Main Aims:

1. Study money as a liquid asset.
2. Develop an OLG model in which individuals live for three periods.
3. Analyze two roles of banks:
   (1.) correcting the mismatch of assets maturities
   (2.) monitor of risky projects.
Money as a Liquid Asset

Money plays two roles in an economy: a medium of exchange and a store of value. So far we have focused mainly on the role of money as a store of value, as it allows young individuals to save for their old age. In the previous chapter, we saw that once we allow for other instruments of savings (e.g. capital), then in order to induce individuals to hold money the rate of return on money should be identical to that of capital. This result crucially depends on the assumption that money and capital (or other assets) are perfectly substitutable. However, in real world the rate of return on money is lower than other assets. Still, individuals hold money. In addition, money is used in the bulk of transactions. In that sense, it is a commonly accepted medium of exchange.
Money as a Liquid Asset

The second feature of money makes it a special asset. It is the most **liquid** asset. It can be exchanged easily, quickly, and at a little cost. Thus, money and capital (or other assets) are not **perfect substitutes**. The important features of money that make it the most liquid asset are: **portability, divisibility, and recognizability**.

Thus, money yields two types of returns: the rate of return as a store of value and the return as a medium of exchange (liquidity service). Individuals hold money even though it yields lower return as a store of value, because of its liquidity.
A Model of Illiquidity

The aim is to develop a model, which is consistent with the following observations about real economies:

1. Fiat money and capital are both valued and held.

2. The rate of return of capital exceeds that of fiat money, \( f_1(k) > \frac{v_{t+1}}{v_t} \).

3. Fiat money is exchanged more frequently than capital.
Main Assumptions: Demographics

1. Individuals live for three periods. Let $c_{1,t}, c_{2,t+1}, c_{3,t+2}$ denote the consumption of an individual born in time $t$ in his/her first, second, and third period of life.

2. Let $N_t$ denote the number of new (young) individuals born in time $t$. The evolution of population is given by $N_t = nN_{t-1}$.

3. A constant supply of money $M$. The stock of money is distributed equally among the initial middle-aged.
Main Assumptions: Technology and Preferences

1. Each individual in his/her first period is endowed with \( y \) units of good.

2. Each individual possess a technology which converts one unit of good saved as capital in the first period in \( X \) units of goods in the third period, with \( X > n^2 \). Linear production function: \( f(k) = Xk \).

3. The utility function of an individual is: \( U(c_1,t, c_{2,t+1}, c_{3,t+2}) \). Any individual would like to consume in all three-periods of life.
Main Assumptions: Two Assumptions to Rule Out Credit (IOU)

1. It is impossible to observe the amount of capital saved by an individual.

2. It is not possible for an individual to enforce a repayment of IOUs.

The above two assumptions rule out any exchanges on credit.
Main Problem

The main problem faced by an individual is to provide for consumption in his second period of life. He/she can consume in the first-period using his/her endowment, and the third-period by saving in terms of capital. Given the absence of credit, only way in which an individual can consume in the second-period is through saving in terms of money. This creates the role for money in this environment.

Note that the second-period consumption can be financed only through money. However, the third-period consumption can be financed by both money and capital. The question now is whether he/she will finance his/her third-period consumption through money or capital or both.
Main Problem

The answer is that the third-period consumption is financed only through capital. The reason is that over two periods capital yields a higher rate of return. One-period rate of return on money is $\frac{v_{t+1}}{v_t} = n$ (constant money supply $z = 1$). Two-period rate of return on money $\frac{v_{t+2}}{v_t} = \frac{v_{t+2}}{v_{t+1}} \frac{v_{t+1}}{v_t} = n^2$. By assumption, the rate of return on capital $X > n^2$, and thus it is a superior medium of saving over two periods. Thus, individuals will finance their consumption in their third-period through capital.

