Chapters 1 and 2
Trade Without Money and A Model of Money

Main Aims:

1. Answer following two questions:
   - Why money is used as a medium of exchange?
   - How and why money improves allocations in an economy and social welfare?

2. Develop a simple overlapping generation model (OLG) of money.
Money as a Medium of Exchange

Paper or fiat money is intrinsically useless in the sense that they do not directly yield utility and/or cannot be used as input in production. They do however lead to higher consumption/production by facilitating trading in real world. Now the question is what feature of real world leads to use of money as a facilitator of trading processes:

Problem of the Double Coincidence of Wants: In the absence of money it is difficult to acquire desired consumption good or input for production or sell goods.
Money as a Medium of Exchange

However, mere presence of the problem of double coincidence of wants is not enough. People should be willing to hold money as an asset (store of value). When a buyer buys good from a seller using money, then the seller is left with money which he/she cannot directly consume. The seller can potentially use the money to buy goods in future. Thus, if a seller is willing to sell his/her good for money, he/she must believe that others will also accept money in exchange for goods in future. A necessary condition for that is that there is always someone who will live in the next period.
Money as a Medium of Exchange

Any model of money (in which money emerges as a medium of exchange) incorporates these three necessary conditions:

1. Frictions (difficulty) in trading
2. Infinitely-lived economy
3. Imperfect credit market
Overlapping Generations Model as a Model of Money

1. Highly tractable and can be used to address many monetary and non-monetary issues

2. Provides a natural physical environment in which trading without money is difficult

3. Dynamic models. Anticipated future events affect current decisions
Environment

– Infinitely-lived economy

– Economy is populated by people who live for two periods. People in the first period of their life are called young and in the second period of their life old.

– In each period \( t \geq 1 \), \( N_t \) individuals are born.

– The economy starts in period 1. The generations born in periods \( t \geq 1 \) are called future generations. In addition, in period 1 there are \( N_0 \) people born in time 0 called the initial old.

– In any period \( t \geq 1 \), two generations live- \( N_t \) young and \( N_{t-1} \) old. Total population at time \( t \) is \( N_t + N_{t-1} \).
Environment

– There is only one perishable or non-storable good.
– Each individual when young receive an endowment of the consumption good equal to $y$. Old receive nothing. Thus the total availability or supply of good at time $t$ is $yN_t$.
– Preferences: Each individual would like to consume in both periods of their life. Let $c_{1,t}$ and $c_{2,t+1}$ denote the consumption of an individual born in time $t$ as a young and as an old respectively. The utility function is given by

$$U(c_{1,t}, c_{2,t+1}), \ U_1 \ & U_2 > 0, \ U_{11} \ & U_{22} < 0$$

where $U_1$ and $U_2$ are partial derivatives of the utility function with respect to $c_{1,t}$ and $c_{2,t+1}$ respectively.
Economic Questions

1. Given the environment (endowments, preferences, population structure), what will be the allocation of goods among different generations with or without money?

2. Does the allocation in an economy with money is better than the allocation without money (barter economy)?

3. Does the existence of money lead to the best outcome possible in the given environment?
Economic Questions

We will answer the above three questions as follows:

1. First, find out the best outcome possible (Golden Rule Allocation/ Allocation by the central planner).

2. Second, derive the allocation in a barter economy and compare with the golden rule allocation.

3. Third, derive the allocation in an economy with money and compare with the golden rule allocation as well as the allocation in a barter economy.
Golden Rule or Central Planner Allocation

Imagine that there exists a central (or social) planner with total knowledge and complete control over economy. Question is what allocation of consumption he/she would choose. In order to answer the above question, we need to know the feasible set of allocations and the objective function or the preference of the planner.

