstract. In this paper, the Waugh-Oi-Massell approach to analysing the welfare effects of price stabilization is extended to consider the case of price uncertainty, not just price instability. It is shown that when producers cannot postpone production until prices are revealed, as is the case in agriculture, results opposite to those obtained under the Waugh-Oi-Massell production-mode assumption occur. In addition, it is shown that a price stabilization scheme may be Pareto optimal. Empirical estimates of the difference between the welfare measures under the two assumptions are also provided.

La stabilisation du prix des denrées quand les prix sont incertains. Dans ce mémoire, les auteurs utilisent l’approche Waugh-Oi-Massell pour analyser les effets de bien-être de mécanismes de stabilisation des prix mais en proposent l’extension au cas où les prix sont incertains et non seulement instables. On montre que dans le cas où les producteurs ne peuvent pas reporter la décision de produire jusqu’à ce que les prix soient connus – ainsi que c’est le cas en agriculture – on obtient des résultats contraires à ceux obtenus par Waugh-Oi-Massell à partir de leurs postulats. On montre aussi qu’un mécanisme de stabilisation des prix peut être optimal au sens de Pareto. Des évaluations empiriques de la différence entre les mesures de bien-être dans les deux cas sont fournies.

INTRODUCTION

Government intervention in markets to stabilize commodity prices and producer incomes appears to be a fact of economic life. A popular approach for analysing the effects of stabilization programs is based on work by Waugh (1944), Oi (1961), and Massell (1969), although modifications have recently been made to this framework (see Schmitz, 1984). In Waugh’s analysis of instability due to supply shifts consumers preferred price instability, while in Oi’s analysis of shifts in demand producers preferred price instability. Massell showed that, upon integrating the analyses of Waugh and Oi, if compensation is allowed, society gains from stabilization, regardless of the source of instability (supply or demand variability). However, it is important to note that in the Massell analysis, price stabilization is not Pareto optimal since

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there are losers and gainers from stabilization. Therefore, compensation is needed before there is an overall improvement in welfare.

The major problem with the Waugh-Oi-Massell studies is that they refer to the case where producers can postpone their production until prices are revealed. In many industries (e.g., agriculture), however, production decisions need to be made before prices are known. Hence, unless perfect futures markets exist, producers are uncertain regarding the price they will finally receive. In this paper we consider the effects of price stabilization schemes where price uncertainty exists (not just price instability, as in the Waugh-Oi-Massell case) and compare this with the Waugh-Oi-Massell production-mode assumption. We show several strong results, one of which is that price stabilization can be Pareto optimal – a result which has not been established in the literature. As in the original papers by Waugh, Oi and Massell, our analysis is a partial equilibrium one. Prices are taken as exogenous to consumers and producers, and price changes are assumed to have no impact on the demands for other commodities or inputs. However, it is important to stress that the Waugh-Oi-Massell studies refer to the case where producers postpone their production until price is revealed. Our study considers the opposite extreme, and the results are quite different. Some empirical estimates of the magnitude of the difference are provided in an appendix to the paper.

WELFARE GAINS IN THE MASSELL FRAMEWORK

The supply shift (Waugh-Massell) and demand shift (Oi-Massell) cases are discussed separately. These models are based on the concept of economic surplus. Although this framework has had a controversial history (e.g., Currie, Murphy, and Schmitz, 1971), it has recently been strongly supported on theoretical grounds by, for example, Willig (1976). However, in our models we still use the concept of consumer’s surplus; on the producer side, we use expected gross revenue as a criterion rather than rents.

Waugh-Massell supply shifts

Consider figure 1. A government-sponsored stabilization authority knows that supply fluctuates randomly about the expected supply curve $S_E$. Following Massell, it is assumed that supply functions $S_0$ and $S_1$ occur with probability 0.5 and that the costs of the stabilization scheme are zero; it is also assumed that consumers are risk-neutral. The buffer stock agency stores $QQ_0$ units of the commodity when supply is $S_0$ and releases $Q,Q = QQ_0$ due to symmetry) units when supply is $S_1$. Price is stabilized at $P$, and $Q$ units of the commodity are purchased in each time period. Massell’s measure of the expected net gain to society over two periods is equal to area $abk + bfg$.

