Risks and Saving

- Income Risk and Precautionary Saving
- Capital Income Risk
Saving Under Certainty

The risk-averse consumer’s problem is

$$\max_{c_1, c_2, s} U(c_1) + \beta U(c_2)$$

subject to

$$c_1 + s = y_1 \quad \&$$

$$c_2 = \bar{R}s + \bar{y}_2$$

where $y_1$, $\bar{y}_2$ & $\bar{R}$ are first period income, second period income, and the rate of interest respectively. Let $y_1 > \bar{y}_2$. The optimal amount of saving is given by

$$U_c(y_1 - s) = \beta \bar{R}U_c(\bar{R}s + \bar{y}_2)$$

Denote the optimal amount of saving by $\bar{s}$. 
Income Risk and Saving

Now suppose that the second period income, $y_2$, of the consumer is uncertain. The consumer has to save in the first period before knowing its second period income. Suppose that

$$Ey_2 = \bar{y}_2$$

where $E$ is the expectation operator. Denote the optimal amount of saving with uncertain income by $s^*$. The question is whether $s^* > \bar{s}$. Will uncertainty about future income will lead to higher saving (precautionary saving) in the first period?
Now the consumer’s problem is

$$\max_{c_1, c_2, s} U(c_1) + \beta E U(c_2)$$

subject to

$$c_1 + s = y_1 \ & (5)$$

$$c_2 = Rs + y_2. \ & (6)$$

The optimal amount of saving, $s^*$, is given by

$$U_c(y_1 - s) = \beta REU_c(Rs + y_2). \ & (7)$$
Now compare (3) and (7). The LHS of both equations has identical expressions. However, the RHS has different expressions. Given the concavity of the utility function (or diminishing marginal utility), it is immediately clear that if

$$EU_c(\bar{R}s + y_2) > U_c(\bar{R}s + \bar{y}_2) \equiv U_c(E(\bar{R}s + y_2))$$  \hspace{1cm} (8)

then

$$s^* > \bar{s}.$$

It is the mathematical property (Jensen’s inequality) that if a function $f(x)$ is a convex function of the random variable $x$ then

$$Ef(x) > f(E(x)) \equiv f(\bar{x})$$  \hspace{1cm} (9)

where $\bar{x} = E(x)$. 
Precautionary Saving

Essentially we need to find out under what condition \( U_c(\bar{Rs} + y_2) \equiv U_c(c_2) \) is a convex function of \( y_2 \). Taking the second-derivative of \( U_c(c_2) \) w.r.t. \( y_2 \) you can see that when the third derivative of the utility function is positive i.e. \( U_{ccc}(c) > 0 \), \( U_c(c_2) \) will be a convex function of \( y_2 \). In this case, \( s^* > \bar{s} \). The difference between \( s^* - \bar{s} \) is known as precautionary saving. This is extra saving due to uncertainty in the future income.

Example 1: Suppose that \( U(c) = c^{1-\mu} \). In this case, \( U_{ccc}(c) = \mu(1 + \mu)c^{-(\mu+1)} > 0 \). Thus, in case of income uncertainty there will be extra-saving.

Example 2: Suppose that we have quadratic utility function, \( U(c) = ac - bc^2 \). Since \( U_{ccc}(c) = 0 \), in this case, there will be no extra saving due to income uncertainty. Thus, the mere fact that an individual is risk-averse does not mean there will be precautionary saving. This is known as certainty-equivalence result.
Now suppose that $y_2$ is certain, but there is uncertainty about the rate of interest $R$. Let $R$ be the random variable with $E(R) = \bar{R}$. Now the optimization problem is

$$\max_{c_1, c_2, s} U(c_1) + \beta EU(c_2)$$

subject to

$$c_1 + s = y_1 \quad \& \quad (10)$$

$$c_2 = Rs + \bar{y}_2 \quad (11)$$

The optimal amount of saving, $s^{**}$, is given by

$$U_c(y_1 - s) = \beta EU_c(Rs + \bar{y}_2)R. \quad (12)$$
We want to know under what condition $s^{**} > \bar{s}$. As before we need to derive the condition under which $U_c(Rs + \bar{y}_2)R \equiv U_c(c_2)R$ is a convex function of $R$. Taking the second derivative, you can show that $U_c(c_2)R$ is a convex function of $R$ if

$$U_{ccc}(c_2)Rs + 2U_{cc}(c_2) > 0 \text{ or } (13)$$

$$-\frac{U_{ccc}(c_2)}{U_{cc}(c_2)}Rs > 2. \quad (14)$$

As is evident $U_{ccc}(c_2) > 0$ is no longer sufficient (though necessary) to ensure extra-saving.
For example, with $U(c) = \frac{c^{1-\mu}}{1-\mu}$, (14) is satisfied only when $(\mu + 1) \frac{R_s}{c_2} > 2$. The reason is that risky capital income affects saving in two opposite ways. Firstly, the precautionary motive $(U_{ccc}(c_2) > 0)$ has positive effect on saving. However, risky capital income also reduces the attractiveness of saving (negative substitution effect), which is captured by the term $U_{cc}(c_2)$. Only if the precautionary motive dominates the negative substitution effect, saving will be higher than the certainty case.