Given the assumptions, the model implies that money is held only for one period in order to finance the second-period consumption. On the other hand, capital is held for two periods in order to finance consumption in the third period. Thus, money changes hands more frequently. Also both money and capital are held, though money yields a lower rate of return.
The Budget Constraint

The budget constraints for different periods are

\[ c_{1,t} + v_t m_t + k_t \leq y. \] \hspace{1cm} (7.1)

\[ c_{2,t+1} \leq v_{t+1} m_t. \] \hspace{1cm} (7.2)

\[ c_{3,t+2} \leq X k_t. \] \hspace{1cm} (7.3)

The life-time budget constraint is

\[ c_{1,t} + \frac{v_t}{v_{t+1}} c_{2,t+1} + \frac{1}{X} c_{3,t+2} \leq y. \] \hspace{1cm} (7.4)
An Individual’s Problem

An individual’s problem is to choose his/her consumption pattern, \( c_{1,t}, c_{2,t+1}, c_{3,t+2} \), and the portfolio allocation, \( m_t, k_t \) in order to maximize his/her utility

\[
\max_{c_{1,t}, c_{2,t+1}, c_{3,t+2}, m_t, k_t} U(c_{1,t}, c_{2,t+1}, c_{3,t+2}) \tag{7.5}
\]

subject to his/her budget constraints (7.1), (7.2) and (7.3).
An Individual’s Problem

By putting (7.1), (7.2), and (7.3) in the objective function (7.5), we can convert the above constrained optimization problem in the unconstrained one. The unconstrained problem is:

\[
\begin{align*}
\max_{m_t, k_t} U(y - v_t m_t - k_t, v_{t+1} m_t, Xk_t).
\end{align*}
\] (7.6)

The first-order conditions are

\[
\begin{align*}
    m_t : \frac{U_1}{U_2} &= \frac{v_{t+1}}{v_t} \quad \text{(7.7)}
\end{align*}
\]

and

\[
\begin{align*}
    k_t : \frac{U_1}{U_3} &= X. \quad \text{(7.8)}
\end{align*}
\]

(7.7) and (7.8), together with the individual’s budget constraint determine optimal choices of an individual. \( m, k \).
Competitive Monetary Equilibrium

We can use market clearing conditions along with the first order conditions and budget constraints to pin down allocations and prices. From the money market clearing condition, we have

\[
\frac{v_{t+1}}{v_t} = n. \tag{7.9}
\]

The goods market clearing condition is

\[
N_t c_{1t} + N_t k_t + N_{t-1} c_{2t} + N_{t-2} c_{3t} = N_t y + N_{t-2} k_t X. \tag{7.10}
\]

In the stationary environment, the goods market clearing condition can be written as

\[
c_1 + k + \frac{c_2}{n} + \frac{c_3}{n^2} = y + \frac{Xk}{n^2}. \tag{7.11}
\]
In the previous section, we saw that money yields a lower rate of return than capital, but still it is held because money is a liquid asset and capital illiquid. Crucial to the argument is that goods cannot be bought and sold on credit due to problems with the enforcement of the repayments. Now, we modify this assumption. Suppose that one individual (or few but not all), call him/her a bank, is such that he/she can issue IOUs. In other words, the bank accepts deposits and returns the deposits with interest on demand. Then as described below the bank can exploit the rate of return-differences between money and capital, called arbitrage, and can make pure profit.
Emergence of Banks: Arbitrage

Suppose that the bank at time $t$ borrows one unit of good for one period (or accepts one period deposit) from a young individual at the gross interest rate $r$. The individual will be willing to lend him/her as long as $r \geq n$ i.e., the rate of return to the lender (depositor) should be at least as great as the rate of return on the alternative asset, namely money. The bank uses the borrowed goods as capital, which yields $X$ units of goods in time $t + 2$.

However, the bank has to pay-back $r$ units of goods to the lender at time $t + 1$. In order to meet the repayment obligations at time $t + 1$, the bank can borrow $r$ units of goods from a young individual at the interest rate $r$ for one period at time $t + 1$. In time $t + 2$, the total repayment obligation for the bank is $r \cdot r = r^2$. The total net profit to the bank from these operations is $X - r^2$. As long as $X > r^2$, which will indeed be the case with one bank (in the case of one bank $r = n$), the bank makes a positive profit.
Competitive Banking

The rate of return differences arising out of the mismatch of asset maturities provide an opportunity to make profits from arbitrage and can potentially lead to the emergence of banks. In the previous case, we considered the case of the monopoly bank (one bank) and we saw that such a bank makes a strictly positive profit. However, if we allow competition among many banks, then the profit will vanish i.e., $X = r^2$. This implies that the rate of return offered, $r = \sqrt{X}$.