In an environment where producers are uncertain regarding eventual prices because exogenous supply shifters are at work, the producer surplus measures used by Massell no longer hold. The appropriate welfare measure of the producers' gains is determined

1 See Turnovsky, Shalit and Schmitz (1980) for a discussion regarding the validity of this assumption.
by comparing the expected gross revenue under price stabilization with that accruing in the absence of stabilization. The net expected gain to producers from stabilization over the two periods is given by the following:

\[
\text{Net producer gain} = (\text{gross revenue with stabilization}) - (\text{gross revenue without stabilization})
\]

\[
= 2PQ - (P_0Q_0 + P_1Q_1)
\]

\[
= \text{area} (Q_1cbQ - P_1acP) + \text{area} (PbeP_0 - QefQ_0).
\]

By symmetry, \(\text{area} (P_1acP) = \text{area} (PcdP_0)\) and \(\text{area} (Q_1deQ) = \text{area} (QefQ_0)\). Therefore,

\[
\text{Net producer gain} = 2 \times \text{area} (cbed).
\]

The consumer gain from stabilization in one period is given by \(\text{area} (P_1abP)\), while the consumer loss is equal to \(\text{area} (PbfP_0)\). The net expected consumer loss is, consequently, given by \(\text{area} (abc - cbfd) = \text{area} (cbed)\), since \(\text{area} (abc) = \text{area} (cbed)\).

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2 The two ex post supply curves in our analysis become vertical \((Q_1Q'_1\) and \(Q_0Q'_0\)).

3 According to Massell, the net expected gain to the producer over the two time periods is given by \(\text{area} (cdhg)\). This area is larger than the area we suggest as the correct measure of producer gain by \(\text{area} (fmgh)\).
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Finally, the net expected gain to society from stabilization over the two time periods is measured by area (cbef).

The Waugh-Massell measure of the expected gain to society in a two-period interval is larger than the measure derived here. The difference is as follows:

\[
\text{Difference} = \text{area (cbef)} - \text{area (abk + bfg)} - \text{area (befin)} - \text{area (abc + ack + bfin + mfg)} - \text{area (ack + mfg)}.
\]

This area is a measure of the amount by which the Waugh-Massell area overestimates the actual expected gain to society from stabilization in the case of random supply shifts. The difference between the measures approaches zero when the supply curve becomes completely inelastic or the demand curve becomes infinitely elastic.

Oi-Massell demand shifts

Now consider figure 2. In this case the supply function is non-random and can be considered the ‘planning’ supply curve, \( S_p \). Instability is due to stochastic demand. Consistent with Massell, it is assumed that the demand schedules \( D_0 \) and \( D_1 \) are equally likely to occur, that is, with probability 0.5, and consumers are assumed to be risk-neutral, as before. In this case, based on Oi’s results, Massell argues that producers prefer price instability. However, he also argues that society as a whole will gain area (qrs + stu) from stabilization since the gain to consumers exceeds the loss to producers.

Massell’s argument relies on producers correctly anticipating which demand function results ex post. For some commodities this assumption is unrealistic. If prices are really unstable, it is not clear that producers are able to react immediately to price changes. As a result, it no longer follows that producers are better off under price instability. Rather, recognizing the random nature of demand, producers make decisions according to the expected or ex ante demand curve \( D_E \). Thus, producers expect \( \hat{P} \) to occur and, hence, decide to produce quantity \( OQ \). Since supply is non-stochastic, planned production equals realized output. The supply curve effectively becomes completely inelastic and measures of welfare are then quite different from those suggested by Massell. Finally, should the expected demand curve shift, producers would move along \( S_p \) to a new equilibrium.

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4 The Waugh-Massell measure of welfare gain overestimates the actual welfare gain by area (fmgh), as the reader can verify. This is equivalent to the amount by which Massell overestimates the expected producer gain from stabilization, as indicated in fn. 3.

