## ECON 482/530: RESOURCE ECONOMICS Homework #2 Due: Friday, January 31, 2020

NOTE: Where I have written R, you could also use Matlab, Excel, Python or some other software; instead of GAMS you could use the solver in Excel, a package in R, or some other mathematical programming software.

1. Solve the following problems in GAMS. Show your GAMS code and answers.

(a) The following is a linear programming (LP) problem

min  $Z = X_1 + 3X_2 + 3X_3$ s.t.  $X_1 + X_2 \ge 3$  $X_2 + X_3 \ge 5$  $X_1 + X_3 = 4$ 

(b) The following is a mixed integer programming (MIP) problem

 $\begin{array}{ll} \max & Z = x + y \\ \text{s.t.} & -3x + \ 2y \geq 1 \\ & -8x + 10y \leq 10 \\ & y \ \in \{0,1\}, \, x \geq 0.3 \end{array}$ 

**2**. Assume reserves of oil evolve according to the following difference equation:  $R_{t+1} = R_t - q_t$ , where  $R_t$  represents the reserves at time *t*, and  $q_t$  represents oil extraction at time *t*. Write R code to answer the following questions.

Consider an economy in which initial reserves of oil amount to  $R_0 = 500$ . The authority decides to extract 40% of the level of reserves at each time period, so  $q_t = 0.4 R_t$ .

(a) What is the path of extraction from t = 0 to t = 5?

(b) What is the path of the level of reserves from t = 0 to t = 5?

Now consider extraction of oil as described above, but assume profits at time *t* are determined by the following function:

$$\pi_t = pq_t - \alpha q_t/R_t^2$$

where p = 40 represents the oil price and  $\alpha = 50,000$  is a cost parameter. Assume operations are allowed from t = 0 to t = 5 (6 time periods), and that the discount rate is r = 0.02.

(c) What is the present value of the sum of profits from oil extraction?

(d) Assume that a new technology can reduce  $\alpha$  from 50,000 to 30,000. What is the present value of the sum of profits from oil extraction under the new technology?

(e) If this new technology is available at the cost of one payment of \$300, should the authority buy the technology?

**3**. Now find the optimal solution to the problem specified in question **2** using GAMS. That is, solve the following mathematical program:

| Maximize | $Z = \sum_{t=0}^{5} \beta^{t} \left[ pq_{t} - \alpha \frac{q_{t}}{R_{t}^{2}} \right]$ |
|----------|---|
| s.t.     | $R_{t+1} = R_t - q_t, \ \forall t = 0,, 5$  |
|          | $q_t \ge 0$ (decision variables)  |

where p = 40,  $\alpha = 50,000$ ,  $R_0 = 500$  (and it cannot be negative). What happens if the technology results in  $\alpha = 30,000$ ?