We will begin with the feasible set of allocations. The feasible set of allocations consists of all the allocations of consumption goods which are equal to or lower than the total availability of goods.
To know feasible allocations, we need to know the total availability/supply of goods and total demand in any period. The total supply in period $t$ is

$$N_{ty}. \quad (1.1)$$

The demand for goods come from young and old individuals at time $t$. Suppose that each young consume $c_{1,t}$ amount of good and each old consume $c_{2,t}$ (symmetric allocation). Then total demand at time $t$ is

$$N_{t}c_{1,t} + N_{t-1}c_{2,t}. \quad (1.2)$$
Feasible Allocations

The set of feasible allocations are given by all the combinations of \( c_{1,t} \) and \( c_{2,t} \) such that

\[
N_t c_{1,t} + N_{t-1} c_{2,t} \leq N_t y. \tag{1.3}
\]

(1.3) is also known as the feasibility constraint. If we assume that population is constant, \( N_t = N, \forall t \), then (1.3) can be written as

\[
c_{1,t} + c_{2,t} \leq y. \tag{1.4}
\]

If we further assume that consumption is stationary i.e., each generation has same consumption pattern, \( c_{1,t} = c_1 \), and \( c_{2,t} = c_2 \ \forall t \), then the set of feasible allocations is given by \( c_1, c_2 \) such that

\[
c_1 + c_2 \leq y. \tag{1.5}
\]
Assume that the central/social planner wants to maximize the utility of future generations (not of initial old). Given feasible allocations and the objective function, now we can derive the optimal allocations chosen by the planner. Such an allocation is known as the Golden Rule Allocation. Focussing on symmetric and stationary allocations, the planner problem reduces to

\[ \max_{c_1, c_2} U(c_1, c_2) \]  \hspace{1cm} (1.6)

subject to the feasibility constraint (1.5).
Golden Rule Allocation

The golden rule allocation can be derived as follows. Since the utility is strictly increasing in consumption, (1.5) will be binding.

\[ c_1 + c_2 = y. \]  

(1.7)

Putting (1.7) in the objective function (1.6), the optimization problem reduces to

\[
\max_{c_1} U(c_1, y - c_1).
\]

(1.8)

The first order condition is

\[ U_1 = U_2 \]

(1.9)

which implies that allocation will be such as to equate the marginal utility of consumption across two periods. (1.7) and (1.9) together give the golden rule allocation \( \{ c_1*, c_2* \} \).
Now we consider allocations achieved when individual agents interact with each other and take their own decisions. First, we consider allocations in the barter economy and then in an economy with money.

Before we do so, we need to make some assumptions regarding the behavior of individuals and the market structure and define the notion of equilibrium. These are given below:
A competitive equilibrium has following properties:

1. Individuals maximize their utilities subject to their budget constraints.

2. They take prices as given.

3. Supply equals demand in each market i.e., markets clear.

We are going to find allocations which satisfy the above three conditions. Allocations and prices which satisfy these conditions constitute the competitive equilibrium.
Equilibrium in a Barter Economy

In the economy, individual agents decide how much to consume when they are young and how much when they are old. Notice that old do not have any endowment. Also no young individual can save in order to finance his/her consumption for old age, as goods are perishable. Thus old can consume if they can trade with the young.

But will any young would trade with old? Answer is NO! An old has no endowment and thus cannot give a young anything in exchange. A young will also not sell good to an old on credit, because the old will not be alive next period to repay. There will be no trade among young because they are all identical.

Thus in a barter economy there will be no trade (autarky). Young will consume their endowment, and old will consume nothing i.e., \( c_1 = y, \ c_2 = 0 \). Since, individuals would like to consume in both parts of their lives, they are worse-off compared to the golden rule allocation.
Equilibrium in a Monetary Economy

Now we consider equilibrium in a monetary economy. Suppose that the government/central bank introduces $M$ units of fiat money in the economy. Fiat money has following characteristics:

1. They are intrinsically useless. In other words, they do not directly serve as an object of consumption or input for production.

2. They are (nearly) costless to produce.

3. They cannot be counterfeited.
A monetary equilibrium is a competitive equilibrium in which (fiat) money is valued or have purchasing power.

Given the characteristics of fiat money, it will be valued only if it enables individuals to trade for something they want to consume. As discussed earlier, there can potentially be trade between the young and the old. But given the age structure of the population such trades do not take place in the barter economy.

The question of whether money will be valued reduces to whether it can facilitate trade between the young and the old. In other words, will a young in the current period give up part of his/her endowment in exchange for money?
To answer the above question, assume that at time 1, the government/central banker distributes $M$ units of fiat money equally among the initial old. Thus, each initial old has $\frac{M}{N}$ units of money.

