ECON 482/530: RESOURCE ECONOMICS
Homework #2
Due: September 26, 2018

1. Consider a fishery with a logistics biological growth function as follows:

$$g(x_t) = \tfrac{1}{2}\, x_t\, (1 - x_t/500)$$

where x_t represents the stock of fish at time t. Assume the stock of fish evolves according to a difference equation:

$$x_{t+1} = x_t + g(x_t) - h_t,$$

where h_t represents the catch of fish at time t. Write R code to solve the following questions.

(a) What is the (net) biological growth of fish if the stock of fish is equal to:
(i). 50?
(ii). 500?
(iii). 600?

(b) Let the initial stock of fish be 100 ($x_0 = 100$). Consider the case of a pristine fishery where harvest is equal to zero for all time periods. Use R to graph the evolution of the stock of fish from time t = 0 to t = 100. (Provide the R code and graph).

(i). What is the fish stock at time t = 2?

(ii). What is the fish stock at time t = 10?

(iii). What is the steady state stock of fish?

(c) Let the initial stock of fish be $x_0 = 80$. Assume the manager of the fishery adopts a constant harvest policy where $h_t = 30$ for all time periods.

(i). What is the stock of fish at time t = 3?

(ii). Is this species of fish heading towards extinction?

(d) Let the initial stock of fish be $x_0 = 60$. Again assume the manager of the fishery adopts a constant harvest policy where $h_t = 30$ for all time periods.

(i). What is the stock of fish at $t = 3$?

(ii). Is this species of fish heading towards extinction?

2. Assume reserves of oil evolve according to the following difference equation: $R_{t+1} = R_t - q_t$, where R_t represents the reserves at time t, and q_t represents oil extraction at time t. Write R code to answer the following questions.

Consider an economy in which initial reserves of oil amount to $R_0 = 500$. The authority decides to extract 40% of the level of reserves at each time period, so $q_t = 0.4\,R_t$.

(a) What is the path of extraction from $t = 0$ to $t = 5$?

(b) What is the path of the level of reserves from $t = 0$ to $t = 5$?

Now consider extraction of oil as described above, but assume profits at time t are determined by the following function:

$$\pi_t = pq_t - \alpha\, q_t/R_t^2$$

where $p = 40$ represents the oil price and $\alpha = 50{,}000$ is a cost parameter. Assume operations are allowed from t = 0 to t = 5 (6 time periods), and that the discount rate is $r = 0.02$.

(c) What is the present value of the sum of profits from oil extraction?

(d) Assume that a new technology can reduce α from 50,000 to 30,000. What is the present value of the sum of profits from oil extraction under the new technology?

(e) If this new technology is available at the cost of one payment of $300, should the authority buy the technology?

3. Now find the optimal solution to the problem specified in question **2** using GAMS. That is, solve the following mathematical program:

Maximize	$Z = \sum_{t=0}^{5} \beta^t \left[pq_t - \alpha\, \frac{q_t}{R_t^2} \right]$
s.t.	$R_{t+1} = R_t - q_t,\ \forall t = 0, \ldots, 5$
	$q_t \geq 0$ (decision variables)

where $p = 40$, $\alpha = 50{,}000$, $R_0 = 500$ (and it cannot be negative). What happens if the technology results in $\alpha = 30{,}000$?