5 Consider, for example, a producer who is an expected profit maximizer over time and who faces a random price in each period of \( P_t \). The problem facing the producer who makes all of his production decisions at time \( t = 0 \) is to

\[
\text{Max. } E(\pi) = E \sum_{t=0}^{T} [P_t q_t - c(q_t)](1 + r)^{-t},
\]

where \( \pi = \) discounted profits, \( q = \) output (nonrandom),
\[c(q) = \text{cost of producing } q \text{ units of output},\]
\[E = \text{expectations operator, and}\]
\[r = \text{discount rate}.\]

Equation (1) can be rewritten as
\[\text{Max. } E(\pi) = \sum_{t=0}^{\infty} [E(P_t)q_t - c(q_t)](1 + r)^{-t}, \quad (2)\]

Assuming the second-order conditions hold, the necessary conditions for an optimum are
\[\frac{\partial E(\pi)}{\partial q} = E(P_t) - c'(q_t) = 0,\]
which implies that
\[q_t^* = c'^{-1}[E(P_t)].\]

Equation (4) indicates that the output in every period will be identical and equal to \(q_t^*.\) That is, producers operating under the above assumptions will react to price uncertainty not by varying production, but rather, will produce a fixed quantity in each period, depending on the expected price in that period.

\[\text{FIGURE 2 Welfare measures in the Oi-Massell stabilization case}\]
To stabilize price at $\bar{P}$, the buffer stock agency must store an amount $qs$ when demand is $D_0$, and release an equal amount $su(=qs)$ when demand is $D_1$. Once again we assume that the costs of the stabilization scheme are zero. With stabilization the producer gains and losses can be determined as in the previous section. In this case, producers are indifferent regarding the stabilization scheme because $2(\bar{P}\bar{Q}) = P_0\bar{Q} + P_1\bar{Q}$; that is, the total revenue is the same under stabilization as without it. In addition, the total producer surplus received under the two regimes is identical since area $[zsvP_1 + (znP_0 - nsw)] = 2 \times \text{area } (zs\bar{P})$. Note that this result is different from that obtained in the Oi-Massell case, where producers clearly prefer price instability.

The consumer gains and losses can also be calculated. When demand is $D_1$, the gain to consumers from stabilization is given by area $(P_1\bar{v}\bar{u}P)$; when demand is $D_0$, the consumer surplus loss is measured by area $(\bar{P}swP_0 - qsw)$. Since area $(P_1\bar{v}\bar{s}\bar{P})$ is equal to area $(\bar{P}swP_0)$, the net expected gain in consumer surplus due to stabilization over the two periods is measured by area $(qxvs) = \text{area } (qv\bar{u})$. Since the gain or loss to producers is zero, the net gain to society is identical to the net positive gain to the consumers. Price stabilization is, therefore, a Pareto optimal policy.

Now compare our measure of the gain to society with that of Oi-Massell. Since area $(stu)$ is common to both measures, the difference is given by area $(qvts - qrs)$. If the supply curve $S_p$ is orthogonal to the demand curves, then area $(qvs) = \text{area } (qys)$, and area $(yvts)$ measures the difference between our measure of the gain to society and that of Massell. In the extreme cases of perfectly elastic demand and completely inelastic $S_p$, the difference between the measures of societal welfare gain is zero; indeed, in the latter case, the welfare gain to society is zero. In general, the Massell measure underestimates the gain to society from stabilization when the cause of instability is demand variability.

Finally, it is interesting that (1) price variability is greater in our model and (2) the storage needed to bring about stability is greater. Consequently, if storage costs are included they will be larger in our model and this will reduce our measure of the welfare gains to a greater extent than in the Oi-Massell case.

**Conclusion**

In this paper it was theoretically shown, along with some empirical estimates (appendix), that the results derived using the Massell framework, where prices are certain, do not hold in an uncertainty environment. In particular, in the Waugh-Massell case of stochastic supply but stable demand, the Massell measure always overestimates the true gain to society from commodity price stabilization. In the Oi-Massell case of demand variability but non-random supply, the strong result emerges which is that stabilization is Pareto optimal, since a rational expectations viewpoint is adopted. In this situation, the Massell measure underestimates the actual gain to society.