In this economy, young individuals can sell part of their endowment to old individuals for money in the current period and then buy consumptions goods next period using money. If such exchanges are possible, then each generation can consume in both parts of their lives - young and old and attain higher utility.
For such exchanges to be possible two conditions are necessary:

1. Supply of fiat money should be limited.

2. Young in each period must believe that next period money will be valuable. If in any period $T$, young believe that money will be not valuable next period, it will be worthless not only in time $T$ but all the previous periods.
With the above comments in the background, we are in a position to derive allocations and prices in the monetary economy. First, we will derive optimal choices made by individuals. Then, we will derive prices implied by the interactions among different individuals.

Let $p_t$ be the price of the consumption good. The purchasing power of one unit of money, $v_t$, is defined as

$$v_t = \frac{1}{p_t}.$$  (1.10)
Optimal Choices

We will analyze optimal choices of individuals under the assumption that money is valued. Then, we will show that it is indeed the case in equilibrium.

Any individual makes three choices: \( c_{1,t}, c_{2,t+1}, m_t \), where \( m_t \) is the amount of money acquired in period \( t \). We assume that these choices are made to maximize the utility of the individual.

\[
\max_{c_{1,t}, c_{2,t+1}, m_t} U(c_{1,t}, c_{2,t+1}).
\]  
(1.11)
Optimal Choices

These choices are made subject to the individual budget constraints. The budget constraint in period \( t \) is

\[
c_{1,t} + v_t m_t \leq y \tag{1.12}
\]

where \( v_t m_t \) is the units of endowment sold. The budget constraint in period \( t + 1 \) is

\[
c_{2,t+1} \leq v_{t+1} m_t. \tag{1.13}
\]

We can combine (1.12) and (1.13) and derive an individual’s life-time budget constraint

\[
c_{1,t} + \left[ \frac{v_t}{v_{t+1}} \right] c_{2,t+1} \leq y. \tag{1.14}
\]
Optimal Choices

An individual’s problem is

$$\max_{c_{1,t}, c_{2,t+1}} U(c_{1,t}, c_{2,t+1}).$$ \hspace{1cm} (1.15)

subject to

$$c_{1,t} + \left[ \frac{v_t}{v_{t+1}} \right] c_{2,t+1} \leq y.$$ \hspace{1cm} (1.16)

While making optimal choices, the individual takes value of money (or prices) $v_t$, $v_{t+1}$ as given.
Optimal Choices

Given the assumptions regarding the utility function, the budget constraints (1.12) and (1.13) will be binding. We can put the budget constraints in the objective function (1.15). The problem now becomes

$$\max_{m_t} U(y - v_t m_t, v_{t+1} m_t).$$ 

(1.17)

The first order condition is

$$\frac{U_1(y - v_t m_t, v_{t+1} m_t)}{U_2(y - v_t m_t, v_{t+1} m_t)} = \frac{v_{t+1}}{v_t}. \quad (1.18)$$

(1.18) equates the marginal rate of substitution between the first and the second period consumption to the ratio of the value of money between the second and the first period.
Optimal Choices

The ratio of the value of money between the second and the first period \( \frac{v_{t+1}}{v_t} \) can be interpreted as the real rate of return of money. By accepting one unit of money in period \( t \), an individual forgoes the opportunity to consume \( v_t \) units of good. But by using money, the individual acquires \( v_{t+1} \) units of consumption good next period. Thus the pay-off for holding money is \( v_{t+1} \) units of consumption good.

In general, one can derive (gross) rate of return on any asset by using the following formula \( \text{Rate of Return} = \frac{\text{Pay-off}}{\text{Price of the Asset}} \).
Using (1.18) together with (1.12) and (1.13), we can derive the optimal choices of an individual, $c_{1,t}$, $c_{2,t+1}$, $m_t$. However, they will be functions of the value of money $v_t$, $v_{t+1}$, which are yet unknown.