Note that with banking no saving is done in terms of fiat money. Individuals save in terms of capital and lending (making deposits to the banks). The banks, in turn, use all the deposits as capital. Thus, an economy with banking has more capital stock and output than an economy without banking. Also future generations are better-off, as they receive a higher return on their savings.
Suppose that a bank offers rate of interest $r > n$ on one-period deposit. What will be the optimal portfolio choice? Let $d_t$ denote the amount deposited by the individual in the bank. Then the budget constraints will be

$$c_{1,t} + d_t + k_t \leq y. \quad (7.12)$$

$$c_{2,t+1} \leq rd_t. \quad (7.13)$$

$$c_{3,t+2} \leq Xk_t. \quad (7.14)$$

The life-time (or inter-temporal) budget constraint is

$$c_{1,t} + \frac{c_{2,t+1}}{r} + \frac{c_{3,t+2}}{X} \leq y. \quad (7.15)$$
An Individual’s Problem

An individual’s problem is to choose his/her consumption pattern, $c_{1,t}$, $c_{2,t+1}$, $c_{3,t+2}$, and the portfolio allocation, $d_t$, $k_t$ in order to maximize his’her utility

$$\max_{c_{1,t}, c_{2,t+1}, c_{3,t+2}, d_t, k_t} U(c_{1,t}, c_{2,t+1}, c_{3,t+2})$$

subject to his/her budget constraints (7.12), (7.13) and (7.14).
An Individual’s Problem

By putting (7.12), (7.13), and (7.14) in the objective function (7.16), we can convert the above constrained optimization problem in the unconstrained one. The unconstrained problem is:

$$\max_{m_t,k_t} U(y - d_t - k_t, rd_t, Xk_t). \quad (7.17)$$

The first-order conditions are

$$d_t : \frac{U_1}{U_2} = r \quad (7.18)$$

and

$$k_t : \frac{U_1}{U_3} = X. \quad (7.19)$$

(7.18) and (7.19), together with the individual’s budget constraint determine optimal choices of an individual. $d_t, k_t$. 
Banks as Monitors of Risky Projects

Banks can also emerge in the case where projects require borrowed funds, pay-offs from the projects are risky (or uncertain), and there is asymmetric information regarding the success or failures of projects. More concretely a borrower knows exactly whether the project has been successful or a failure. One the other hand, lenders must incur some cost, called **monitoring cost**, in order to find out whether the project has failed or been successful.

In such a setting, banks can intermediate among borrowers and lenders. They can monitor the projects on behalf of lenders (or depositors), offer them a risk-free way to invest in the risky assets, and reduce the overall cost of monitoring.
Banks as Monitors of Risky Projects

In this section, we build a model of banking which displays following four features:

1. Each bank deals with large numbers of depositors (lenders) and entrepreneurs (borrowers).
2. Loans that must be monitored are made by banks rather than individuals.
3. Banks offer depositors a risk-free return while holding risky assets.
4. Depositors do not monitor the banks’ performance.
Main Assumptions: Nature of Projects and Information Structure

1. There are large number of entrepreneurs. Each entrepreneur has one project. Entrepreneurs require capital to start their projects. Suppose that each project requires $\mu k$ units of capital with $\mu > 1$.

2. There are equal number of risk-averse lenders each with capital $k$.

3. Projects are risky. With probability $2/3$ a project succeeds and with probability $1/3$ it fails. In the case of success, the pay-off is $x\mu k$ and in the case of failure the pay-off is zero.

4. Only after the investment has been done, an entrepreneur knows whether the project has been successful or not.

5. The success and failure of a project is not observed costlessly to others. In particular, it costs $\theta$ units of goods to any individual other than the entrepreneur to find out about the success or failure of the project.
Nature of Contract

1. In the case of failure, an entrepreneur has no resources to repay the loans. Thus, a contract cannot require an entrepreneur to make payments when the project fails.

