

GAMS for Solving Mathematical Programming Problems

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- Powerful software for solving LP, QP, NLP, IP, MIP, MINLP and CGE models.
- Easy to use when setting up large problems: much easier than Excel, Matlab, etc.
- More powerful than Excel, Matlab, Maple and other packages for solving very large linear and non-linear programs
- Enables researcher to access a variety of powerful solvers using the same written code.
- Solvers include Minos, CPLEX, ConOpt, Lindo, XA, and any other commercial and non-commercial solver that is available.
- Excellent documentation (e.g., McCarl's User Manual) at www.gams.com
- Numerous sample programs available in model library at <http://www.gams.com/modlib/modlib.htm>

GAMSIDE: Windows graphical interface to run GAMS

- **GAMS:** General Algebraic Modeling System

plus

- **IDE:** Integrated Development Environment

Summary steps for using GAMS

1. Install GAMS &IDE on your computer making an icon



2. Open the IDE through the icon

3. Create a project by going to the file selection in the upper left corner.

What is a project? GAMSIDE employs a ‘Project’ file (1) to identify a location where all saved files are to be placed (to place files elsewhere use ‘save as’ dialogue) and where GAMS looks for files when executing; and (2) to which project files, program option files and output files are saved. Recommended you define a new project every time you wish to change the file storage directory.

Summary steps for using GAMS (cont)

4. Define a project name and location. Put it in a directory you want to use. Choose a suitable subdirectory and create a subdirectory with the name of the project. In turn, call your project by that name so that it is called *project.gpr*, where gpr stands for GAMS project.
5. Create or open an existing file of GAMS instructions
6. Prepare the file so you think it is ready for execution
7. Run the file with GAMS by clicking on the run button
8. Open and navigate around the output

A GAMS Program

INPUTS:

SETS

- declaration
- assignment of members

DATA (Parameters, Tables, Scalars)

- declaration
- assignment of values

A GAMS program (cont)

VARIABLES

- declaration
- assignment of type

Assignment of bounds and/or initial values
(OPTIONAL)

A GAMS program (cont)

EQUATIONS

- declaration
- definition

MODEL and SOLVE Statements

DISPLAY Statement (OPTIONAL)

Equations in GAMS

Two types of equations in GAMS:

1. Define parameters
2. Model equations (may be inequalities)

Not indexed: Objective is a single value

$$Z = \text{sum}((i,j), a(i,j)*x(i,j));$$

Indexed: Examples

$$R(i) = \text{sum}(j, c(j)*x(i,j));$$

$$R(\text{item}) = \text{sum}(\text{city}, c(\text{city})*x(\text{item},\text{city}));$$

Example problem: Minimize transportation costs

$$\min \sum_i \sum_j c_{ij} x_{ij} \quad (\$)$$

$$s.t. \sum_j x_{ij} \leq a_i \quad (\text{supply limit at plant } i)$$

$$\sum_i x_{ij} \geq b_j \quad (\text{satisfy demand at market } j)$$

See <http://www.gams.com/docs/example.htm>

SETS

i plants /Seattle, San-Diego/;

j market /NY, Chicago, Kancity/;

PARAMETERS

a(i) capacity at plant i

/ Seattle 350

San-Diego 600 / ;

b(j) demand at market j

/ NY 325

Chicago 300

Kancity 275 / ;

TABLE $d(i, j)$ distance in '000s miles;

	NY	Chicago	Kancity
Seattle	2.5	1.7	1.8
San-Diego	2.5	1.8	1.4

SCALAR f freight cost in \$ per '000s miles /8/ ;

PARAMETER $c(i, j)$ transport cost in k\$ per unit ;

$$c(i, j) = f * d(i, j) / 1000;$$

VARIABLES

$x(i, j)$ shipment quantities

z total costs in k\$;

Positive variable x ;

EQUATIONS

supply(*i*) supply limit at plant *i*
demand(*j*) satisfy demand at market *j*
cost objective function is to minimize cost
;

cost.. $z = E = \text{sum}((i,j), c(i,j)*x(i,j));$
supply(*i*).. $\text{sum}(j, x(i,j)) = L = a(i);$
demand(*j*).. $\text{sum}(i, x(i,j)) = G = b(j);$

