Examples of Matlab and GAMS to Solve Three LP/ NLP Problems

1. Fisheries example on NLP problem solved in GAMS. Problem and solution are given here.

2. The forester responsible for managing of 3,038 ha of southern hardwoods wants to convert this land to a regulated pine plantation. The pine plantation will be managed with a 20-year rotation and the conversion is to be done in 20 years. The forester intends to maximize pulpwood production while doing this conversion.

The conversion will require that one-quarter of the total forest area be harvested and replanted in each of four (5-year) planning periods. The initial forest consists of five 'compartments'. The area and expected yield of hardwood pulpwood by compartment is shown in the following table.

Projected Hardwood Pulpwood Production (Green Tons/ha)

	Area	Period			
Compartment	(ha)	1	2	3	4
1	722	35.6	52.2	74.2	101.8
2	621	53.4	73.3	104.9	148.4
3	469	113.9	162.7	225.3	303.3
4	545	92.8	135.0	185.4	258.2
5	681	39.9	56.5	76.7	102.0

Use Matlab's *linprog* function to solve this LP problem.

Here is the LP problem statement:

$$
\begin{aligned}
\text{Max}\quad Z = {} & (35.6\, x_{11} + 53.4\, x_{21} + 113.9\, x_{31} + 92.8\, x_{41} + 39.9\, x_{51}) + \\
& (52.2\, x_{12} + 73.3\, x_{22} + 162.7\, x_{32} + 135.0\, x_{42} + 56.5\, x_{52}) + \\
& (74.2\, x_{13} + 104.9\, x_{23} + 225.3\, x_{33} + 185.4\, x_{43} + 76.7\, x_{53}) + \\
& (101.8\, x_{14} + 148.4\, x_{24} + 303.3\, x_{34} + 258.2\, x_{44} + 102.0\, x_{54})
\end{aligned}
$$

$$
\begin{array}{llll}
\text{s.t.} & x_{11} + x_{12} + x_{13} + x_{14} & \leq 722 & \text{(Area of compartment \#1)} \\
& x_{21} + x_{22} + x_{23} + x_{24} & \leq 621 & \text{(Area of compartment \#2)} \\
& x_{31} + x_{32} + x_{33} + x_{34} & \leq 469 & \text{(Area of compartment \#3)} \\
& x_{41} + x_{42} + x_{43} + x_{44} & \leq 545 & \text{(Area of compartment \#4)} \\
& x_{51} + x_{52} + x_{53} + x_{54} & \leq 681 & \text{(Area of compartment \#5)} \\
& x_{11} + x_{21} + x_{31} + x_{41} + x_{51} & = 759.5 & \text{(Even harvest period \#1)} \\
& x_{12} + x_{22} + x_{32} + x_{42} + x_{52} & = 759.5 & \text{(Even harvest period \#2)} \\
& x_{13} + x_{23} + x_{33} + x_{43} + x_{53} & = 759.5 & \text{(Even harvest period \#3)} \\
& x_{14} + x_{24} + x_{34} + x_{44} + x_{54} & = 759.5 & \text{(Even harvest period \#4)}
\end{array}
$$

$x_{11}, x_{21}, x_{31}, x_{41}, x_{51}, x_{12}, x_{22}, x_{32}, x_{42}, x_{52}, x_{13}, x_{23}, x_{33}, x_{43}, x_{53}, x_{14}, x_{24}, x_{34}, x_{44}, x_{54} \geq 0$
(non-negatively)

Matlab code provided below. Here are the answers.

When simplex algorithm is NOT used:

exitflag = 1

Value of the objective function = 3.8947e+005

Harvest by compartments and time measured across a row

391.0411	0.0000	0.0000	0.0000	368.4589
330.9589	116.0000	0.0000	0.0000	312.5411
0.0000	505.0000	0.0000	254.5000	0.0000
0.0000	0.0000	469.00	290.5000	0.0000

Restriction that annual harvest amounts to 759.5 ha

759.5000 759.5000 759.5000 759.5000

Restriction that no more than total compartment area is harvested
722.0000 621.0000 469.0000 545.0000 681.0000

When 'simplex' is 'on', the answers are different, indicating multiple solutions.

exitflag = 1

Value of the objective function = 3.8947e+005

Harvest by compartments and time measured across a row

78.5000	0	0	0	681.0000
643.5000	116.0000	0	0	0
0	505.0000	0	254.5000	0
0	0	469.0000	290.5000	0

Restriction that annual harvest amounts to 759.5 ha
759.5000 759.5000 759.5000 759.5000

Restriction that no more than total compartment area is harvested
722 621 469 545 681

NOW assume a discount rate of 5% per year and a pulpwood price of $20 per green ton.
The harvest plan differs because current harvests are valued higher than later ones!!

exitflag = 1

Value of the objective function = 4.0676e+006

Harvest by compartments and time measured across a row

0	116.0000	0	0	643.5000
722.0000	0	0	0	37.5000
0	505.0000	254.5000	0	0
0	0	214.5000	545.0000	0

