Homework #4

Simulation and Optimization

Due: October 9, 2018

1. There is often more than one way to solve an optimization problem. Consider Question E2.4 on p.71: Consider the optimal escapement problem for salmon on a particular stream in Alaska. Suppose $x_{t+1} = S_t e^{\gamma(1-\frac{S_t}{K})}$, where $S_t = min[x_t, S^*]$ is escapement in period t, $h_t = max[0, x_t - S^*]$ is the allowed harvest in period t, γ is the intrinsic growth rate, and K > 0 is the carrying capacity of the stream. The net refvenue in period t is given by $\pi_t = ph_t - \frac{c}{2}(\frac{h_t^2}{x_t})$, where p>0 is the unit price for fish on the dock and c>0 is a cost parameter. The optimization problem is as follows:

$$\max_{S^*} \quad \sum_{t=0}^{19} \beta^t \, [ph_t - \frac{c}{2}(\frac{h_t^2}{x_t})]$$

$$\text{s.t.} \quad x_{t+1} = S_t e^{\gamma(1-\frac{S_t}{K})}$$
$$S_t = min[x_t, S^*]$$
$$h_t = max[0, x_t - S^*]$$

when $K = 300{,}000$ (salmon), $\gamma = 1$, p = 5, c = 1, $x_0 = 100{,}000$ (salmon) and the interest rate is 2%. Suppose the initial policy is $S^* = 150{,}000$.

(a) Using R, find the path of salmon stock, harvests, escapement and (discounted) profit in each period. (Essentially duplicate the excel results on p. 72 of the text. Note: The results will not be exactly the same.)

(b) What is the value of S^* that maximizes the net revenue for this problem? That is, solve the problem using a Monte Carlo type simulation method in R. (I used 100 iterations with values of S* between 2,500 and 250,000 and 40 periods rather than 20 in the text.)

2. Solve the problem using GAMS so that one gets an optimal harvest, stock and escapement in each period. How different are your answers?

3. Consider the dynamic open-access system and specify a catch per unit of effort (CPUE) production function so that one gets the following system of equations (Conrad, pp.86-87):

$$x_{t+1} = \left[1 + \mathrm{g} - \frac{\mathrm{g}\mathrm{x}_{\mathrm{t}}}{K} - qE_t\right] x_t$$

$$E_{t+1} = [1 + \eta(\mathrm{pq}x_t - c]E_t$$

Here g is the intrinsic growth rate, K is the carrying capacity, p refers to price, q is the catchability coefficient, c is the cost parameter, E is effort and *x* is the fish stock. For the following parameter values, reproduce the two curves in Figure 3.5 on p.87: c=K=1, p=200, q=0.01 and g=0.1. Let $x_0 = 0.5$ and $E_0 = 1$. For figure (a) let η=0.3, and for figure (b), let η=1.0.