Lecture 1
Growth

There are vast differences in the per-capita income and the standards of living across countries. The average per-capita income of industrialized countries is about ten-times the average per-capita income of poor countries. There are also considerable variations in the cross-country growth rates of income. Currently, countries like China and India are experiencing very high rates of growth. On the other hand, there are many African countries which are experiencing stagnation or even decline in per-capita income. Industrialized countries have experienced sustained increase in their real per-capita income of 2-3% per annum over the last 100 years. The growth experience of other countries has been quite uneven.

In this chapter, we analyze the growth process and its determinants. We address questions such as (i) Why are some countries poor and some rich? (ii) What is the role of human capital in the growth process? (iii) Will poor countries catch up with rich countries? (iv) What are the main determinants of productivity? We will address these questions using Solow or Neo-classical growth model. This model is a starting point for any analysis of growth. It relates long run per-worker consumption and output to savings rate, labor force growth rate, and technical progress. We will analyze three versions of the model: (i) Solow model with no technical progress (ii) Solow model with technical progress and (iii) Augmented Solow growth model. In the basic Solow model, production is assumed to depend on capital and labor. In the Augmented Solow model, human capital (education, health or both) is introduced as an additional input in production.

The growth literature distinguishes between (i) proximate determinants of growth and (ii) deep determinants of growth. The Solow model identifies proximate determinants of growth. The deep determinants literature analyzes the effects of geography, institutions, and social and political systems on growth.

1. Solow Growth Model with no Technical Progress

Assumptions

1. Constant returns to scale production technology, $Y = F(K, L)$, where K and L are aggregate stocks of capital and labor;
2. Diminishing marginal productivity of capital and labor;
3. Constant rate of savings (s), thus total savings is $S(t) = sY(t)$;
4. Constant labor force growth rate (n); $\frac{dL(t)}{dt} = \dot{L} = nL(t)$;
5. Constant depreciation rate (δ);
6. Competitive markets;
7. No technical progress (Temporary Assumption).

Let $I(t)$ be the gross investment at time t, then by definition, the growth rate of capital stock is given by

$$\dot{K} = I(t) - \delta K(t) \tag{1}$$

where $\dot{K} = dK(t)/dt$. Since in equilibrium, savings equal investment, $S(t) = I(t)$, we have

$$\dot{K} = sY(t) - \delta K(t). \tag{2}$$

Now define capital-labor ratio as, $k = \frac{K}{L}$. Then, given constant returns to scale, per-worker output can be written as

$$\frac{Y(t)}{L(t)} = F(\frac{K(t)}{L(t)}, 1) \equiv f(k(t)). \tag{3}$$

Also

$$\frac{\dot{K}}{L(t)} = \frac{sY(t) - \delta K(t)}{L(t)} = sf(k(t)) - \delta k(t). \tag{4}$$

$$\dot{k} = \frac{\dot{K}}{L(t)} - k(t)\frac{\dot{L}}{L(t)}. \tag{5}$$

By combining (4) and (5), we get a non-linear differential equation in the capital-labor ratio, k, which characterizes Solow growth model.

$$\dot{k} = sf(k(t)) - (\delta + n)k(t). \tag{6}$$

Solution of (6) tells us the implications of the Solow model. The first term in the RHS of (6) is the actual investment per worker. The second term, $(\delta + n)k$, is the break-even investment per worker. This is the level of investment, which keeps capital-stock per worker constant. If actual investment exceeds break-even investment, capital per worker rises and $\dot{k} > 0$. On the other hand, if actual investment is lower than the break-even investment, capital per worker falls and $\dot{k} < 0$. When two are equal, capital per worker is constant and $\dot{k} = 0$.

(6) is an example of an autonomous, nonlinear, first-order differential equation. We will use phase diagram to draw its qualitative properties.

Autonomous Differential Equation

The initial-value problem for an autonomous, nonlinear, first-order differential equation has the following form:

$$\dot{z} = g(z(t)) \;\&\; z(t_0) = z_0 \tag{7}$$

where $\frac{dg(z)}{dz}$ $\&\frac{d^2g(z)}{dz^2} \neq 0$.

Phase Diagram

Although, it is known that solution to (7) exists under the condition that $\frac{dg(z)}{dz}$ is continuous in the neighborhood around t_0, in most cases it is not possible to derive the explicit solution. Often qualitative properties of the differential equation are derived by plotting it. Such plots are known as **phase diagram**.