MODEL transport /all/;

SOLVE transport using LP minimizing z;

DISPLAY x.l, x.m ;

Specify upper and lower bounds in lieu of constraints

$x.\text{up}(j) = 50;$

Same as an $x_j \leq 50$ constraint

$y.\text{lo}(\text{"2"})=20; \rightarrow y_2 \geq 20$ constraint

(Note reference to index 2)

$\text{cows}.\text{up}(\text{'3'}) = 200 + \frac{1}{2} x.\text{up}(\text{'3'});$

$\rightarrow \text{cows}_3 \leq 200 + \frac{1}{2} (50)$ constraint

Report Writing in GAMS

DISPLAY can be used after SOLVE statement in GAMS to print out calculation results:

- 1) variable name followed by .L → value of solution for that variable
- 2) variable name followed by .M → reduced cost associated with that variable
- 3) equation name followed by .L → left-hand side value of equation
- 4) equation name followed by .M → shadow price (dual variable) associated with equation

Dual slack variables = RHS of equation – associated .L value

Example displays level and marginal values of X and shadow price of CON1:

```
DISPLAY X.L, X.M, CON1.M;
```

PUT option and Report Writing

Define output file name and identify where output is to be written. If no path is given, default is project subdirectory and file is file name with .put extension

file Output /D:\work\output.txt/ ;

*Set page width

Output.pw=5000;

*Number of decimal places in output file

Output.nd=0;

Solve *modelname* using LP maximizing z;

Now comes the writing component. A comma is needed to create a comma separated value file, and a / is used to indicate a carriage return.

PUT option (cont)

```
put Output;
```

```
  put "Solver status ,", modelname.solvestat/;
```

```
  put "Model status ,", modelname.modelstat/;
```

```
  put "Objective value ,", z /;
```

```
  put "Period" ", " "Price" ", " "Quantity" ", "/;
```

```
    loop(t, put t.tl ", " price.l(t) ", " quan.l(t) ", "/);
```

```
  put/;
```

```
putclose Output;
```

Note how the index `t` needs to be identified in the loop with a `.tl` as opposed to only `.l`.

An example of how to write output is provided in the next two slides for an electricity grid model.



Alberta_model2.gms

```

*****
** Output writing section
*****

emissions.l(h)=sum(p,total.l(p,h)*emfactor(p)/maxefficiency(p)/0.2778);

cost.l(p,h)=fuelcost.l(p,h)+OMcost(p);

peakcosttotal.l=(sum(h,cost.l('peak',h))+ peakcap.l*fixedcost('peak'))/1e6;

total_all.l(h)=sum(p,total.l(p,h));

file result / result.txt /;
result.pw=10000;
put result 'Alberta Electricity Grid Model' ///;
put result 'Run on ' system.date ' using source file ' system.ifile ///;
put "Model status ",energy.modelstat/;
put "Solver status ",energy.solvestat/;
put "Objective ," z.l / "Peak capacity ," peakcap.l /
    "Fixed cost of wind," fixedcost('wind')/
    "Wind capacity," capacity('wind') ///;
put ",," "POWER OUTPUT BY SOURCE (MW)" ",," ",," ",," ",," ",," ",," ",," ",,"
    "COST - $ PER MW" ",," ",," ",," ",," "UPPER RESERVOIR"
    ",," ",," ",," "LOWER RESERVOIR" ",," /
put "Hour" ",," "Total" ",," "Coal" ",," "Gas" ",," "Peak" ",," "Wind" ",,"