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6 A downward sloping supply curve is ruled out, since it is associated with monopoly, not perfect competition. With monopoly there is no need for a government-sponsored stabilization authority.

A vertical demand curve, on the other hand, is considered unrealistic in this context.
APPENDIX: EMPIRICAL ESTIMATES OF THE EFFECTS OF PRICE STABILIZATION

We consider two empirical examples to illustrate the magnitude of the Massell measures of income distribution and societal welfare gain, our suggested measures, and the difference between them. The first example relates to the Canadian beef industry where empirical research has shown that demand is much more stable than supply. Dunn and Heien (1982) argue that retail beef prices vary more than the prices of all other food items in the United States. For Canada, the coefficient of variation of annual beef prices was 0.136 for the period 1967 to 1981. By contrast, the coefficient of variation for the annual index of all food prices was 0.076 during the same period. Further, the price elasticities of short run supply have been estimated to fall in the range 0.03 to 0.63, while demand has been relatively stable (Kulshreshtha, Wilson and Brown, 1971; Rosaasen, 1978; Reutlinger, 1966). Therefore, it is assumed that demand is stable while supply fluctuates.

During the period 1973 to 1981, the real price of beef fluctuated between $39.04 and $57.05 per cwt of cold dressed meat. When the price was $39.04/cwt, annual per capita consumption was 106.12 pounds; when the price was $57.05/cwt, annual per capita consumption was 85.45 pounds. Based on these figures and the assumption that the demand and supply curves are linear, it is possible to compare estimates of the welfare measures used by Massell with those suggested here. The elasticity of demand (ED) was chosen to be -0.50, -0.75, and -1.00, while the elasticity of supply (ES) was assumed to be 0.15 and 0.60. The magnitude of the welfare measures and income distributional effects are provided in table 1.7

The results indicate that, for this example, the redistributional effects and welfare gains to society from stabilization are substantial. Given our assumptions, the annual per capita allocative efficiency gains are estimated at between $0.55 and $2.20 depending on which measure of gain is employed, the one in this study or Massell’s. This translates into an annual gain of $12.65 to $50.60 million for Canadian society. Based on the measures derived in the current study, however, the annual gains to Canadian society are much smaller ($12.6 to $25.3 million), although still substantial. It is evident from table 1 that Massell’s analysis implies a much greater gain to producers than suggested here, although the loss to consumers is the same.8 Therefore, it is the gain to producers that accounts for the difference in net welfare gain to society.

The second example pertains to the Canadian broiler industry where supply is much more stable than demand. In a supply-management study, Veeman (1982) estimated that the free-market price of broilers was 41.52¢ per pound. With supply

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7 The symmetry of results in the columns of table 1 are due to the symmetry in figure 1. As a result, when $\text{ED} = -1.00$ and $\text{ES} = 1.00$, the difference in net welfare gain between our procedure and Massell’s would be equal to our measure of net welfare gain.

8 The estimates of annual loss to consumers obtained from the linear models presented in this paper compare favourably with those estimated by Dunn and Heien (1982), who assume an ordinal, translog utility function. Using Monte Carlo methods, they estimate an average loss to U.S. consumers of beef of $1.14 per year (579). However, they do not provide estimates of the gain to producers.
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TABLE 1
Annual per capita expected welfare gain due to stabilization in the Canadian beef industry: an example of instability caused by supply shifts

\[ P_0 = 39.04 \text{cwt} \quad Q_0 = 106.12 \text{ lbs} \quad P = 48.045 \text{cwt} \]