2. In such a setting, in the absence of monitoring a successful entrepreneur has an incentive to lie as he/she can claim the entire pay-off.

3. In order to deter a successful entrepreneur from lying, the contract between a lender and an entrepreneur would involve investigation of the claim of failure by an entrepreneur. Assume that a lender investigates every claim of failure.

4. Also the contract would require repayment contingent on the outcome of the project. In the case of failure, there will be no repayment. But in the case of success, there will be repayment, the amount of which is determined below.
Let \( N \) be the total number of entrepreneurs and thus total number of potential projects. Since, the number of lenders is identical to the number of entrepreneurs, there are also \( N \) number of lenders. Total fund available for lending then is \( Nk \). Total potential demand for borrowing is \( \mu Nk \). Since, \( \mu > 1 \) this implies that not all potential projects are funded. Only \( \frac{1}{\mu} \) fraction of potential projects are funded. Since, each lender has at most \( k \) amount of capital to lend, given \( \mu > 1 \), a project requires borrowing from more than one lender.

In the case, a project receives funding, the expected pay-off from the project is

\[
\frac{2}{3}x\mu k + \frac{1}{3}0 = \frac{2}{3}x\mu k. \tag{7.20}
\]

The rate of return to an entrepreneur in the case of successful project is \( \frac{x\mu k}{\mu k} = x \).
Since only a fraction of projects are funded, the competition among entrepreneurs ensures that the rate of return received by a lender in the case of successful project is equal to $x$. As mentioned earlier, lenders are risk-averse and thus would like to avoid the risk of project failure. In the present environment, it can reduce the risk of failure by lending to many entrepreneurs rather than to one (diversification of risks). On the other hand, lending to more than one entrepreneur increases the expected monitoring cost. The trade-off between the monitoring cost and the risk-diversification enures that a lender lends to more than one entrepreneur.

Suppose that a lender lends to $J$ entrepreneurs and thus lending-per-entrepreneur is $K_J$. This also implies that a project is financed by $\mu J$ lenders. The expected total return to the lender from such a strategy is
Investment Without Banks/Intermediation

$$\frac{2}{3} xk + \frac{1}{3} (-\theta)J$$

(7.21)

where second term in (2.21) comes from the fact that a lender investigates each case of reported failure. The average return to a lender per-unit of investment is

$$\frac{2}{3} x + \frac{1}{3} (-\theta) \frac{J}{k}.$$  

(7.22)

There are two types of inefficiencies in this economy. Firstly, a lender who lends to many entrepreneurs incurs on average higher monitoring cost compared to the case where he/she lends to just one entrepreneur. Secondly, there is a great deal of duplication of monitoring. As each entrepreneur borrows from $\mu J$ lenders, in the case of a failure of a project it is investigated by all the $\mu J$ lenders. As we will see below, the presence of banks minimizes these monitoring costs and also allows lenders to completely diversify their risks.
Now suppose there is a bank which accepts deposits from the lenders and lends those deposits to entrepreneurs. Since a bank is large relative to an entrepreneur, it can lend to a large number of entrepreneurs. This way it can completely diversify its risks in the sense that the return or the pay-off from its investment will be certain (not uncertain). The average rate of return from each project will be

\[
\frac{2}{3} x \mu k + \frac{1}{3} (-\theta). \tag{7.23}
\]

Note that in the case of failure, a project is investigated only once by the bank. Since lending is equal to \( \mu k \) per project, it can offer the rate of return to the depositor per unit of investment up to

\[
\frac{2}{3} x + \frac{1}{3} \left( \frac{-\theta}{\mu k} \right). \tag{7.24}
\]
In fact if we allow, competition among banks, then (7.24) will be the return which a lender (depositor) will get. The comparison of (7.22) with (7.24) shows the lenders receive a higher rate of return with banking. Also the resources lost in monitoring failed projects are also lower as each failed project is investigated only once.