```



Alberta_model2.gms

```

put "Objective ," z.l / "Peak capacity ," peakcap.l /
      "Fixed cost of wind," fixedcost('wind')/
      "Wind capacity," capacity('wind') ///
put "," "POWER OUTPUT BY SOURCE (MW)" "," "," "," "," "," "," "," "," ","
      "COST - $ PER MW" "," "," "," "," "," "UPPER RESERVOIR"
      "," "," "," "LOWER RESERVOIR" "," /
put "Hour" "," "Total" "," "Coal" "," "Gas" "," "Peak" "," "Wind" ","
      "Hydro" "," "Demand/load" "," "Coal" "," "Gas" "," "Peak" ","
      "Wind" "," "Hydro" "," "Volume" "," "Outflow" ","
      "Inflow" "," "CO2 Output" "," "PumpFlow" ","
      "Volume" "," /;
loop (h, put h.tl "," total_all.l(h) "," total.l('coal',h) ","
total.l('gas',h) "," total.l('peak',h) ","
total.l('wind',h) "," total.l('hydro',h) "," demand(h)
"," cost.l('coal',h) "," cost.l('gas',h) ","
cost.l('peak',h) "," cost.l('wind',h) ","
cost.l('hydro',h) "," volume.l(h) "," outflow.l(h) ","
inflow(h) "," emissions.l(h) "," reflow.l(h) ","
volume2.l(h) /;);
put /;
putclose;

```

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Insert

Calling external Files from within GAMS

- \$Include *externalfilename*

where *externalfilename* can include full path or just file name if located in the current working directory as declared in GAMS project.

The file to include consists of GAMS code.

- The ‘master’ GAMS file treats the statements in the include file as a continuation of the program code precisely at the \$Include point.
- Other commands for including external files are:
\$Batinclude, \$Libinclude, \$Sysinclude
- \$Oninclude and \$Offinclude turn the echo print of the include file in the LST file on and off (default is ‘on’)

Example from grid model

PARAMETER inflow(h) hourly inflow of water m³ per s
\$Include inflow

PARAMETER demand(h) hourly demand for electricity
\$Include demand2003

PARAMETER wpower(h) hourly wind power (MW)
\$Include wpowers

Data are found in **inflow.gms**, **demand2003.gms** and **wpowers.gms**

Comma Separated Value (CSV) Files

- Spreadsheets and other programs (e.g., Matlab) can read and write CSV files
 - Commas separate fields
 - Text items can be in quotes
- \$ondelim – tells GAMS that entries which follow are in CSV format
- \$offdelim – tells GAMS to ‘turn off’ use of CSV format

Examples using CSV format

SETS

K index k /K1*K3/

J index j /J1, J2, J3/;

TABLE CSVEX1(K,J) Data in csv Format

\$ondelim

dummy, J1, J2, J3

K1, 16, 34, 12

K2, 3, 11, 5

K3, 52, 16, 4

\$Offdelim

Note use of 'dummy' as the 'placeholder' in a table (see also below)

Alternatively an external file
can contain the table:

SETS

K index k /K1*K3/

J index j /J1*J3/;

TABLE CSVEX2(K,J) data in csv format

\$ondelim

\$Include externalcsvdatafile

\$offdelim

The file *externalcsvdatafile.gms* contains:

dummy, J1, J2, J3

K1, 16, 34, 12

K2, 3, 11, 5

K3, 52, 16, 4

Note: ‘dummy’ is needed in the csv format to use up the space over the set elements defining the table rows – in the first row, first column.

Consider this example in GAMS and Excel

$$\begin{aligned} \text{Max} \quad & Z = 3 x_1 + 2 x_2 + 0.5 x_3 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 \leq 10 \\ & x_1 - x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Three equations; x_3 is not constrained ('free'). The GAMS setup and solution are provided on the next slides.

GAMS Model

\$Title McCarl & Spreen Chapter 3 Example

SETS

i index 1 /1*3/

j index 2 /1, 2/

;

PARAMETER b(j) rhs constraint values

/ 1 10

2 3 /

;

TABLE a(j,i) Table of technical constraints

	1	2	3
1	1	1	1
2	1	-1	0

;

PARAMETER $c(i)$ coefficients in obj

/1 3

2 2

3 0.5/

;

VARIABLES

z objective value

$x(i)$ activities

;

EQUATIONS

OBJ Objective function

CON(j) Constraint

;

* -----
* Construction of the actual LP model
* -----

OBJ .. z =E= **sum**(i, c(i)*x(i));

CON(j).. **sum**(i, a(j,i)*x(i)) =L= b(j);

MODEL McSpreen /all/;

x.lo('1')=0; x.lo('2')=0;

SOLVE McSpreen using LP maximizing z;