Steps in Drawing Phase Diagram

Our goal is to plot $\dot{z}$ or $g(z(t))$.

Step 1 Take $\dot{z}$ or $g(z(t))$ on y-axis and $z(t)$ on x-axis.

Step 2 Take the first and the second derivative of $\dot{z}$ or $g(z(t))$ with respect to z, $\frac{dg(z)}{dz}$ and $\frac{d^2g(z)}{dz^2}$. This gives you the shape of the curve (increasing, decreasing, concave, or convex).

Step 3 Derive the steady-state points by setting

$$\dot{z} = g(z(t)) = 0. \tag{8}$$

Steady-state or equilibrium points are the points at which the curve of $\dot{z}$ or $g(z)$ intersects the x-axis. There can be more than one steady-state point (multiplicity of equilibria).

Stability Analysis

Stability analysis tells us about the convergence property of the differential equation. Steady-state points can be **stable** or **unstable**. A steady state point is stable, if the differential system converges to that point. Otherwise, it is unstable.

Theorem: A steady-state equilibrium point of a nonlinear first-order differential equation is stable if the derivative $\frac{d\dot{z}}{dz} < 0$ at that point and unstable if the derivative is positive at that point

With this digression we are now ready to derive the implications of the Solow model.

Implications of the Solow Model without Technical Progress

The steady state points satisfy

$$sf(k(t)) = (\delta + n)k(t). \tag{9}$$

To ensure that steady-state points exist, assume that the production function is such that $f(0) = 0$, $\lim_k \to 0 \; f_k(k) = \infty$ & $\lim_k \to \infty \; f_k(k) = 0$. These assumptions are known as Inada conditions. Under these assumptions, one can easily show that there are two steady-states: one at $k^* = 0$ and other at $k^* > 0$.

Given Inada condition, the slope of $\dot{k}$ is strictly positive (in fact ∞) at $k^* = 0$ and strictly negative at $k^* > 0$. Thus, steady state $k^* = 0$ is unstable and the steady state with $k^* > 0$ is stable. It implies that if the economy starts from any $k > 0$, ultimately it converges to the stable steady stable $k^* > 0$. This has following implications:

1. In the long run economy reaches a stable steady state equilibrium.
2. There will be no growth in per-worker output, $\frac{Y}{L}$, consumption, $\frac{C}{L}$, and capital stock, $\frac{K}{L}$, in the long run.
3. Output, Y, consumption, C, and capital stock, K, will grow at the labor-force growth rate, n, in the long run.
4. Per-worker output, $\frac{Y}{L}$, consumption, $\frac{C}{L}$, and capital stock, $\frac{K}{L}$, depend on the savings rate, s, and the labor force growth rate, n. An economy with a higher savings rate and a lower labor force growth rate will have higher steady-state per-worker output, $\frac{Y}{L}$, consumption, $\frac{C}{L}$, and capital stock, $\frac{K}{L}$.

The Solow model implies that the economy converges to a **balanced growth path** – a situation where each variable of the model is growing at a constant rate. Next, we consider a Solow model with technical progress.

2. Solow Growth Model with Technical Progress

In the previous section, we considered Solow growth model without technical progress. Now, we introduce technical progress. Suppose that production depends on capital, K, and **effective labor**, EL defined as

$$EL(t) = E(t)L(t) \tag{10}$$

where $E(t)$ is a measure of technology. $E(t)$ is also known as the **total factor productivity (TFP)**. Thus, the production function is

$$Y(t) = F(K(t), E(t)L(t)). \tag{11}$$

Suppose $E(t)$ evolves as follows

$$\dot{E} = \gamma E(t), \ \gamma > 0. \tag{12}$$

Such technical change is called **labor augmenting** or **Harrod-Neutral**. Rest of the model is identical to the previous one.