Restriction that annual harvest amounts to 759.5 ha

759.5000 759.5000 759.5000 759.5000

Restriction that no more than total compartment area is harvested

722 621 469 545 681

(Main Program)

```
% Forest management problem @ G. Cornelis van Kooten

tic
c1 = [35.6 52.2 74.2 101.8; 53.4 73.3 104.9 148.4;
    113.9 162.7 225.3 303.3; 92.8 135.0 185.4 258.2; 39.9 56.5 76.7 102.0];
% Multiply rows of c1 by $20 times discount factors 0.885, 0.6936 ...
discount = [1/(1.05^2.5) 1/(1.05^7.5) 1/(1.05^12.5) 1/(1.05^17.5)];

c2=20*discount';
c3=repmat(c2,1,5).*c1';

A = [ones(1,4), zeros(1,16); zeros(1,4), ones(1,4), zeros(1,12);
    zeros(1,8), ones(1,4), zeros(1,8);
    zeros(1,12), ones(1,4), zeros(1,4); zeros(1,16), ones(1,4)];

Aeq = [1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0;
    0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0;
    0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0;
    0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1];

beq = [759.5 759.5 759.5 759.5];

b = [722 621 469 545 681];

lb = zeros(1, 20);

% Creating the correct objective function coefficient vector
% Substitute c3 for c1' (NOTE c3 has already been transposed)
f = -reshape(c3, 1, 20);
options = optimset('Largescale', 'off', 'simplex', 'on');

%Calling the LP function
[x,fval,exitflag] = linprog(f,A,b,Aeq,beq,lb,[],[],options);

y=reshape(x,4,5);
exitflag

'Value of the objective function'
-fval

'Harvest by compartments and time measured across a row'
y
'Restriction that annual harvest amounts to 759.5 ha'
sum(y')
'Restriction that no more than total compartment area is harvested'
sum(y)
```

3. A rural hospital wants to maximize the profit (revenue minus expenses) that it gets from patients hospitalized under its care. Annual revenue depends on the numbers of medical (x_1) and surgical (x_2) patients admitted. Since medical and surgical patients impose costs upon each other, the profit function is nonlinear. The hospital identifies three constraints. The mathematical programming problem is as follows:

Max $\pi = 13\, x_1 + 6\, x_1 x_2 + 5\, x_2 + 1/x_2$ (profit in ‘000s \$)

Subject to

$2\, x_1^2 + 4\, x_2 \leq 90$ (nursing capacity in ‘000s labor-days)

$x_1 + x_2^3 \leq 75$ (X-ray capacity in ‘000s)

$8\, x_1 - 2\, x_2 \leq 61$ (materials budget in ‘000s \$)

$x_1, x_2 \geq 0$

Solution using Matlab.

The answer is: $x_1 = 6.0663$ $x_2 = 4.1003$. These values need to be multiplied by 1,000; thus, 6066 medical and 4100 surgical patients. The exit flag is 1. The objective value is 248.8457, or \$248,846 in profit.

GAMS leads to the following: (GAMS code here)

Objective value = 248.8457; $x_1 = 6.066$ $x_2 = 4.100$; and the shadow prices for the three constraints

Nursing capacity = 1.521

X-ray capacity = 0.699

Materials budget = 0 (excess budget)

Main Program

```
% Rural hospital. Nonlinear programming @G. Cornelis van Kooten

clear all;
x0=[1 1];
lb = [0 0];
%A = [-0.5 0.1 -1; -1 0.1 -1]; b = [-60, -40];
options=optimset('Largescale','off');
%options=optimset('Largescale','off', 'TolCon', 1.0000e-15);

%[x,fval,exitflag]=fmincon('profit',x0,A,b,[],[],lb,[],[],options);
[x,fval,exitflag]=fmincon('objective',x0,[],[],[],[],lb,[], ...
    @constraint,options);

exitflag
'Values of activity levels'
x
'Objective value'
-fval
```

Two Function Files (objective.m and constraint.m)

```
% Profit function for HW#2 (2009)

function f=objective(x)
    f=-(13*x(1)+6*x(1)*x(2)+5*x(2)+1/x(2));


%This function contains the inequality and equality constraint equations
% Note that inequality constraints must by of <= form

function [cin,ceq]=constraint(x)
    cin=[2*x(1)*x(1)+4*x(2)-90; x(1)+x(2)^3-75; 8*x(1)-2*x(2)-61];
    ceq=[];
```

GAMS Code

```
$Title Nonlinear Programming Example

SETS
  i    index 1  /1*2/ ;

VARIABLES
  z     objective value
;

Positive VARIABLE x;

EQUATIONS
OBJ      Objective function
CON1     Constraint 1
CON2     Constraint 2
CON3     Constraint 3
;

* -------------------------------------------------------------
*    Construction of the actual NLP model
* -------------------------------------------------------------

OBJ ..  z =E= 13*x('1') + 6*x('1')*x('2') + 5*x('2') + 1/x('2');

CON1.. 2*x('1')**2 + 4*x('2') =L= 90;

CON2.. x('1') + x('2')**3 =L= 75;

CON3.. 8*x('1') - 2*x('2') =L= 61;

x.l(i) = 1;

Model Nurse /all/;

solve Nurse using NLP maximizing z;
```