Solution to previous GAMS problem



McSreen_Chap3_example.gms McSreen_Chap3_example.lst

- Compilation
- Equation Listing S
- Equation
- Column Listing S
- Column
- Model Statistics S
- Solution Report S
- SolEQU
- SolVAR

```

S O L V E            S U M M A R Y

MODEL    McSreen            OBJECTIVE    z
TYPE     LP                 DIRECTION    MAXIMIZE
SOLVER   CPLEX              FROM LINE    48

**** SOLVER STATUS        1 NORMAL COMPLETION
**** MODEL STATUS         3 UNBOUNDED
**** OBJECTIVE VALUE                    26.5000

RESOURCE USAGE, LIMIT            0.015        1000.000
ITERATION COUNT, LIMIT            2            10000

GAMS/Cplex    Apr 21, 2006 WIN.CP.CP 22.2 031.034.041.VIS For Cplex 10.0
Cplex 10.0.1, GAMS Link 31

Presolve found the problem infeasible or unbounded.
Rerunning with presolve turned off.
Dual infeasible or unbounded. Switching to primal to aid diagnosis.
Model has an unbounded ray.

**** ERRORS/WARNINGS IN VARIABLE x(3)
      1 error(s): 1 Unbounded variable
    
```

SinkCapacity [a] [Print] [Run]

McScreen_Chap3_example.gms | McScreen_Chap3_example.lst

- Compilation
- Equation Listing
- Equation
- Column Listing
- Column
- Model Statistics
- Solution Report
- SolEQU
- SolVAR

```

**** ERRORS/WARNINGS IN VARIABLE x(3)
      1 error(s): 1 Unbounded variable

      LOWER      LEVEL      UPPER      MARGINAL
---- EQU OBJ
      .           .           .           1.000

      OBJ  Objective function

---- EQU CON  Constraint

      LOWER      LEVEL      UPPER      MARGINAL
1      -INF      10.000    10.000    2.500
2      -INF      3.000     3.000    0.500

      LOWER      LEVEL      UPPER      MARGINAL
---- VAR z
      -INF      26.500    +INF      .

      z  objective value

---- VAR x  activities
    
```

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- Compilation
- Equation Listing
- Equation
- Column Listing
- Column
- Model Statistics
- Solution Report
- SolEQU
- SolVAR

```

LOWER      LEVEL      UPPER      MARGINAL
---- VAR z          -INF      26.500      +INF      .
z objective value
---- VAR x activities
LOWER      LEVEL      UPPER      MARGINAL
1          .          6.500      +INF      .
2          .          3.500      +INF      .
3          -INF      .          +INF      -2.000 UNBND

**** REPORT SUMMARY :
0          NONOPT
0 INFEASIBLE
1 UNBOUNDED (UNBND)

EXECUTION TIME      =      0.000 SECONDS      2 Mb      WIN222-145 Apr 21,

```

Notice that GAMS provides a solution

Now consider the solution in Excel

Consider both the PRIMAL and the DUAL!

PRIMAL Max $Z = 3 x_1 + 2 x_2 + 0.5 x_3$

s.t. $x_1 + x_2 + x_3 \leq 10$

$x_1 - x_2 \leq 3$

$x_1, x_2 \geq 0$

DUAL Min $R = 10 y_1 + 3 y_2$

s.t. $y_1 + y_2 \geq 3$

$y_1 - y_2 \geq 2$

$y_1 = 0.5$

$y_1, y_2 \geq 0$

Any attempt to solve either the Primal or the Dual problem within Excel resulted in “The SET CELL values do not converge”. Attempts to solve it as a NLP rather than LP did not help in this regard.

Excel does not appear able to solve this simple LP. Actually, since the problem is unbounded it gives infinite values in the cells that are to be changed.

Recall: Unbounded PRIMAL means an infeasible DUAL, and vice versa

Final Comments

- Excel does not appear to be a reliable solver
- Neither can Matlab solve the above problem
- Matlab and Excel are generally okay for mid-size LPs and small NLPs, but not larger models (but in this case they cannot even ‘solve’ a small LP)
- GAMS is a preferred platform because of its power to solve even the smallest and extremely large problems (with millions of constraints)