<table>
<thead>
<tr>
<th>Elasticity [^{b}]</th>
<th>Quantity</th>
<th>Gain to producers</th>
<th>Loss to consumers [^{c}]</th>
<th>Net welfare gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ED ) (^{(\text{lbs})})</td>
<td>( Q ) (^{(\text{lbs})})</td>
<td>( Q_0 ) (^{(\text{lbs})})</td>
<td>( \text{Current study} ) (^{(\text{c})})</td>
<td>( \text{Massell} ) (^{(\text{c})})</td>
</tr>
<tr>
<td>(-0.50)</td>
<td>93.91</td>
<td>109.79</td>
<td>110.0</td>
<td>126.5</td>
</tr>
<tr>
<td>(-0.50)</td>
<td>93.91</td>
<td>120.81</td>
<td>110.0</td>
<td>176.1</td>
</tr>
<tr>
<td>(-0.75)</td>
<td>87.76</td>
<td>109.79</td>
<td>165.4</td>
<td>181.9</td>
</tr>
<tr>
<td>(-1.00)</td>
<td>81.64</td>
<td>109.79</td>
<td>220.4</td>
<td>236.9</td>
</tr>
<tr>
<td>(-1.00)</td>
<td>81.64</td>
<td>120.81</td>
<td>220.4</td>
<td>286.5</td>
</tr>
</tbody>
</table>

\[^{a}\]For notation, refer to figure 1. 
\[^{b}\]Elasticities are given for the point \((Q_0, P_0)\). 
\[^{c}\]Loss to consumers is identical in both studies.

管理，价格上升至48.36c每磅。使用这些价格作为较低和较高限度，并且一个平均年度消费量为945.80亿磅（见Veeman, 1982, 29），是可能的 pregnancy to derive the annual distributional effects and welfare gains suggested in this study and by Massell. A planned price \((\hat{P})\) of 44.94c per pound and elasticity of supply equal to 1.0 were assumed, while demand elasticity was chosen to be \(-0.25\), \(-0.50\), \(-0.75\), and \(-1.00\).

9 Veeman (1982, 29) uses a short-run elasticity of supply \((ES)\) equal to 1.0 and elasticity of demand \((ED)\) equal to \(-0.56\) in her study of the Canadian broiler industry.

TABLE 2
Annual welfare gain to society from stabilization in the Canadian broiler industry: an example of instability caused by demand shifts

\[ \hat{P} = 44.94c/\text{lb} \quad \hat{Q} = 945.80 \text{ million pounds} \quad ES = 1.0 \]

<table>
<thead>
<tr>
<th>Elasticity of demand ({^{(\text{ED})}}) [^{b}]</th>
<th>Price (P_0 ) (^{(\text{c/lb})})</th>
<th>Quantity</th>
<th>Net Welfare Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ES ) (^{(\text{mil. lbs})})</td>
<td>( Q_0 ) (^{(\text{lbs})})</td>
<td>( Q'_0 ) (^{(\text{lbs})})</td>
<td>( \text{Current study} ) (^{(\text{c})})</td>
</tr>
<tr>
<td>(-0.25)</td>
<td>44.26</td>
<td>927.81</td>
<td>931.40</td>
</tr>
<tr>
<td>(-0.50)</td>
<td>43.80</td>
<td>909.81</td>
<td>921.81</td>
</tr>
<tr>
<td>(-0.75)</td>
<td>43.47</td>
<td>891.82</td>
<td>914.95</td>
</tr>
<tr>
<td>(-1.00)</td>
<td>43.23</td>
<td>873.82</td>
<td>909.81</td>
</tr>
</tbody>
</table>

\[^{a}\]For notation, refer to figure 2. 
\[^{b}\]Elasticities are given for the point \((\hat{Q}, \hat{P})\).
The results indicate that the net welfare gains to society from stabilization in the Canadian broiler industry are positive but not very significant. Indeed, depending on the measure employed, the gains represent between 0.01 and 0.58 per cent of total industry sales. However, the data do support our contention that the Oi-Massell welfare measure underestimates the actual gains to society from stabilization. The amount by which the gain is underestimated is substantial when compared to the gains suggested in this study; in the current study, the measure of welfare gain exceeds that of Massell by 2 to 50 per cent. Finally, recall that price stabilization is a Pareto optimum—consumers gain from stabilization while producers neither gain nor lose.

REFERENCES

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