Fundamental Equation of Resource Economics

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Resource example with discounting:

Fishery management problem. Harvest is denoted *h* and the stock (density) of fish, *x*. Objective is to maximize discounted net returns over infinite time. The problem is as follows:

$$\max_{\{h_t\}} \sum_{t=1}^{\infty} \beta^t \pi(x_t, h_t)$$

$$\text{s.t.} \quad x_{t+1} - x_t = g(x_t) - h_t$$

The Lagrangian for this problem is:

$$L = \sum_{t=1}^{\infty} \beta^t \{\pi(x_t, h_t) + \beta\lambda_{t+1}[x_t + g(x_t) - h_t - x_{t+1}]\}$$

The first-order conditions:

$$\frac{\partial L}{\partial h_t} = 0 \Rightarrow \beta^t \left[\frac{\partial \pi}{\partial h_t} - \beta \lambda_{t+1}\right] = 0$$

$$\frac{\partial L}{\partial x_t} = 0 \Rightarrow \beta^t \left\{\frac{\partial \pi}{\partial x_t} + \beta \lambda_{t+1} \left[1 - \frac{dg}{dx_t}\right]\right\} - \beta \lambda_t = 0$$

$$\frac{\partial L}{\partial (\beta \lambda_{t+1})} = 0 \Rightarrow \beta^t [x_t + g(x_t) - h_t - x_{t+1}] = 0$$

We can rewrite these equations so they have an economic interpretation:

The first can be written as: $\frac{\partial \pi}{\partial h_t} = \beta \lambda_{t+1}$

This states that the marginal benefit of current harvest of fish equals marginal benefit of leaving fish *in situ*. Recall $\beta \lambda_{t+1}$ is referred to as the ***user cost***.

The 2nd equation can be rewritten as:

$$\lambda_t = \frac{\partial \pi}{\partial x_t} + \beta \lambda_{t+1} \left[1 - \frac{dg}{dx_t}\right]$$

The LHS gives the *in situ* value of an additional unit of the resource. It equals the current period marginal net benefit plus the marginal benefit that the unharvested fish give in the next period. When rewritten it gives the change in the co-state variable over time and is a sort of arbitrage condition that assures that, for an optimal solution, the resource owner is indifferent between current and future exploitation. Otherwise, someone can gain by inter-temporally (re)allocating supply.

Finally, the third equation is simply a rewrite of the subject to constraint – the equation of motion of the fish stock. We rewrite it as: $x_{t+1} = x_t + g(x_t) - h_t$

To find a steady state, we set $x_{t+1} = x_t = x^*$; $h_{t+1} = h_t = h^*$; $\lambda_{t+1} = \lambda_t = \lambda^*$. Then we get:

$$\beta\lambda = \frac{\partial \pi}{\partial h_t}$$

$$\beta\lambda[1 + \frac{dg}{dx_t} - (1+r)] = \frac{-\partial \pi}{\partial x_t}$$ [recall: $\beta = 1/(1+r)$]

$$h = g(x)$$

Fundamental Equation of Renewable Resources

Substitute $\beta\lambda$ from the first equation into the second equation to get:

$$\frac{dg}{dx} + \frac{\frac{\partial \pi}{\partial x}}{\frac{\partial \pi}{\partial h}} = r$$

The 1st term on the LHS is marginal net growth rate of the in-situ stock of fish;

The 2nd term is *marginal stock effect* that measures marginal value of stock relative to marginal benefit from harvest.

Two terms on LHS together are the internal rate of return to *in-situ* stock and must equal the return (rate of interest) available elsewhere in the economy.

We need to solve the above equation and the third equation from the previous slide to find the optimal values for x^* and h^*. We next provide an example.

An application (Conrad, pp.45-47):

Let $g(x) = g\,x \ln(K/x)$, where g is the intrinsic growth rate and K the carrying capacity of the fish habitat.

Let $\pi(x, h) = p\,h - \frac{1}{2}\,ch^2/x$, where p is price and c is a cost parameter

Then: $$\frac{dg(x)}{dx} = g\,[\ln(K/x) - 1]$$

$$\frac{\partial \pi}{\partial x} = \frac{1}{2}\,c\,(h/x)^2$$

$$\frac{\partial \pi}{\partial h} = p - c\,(h/x)$$

Substitute $h = g\,x \ln(K/x)$ into the lower two equations and enter the results into the fundamental equation of resource economics. This gives (next slide):

$$g\,[\ln(K/x) - 1] + \frac{\tfrac{1}{2}\,c\,\left[\mathrm{g}\ln\left(\frac{K}{x}\right)\right]^2}{p - c\mathrm{g}\;\ln\left(\frac{K}{x}\right)} = r$$

We can now move toward solving this equation numerically:

$$[\ln(K/x) - (g+r)][\mathrm{p} - c\,g\,\ln(K/x)] + \tfrac{1}{2}\,c\,[\mathrm{g}\ln(K/x)]^2 = 0$$

Now assume: $g = 1$, $K = 1$, $p = 5$, $c = 11$ and $r = 0.02$

How to solve? Conrad uses solver in Excel. In R, you install ‘Quadratic Equation’ package. To find the roots of $Ax^2 - Bx + C$: write quadratic(A, -B, C), hit return and there’s the answer. Above equation is not simple as we want to solve for *x*.