In order to find $v_t$, $v_{t+1}$, we turn to market clearing condition. The market clearing condition for money is that the demand for money should equal its supply.
Rate of Return on Money

The supply of money is equal to \( M_t \). What is the demand? The demand for money comes from young who would like to sell some of their endowments to the old. The demand is \( N_t m_t \). Thus market clearing requires that

\[
M_t = N_t m_t. \tag{1.19}
\]

Using the budget constraint of the first period (1.12), we can derive the expression for \( v_t \) as

\[
v_t = \frac{N_t(y - c_{1,t})}{M_t}. \tag{1.20}
\]

Similarly,

\[
v_{t+1} = \frac{N_{t+1}(y - c_{1,t+1})}{M_{t+1}}. \tag{1.21}
\]
Rate of Return on Money

Using (1.20) and (1.21), we can derive the (real) rate of return on money

\[
\frac{V_{t+1}}{V_t} = \frac{N_{t+1}(y-c_{1,t+1})}{M_{t+1}}.
\]

(1.22)

In the stationary environment where \(c_{1,t} = c_1\) and \(c_{2,t} = c_2\), the (real) rate of return on money is

\[
\frac{V_{t+1}}{V_t} = \frac{N_{t+1}}{M_{t+1}}.
\]

(1.23)

If we assume constant population \(N_t = N_{t+1} = N\) and constant money supply \(M_t = M_{t+1} = M\), then the above expression simplifies to

\[
\frac{V_{t+1}}{V_t} = 1
\]

(1.24)

implying constant value of money.
Goods Market Clearing Condition

In the economy apart from money market, there is also goods market. Similar to money market, this market should also clear in each time period, $t$. This requires that market demand for goods should equal market supply of good for each $t$.

$$N_t c_{1,t} + N_{t-1} c_{2,t} = N_t y. \quad (1.25)$$

The left hand side is total demand (from young and old) and the right hand side is total supply. As you can see, it is nothing but feasibility constraint discussed earlier. In the stationary environment with constant population (1.25) can be written as

$$c_1 + c_2 = y. \quad (1.26)$$
Competitive Monetary Equilibrium consists of optimal choices $c_{1,t}$, $c_{2,t+1}$, $m_t$ characterized by (1.12), (1.13), and (1.18), the real rate of return ($\frac{v_{t+1}}{v_t}$) given by (1.22), and the goods market clearing condition given in (1.26).
Recall that the golden rule allocation is given by (1.9)

$$\frac{U_1(c_1)}{U_2(c_2)} = 1 \quad (1.27)$$

and the resource constraints (1.7). With constant population and money supply, allocations in the monetary equilibrium are also characterized by the same equations (just put 1.24 in 1.18 and use budget constraints). Thus the competitive monetary equilibrium achieves the golden rule allocation.
Suppose now that population is growing at the constant growth rate, \( n \). Thus

\[ N_t = nN_{t-1}. \]  \tag{1.28}

Rest of the environment remains the same. Now we want to find out the golden rule allocation and the monetary equilibrium. The set of feasible allocations is given by

\[ N_t c_1 + N_{t-1} c_2 \leq N_t y \]  \tag{1.29}

which simplifies to

\[ c_1 + \left[ \frac{1}{n} \right] c_2 \leq y. \]  \tag{1.30}
The Golden Rule Allocation

The golden rule allocation can be found by solving the following problem:

\[
\max_{c_1, c_2} U(c_1, c_2)
\]

subject to (1.30). The first order condition is

\[
\frac{U_1(c_1)}{U_2(c_2)} = n.
\]

The central/social planner equates the marginal rate of substitution between the current and the future consumption to the growth rate of population. (1.31) together with (1.30) give the golden rule allocations.
The Competitive Monetary Equilibrium

An individual faces the same problem as he/she did in an economy without population growth. Thus the optimal choices continue to be characterized by (1.12), (1.13), and (1.18). Only thing changes are expressions for market clearing conditions. From (1.23) we have

$$\frac{v_{t+1}}{v_t} = n. \quad (1.32)$$

and the goods market clearing condition is given by

$$c_1 + \frac{c_2}{n} = y. \quad (1.33)$$

Finally, (1.12), (1.13), (1.18), (1.32) and (1.33) imply that allocations in the monetary equilibrium are identical to the golden rule allocations.