Assignment 1: ECON 420

Due Date: Thursday, 8th February, in Class

Total Marks: 100

This assignment is based on Solow model and examines the issue of cross-country variations in the per-capita income and the speed of convergence. Below I briefly describe the questions you should address and the methodology and the data-set you should use.

Suppose that the production function is given by

\[ Y_{i,t} = A_{i,t} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha} \]

with \( 0 < \alpha < 1 \), where \( Y_{i,t} \), \( A_{i,t} \), \( K_{i,t} \), and \( L_{i,t} \) are the real GDP, total factor productivity (TFP), capital stock, and labor force in country \( i \) at time \( t \) respectively. Let \( y_{i,t} \), \( s_{i} \), and \( n_{i} \) be the per-worker real GDP at PPP (Purchasing Power Parity) at time \( t \), the average savings rate, and the labor force growth rate in country \( i \) respectively. Let \( A_{i,t} = A_{0} \exp^{g_{t}} \exp^{\xi_{i,t}} \), where \( g \) is the growth rate of TFP and \( \xi_{i,t} \), is the error term. Then, the Solow model predicts that (see Mankiw, Romer and Weil 1992 for derivation)

\[
\ln y_{i,t} = \ln A_{0} + gt + \frac{\alpha}{1-\alpha} \ln s_{i} - \frac{\alpha}{1-\alpha} \ln(n_{i} + g + \delta) + \xi_{i,t}. \tag{1}
\]

Using (1), one can test the implications of the Solow model. For the regression purpose (1) can be written as

\[
\ln y_{i,t} = \beta_{0} + \beta_{1} \ln s_{i} + \beta_{2} \ln(n_{i} + g + \delta) + \xi_{i,t}. \tag{2}
\]

For estimating (2), one needs data for \( y_{i,t}, s_{i} \) and \( n_{i} \) for different countries. We will use the Penn World Tables (PWT) version 8.1 data for the period 1990-2011, which can be accessed at https://www.rug.nl/ggdc/productivity/pwt/pwt-releases/pwt8.1 (see the link at the course web-page). This data set is widely used in the empirical literature.

In the sample, we will include only those countries for which data is available for the entire period: 1990-2011. From PWT, we will use following data series \texttt{rgdpe} (Expenditure-side real GDP at chained PPPs (in mil. 2005US$)), \texttt{emp} (Number of persons engaged (in millions)), and \texttt{csh.i} (share of gross capital formation at current PPPs).

We will use following notations. Let the initial period \( t-1 \) be 1990 and the terminal period \( t = 2011 \). Then,

\begin{align*}
  y_{i,t} &= \text{Per-worker real GDP in country } i \text{ in year } 2011 \\
  y_{i,0} &= \text{Per-worker real GDP in country } i \text{ in year } 1990 \\
  s_{i} &= \text{Average savings/investment rate for country } i \text{ over the period } 1990-2011
\end{align*}
\( n_i = \) The labor force growth rate in country \( i \) over the period 1990-2011

For the estimation purpose, we will need \( \text{rgdpe} \) for two years 1990 and 2011. We will need other data series for the entire period (1990-2011). First divide \( \text{rgdpe} \) by \( \text{emp} \) to derive per-worker real GDP for 1990 and 2011 \((y_{i,t} \text{ for } t = 1990, 2011)\). Then take the average of \( \text{csh}_i \) for the entire period to derive the average savings rate, \( s_i \).

Penn World Table does not provide information on the labor force growth rate. We will proxy the labor force growth rate by the employment growth rate. To calculate the employment growth rate run the following regression:

\[
\ln \text{emp}_{it} = a + b \times \text{time}, \ t = 1990...2011. \tag{3}
\]

The estimated \( b \) can be interpreted as the annual growth rate in employment. Finally assume that \( g + \delta = 0.05 \) as in (Mankiw, Romer and Weil 1992).

(i) Once you have generated all the relevant data, discuss and describe broad patterns of per-capita real income, savings rate and the employment growth rate. You may find it useful to calculate summary statistics (e.g. mean, median, standard deviation, maximum, minimum) of these variables and use graphs. [Marks 30]

(ii) Estimate (2) using Ordinary Least Squares and discuss the results. [Marks 20]

(iii) The Solow model puts the restriction that \( \beta_1 = -\beta_2 \). Estimate the following restricted version of the Solow model for the time period 1990-2011:

\[
\ln y_{i,t} = \mu_0 + \mu_1 (\ln s_i - \ln (n_i + g + \delta)) + \xi_{i,t} \tag{4}
\]

and discuss the results. [Marks 10]

**Note:** You can test the implied restriction using Wald test. If the implied restriction is rejected, one should not estimate the restricted version of the model. However, for this project we will ignore this issue.

### Convergence

(iv) Estimate the following regression model:

\[
\ln y_{i,t} - \ln y_{i,t-1} = \beta_0 + \beta_1 \ln y_{i,t-1} + \xi_i \tag{5}
\]

and discuss whether there is evidence of unconditional convergence. [Marks 10]

(v) Now estimate the following regression:
\[
\ln y_{i,t} - \ln y_{i,t-1} = \beta_0 + \beta_1 \ln y_{i,t-1} + \beta_2 \ln s_i + \beta_3 \ln(n_i + g + \delta) + \xi_i \quad (6)
\]

and discuss whether there is evidence of conditional convergence. [Marks 10]

(vi) Discuss other factors and variables which may affect growth and speed of convergence. [Marks 20]

Some suggestions for further reading:

References