Solving nonlinear equations in R

- R code using ‘nleqslv’ package (could use ‘rootsolve’)

```
library(nleqslv)

fundamental <- function(x) {
    g = 1;  K = 1 ;  p = 5;  c = 11 ; r = 0.02
    y = (log(K/x) - (g+r))*(p - c*g*log(K/x)) + 0.5*c*(g*log(K/x))^2
    return(y)
}

startfun = 0.75  # Guess at the initial solution. Conrad uses 0.75 and 0.07
nleqslv(startfun, fundamental, jac=NULL, jacobian=FALSE)
```

Asymptotic Approach to Bioeconomic Optimum (Conrad, pp.58-63)

Whales where social return is due to eating meat and watching:

$$\pi(x_t, h_t) = \ln(h_t) + v \ln(x_t),$$

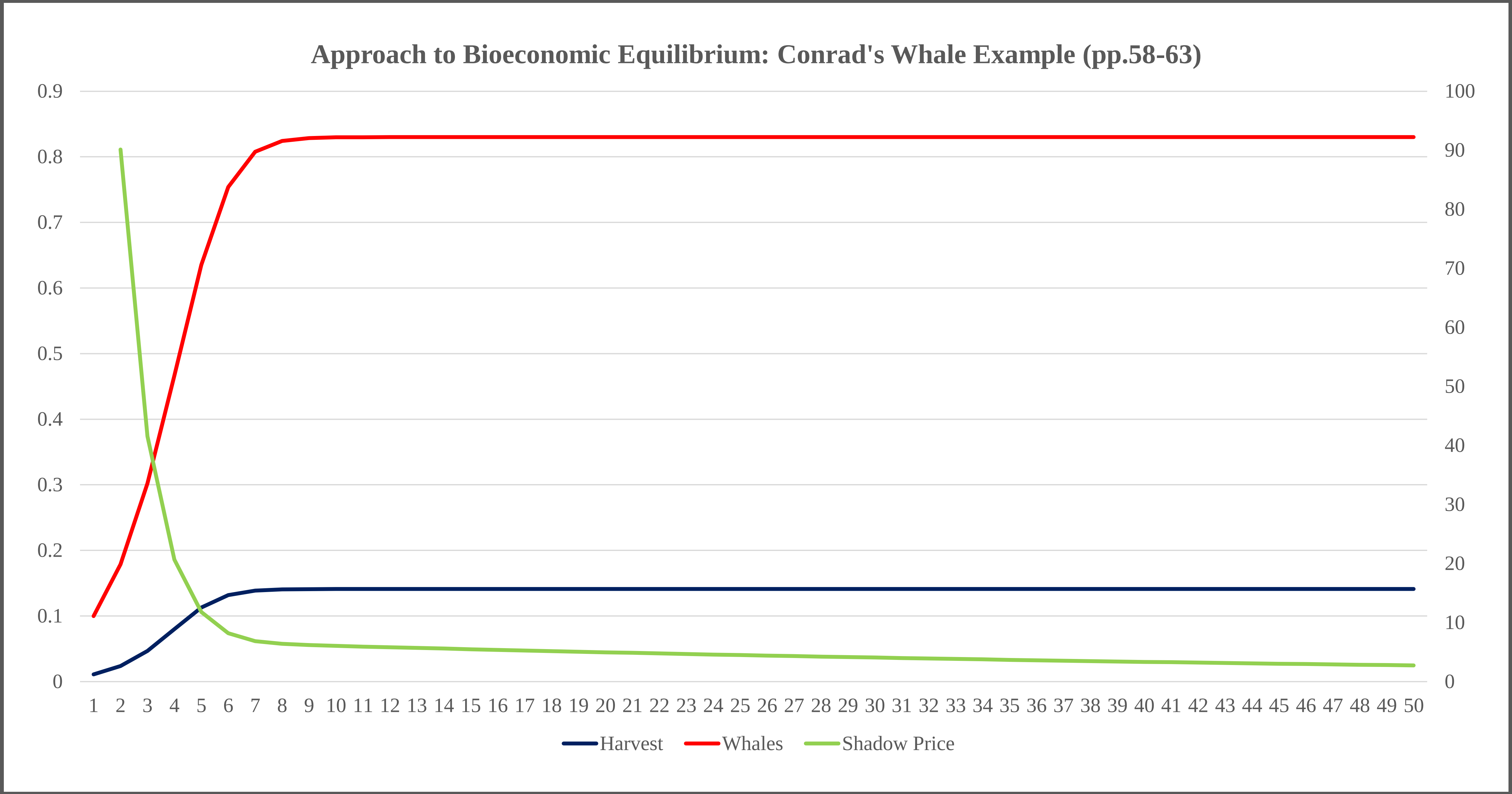
where $v \geq 0$ weights the benefits of whale watchers relative to meat eaters.

Assume logistic growth function: $g(x_t) = g\, x_t(1 - x_t/K)$

Using the fundamental equation (steady state), we can solve for x^*:

$$x^* = K[g(1+v) - r]/[g(v + 2)]$$

g=1, K=1, v=4 and r=0.02 implies x^*=0.83 and h^*=0.1411.


Approach to Bioeconomic Equilibrium: Conrad's Whale Example (pp.58-63)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
100
90
80
70
60
50
40
30
20
10
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
Harvest
Whales
Shadow Price