Now define capital-effective labor ratio as, $\kappa = \frac{K}{EL}$. Then, given constant returns to scale, output per effective labor unit can be written as

$$\frac{Y(t)}{EL(t)} = F(\frac{K(t)}{E(t)L(t)}, 1) = f(\kappa(t)). \tag{13}$$

Also

$$\frac{\dot{K}}{E(t)L(t)} = \frac{sY(t) - \delta K(t)}{E(t)L(t)} = sf(\kappa(t)) - \delta\kappa(t). \tag{14}$$

$$\dot{\kappa} = \frac{\dot{K}}{E(t)L(t)} - \kappa(t)\frac{\dot{E}L}{E(t)L(t)}. \tag{15}$$

Combining above two equations, we get a differential equation in the capital-effective labor ratio which characterizes the Solow growth model with technical change.

$$\dot{\kappa} = sf(\kappa(t)) - (\delta + \gamma + n)\kappa(t). \tag{16}$$

(16) is similar to (6). Now the break-even investment is given by $(\delta + \gamma + n)\kappa(t)$. As before there are two steady states, $\kappa^* = 0$, and $\kappa^* > 0$. $\kappa^* = 0$ is unstable and $\kappa^* > 0$ is stable. Thus, if the economy starts from any $\kappa > 0$, ultimately it converges to the stable steady stable $\kappa^* > 0$. This has following implications:

1. Ultimately economy converges to the balanced growth path just as in the case of no technical progress.
2. On the balanced growth path, output, Y, consumption, C, and capital stock, K, grow at the rate of $\gamma + n$.
3. At the steady state per-worker output, $\frac{Y}{L}$, consumption per worker, $\frac{C}{L}$, and capital stock per worker, $\frac{K}{L}$, grow at the rate of γ.
4. Per-effective labor output, $\frac{Y}{EL}$, consumption, $\frac{C}{EL}$, and capital stock, $\frac{K}{EL}$, depend on the savings rate, s, the labor force growth rate, n, and the rate of technical progress, γ. An economy with a higher savings rate and a lower labor force growth rate and rate of technical progress will have higher steady-state per-effective labor output, $\frac{Y}{EL}$, consumption, $\frac{C}{EL}$, and capital stock, $\frac{K}{EL}$.

We can also derive implications with regard to real wage, w, and the real rate of interest, r. By assumption, markets are competitive and thus factors of production are paid their marginal product. Thus, $w = F_{EL}(K, EL)E$ and $r = F_K(K, EL) - \delta$. Given constant returns to scale, this implies that real wage, w, will grow at rate γ on the balanced growth path. On the other hand, the real rate of interest, r, will remain constant. Also, an economy with a higher capital-effective labor ratio, κ, will have higher real wage, w, and lower real interest rate, r.

Speed of Convergence

We know that ultimately an economy converges to its balanced growth path. But how fast an economy converges to its balanced growth path? This is an important issue, since we are not only interested in the eventual effects of some changes, for example changes in savings rate, but also how rapidly those effects occur. To answer this question, let us take first-order Taylor approximation of (16) around stable steady state, $\kappa^* > 0$. We have

$$\dot{\kappa} \approx \frac{d\dot{\kappa}}{d\kappa}\Big|_{\kappa=\kappa^*}(\kappa(t) - \kappa^*). \tag{17}$$

Let $\lambda \equiv -\frac{d\dot{\kappa}}{d\kappa}\Big|_{\kappa=\kappa^*}$. Then (17) can be written as

$$\dot{\kappa} \approx -\lambda(\kappa(t) - \kappa^*). \tag{18}$$

Since, $\dot{\kappa}$ is positive when κ is below k^* and is negative when κ is above k^*, $\frac{d\dot{\kappa}}{d\kappa}\Big|_{\kappa=\kappa^*}$ is negative in the vicinity of the steady state. Thus, λ is positive. (18) implies that in the vicinity of steady state, κ moves towards κ^* at a speed approximately proportional to its distance from κ^*.

Note that (18) is an example of a **first order autonomous differential equation**. By solving this equation, one can derive an explicit path of $\kappa(t)$. The general form of the first order autonomous differential equation is

$$\dot{x} + ax(t) = b \tag{19}$$

where a and b are two constants. The steady state value of x is given by, $x^* = b/a$. For steady state to exist, we must have $a \neq 0$. Let $x(0)$ be the initial value of x, then the solution of (19) is given by

$$x(t) = \frac{b}{a} + \exp^{-at}(x(0) - x^*). \tag{20}$$

Using (20), we can also address the question, whether $x(t)$ will converge to its steady state value, x^* in the long run. This is equivalent to asking the question:

$$\lim_{t\to\infty} = \frac{b}{a} + \exp^{-at}(x(0) - x^*) = x^*? \tag{21}$$

If $\lim_{t\to\infty} = x^*$, then $x(t)$ will converge to its steady state value in the long run. The condition for convergence is the $a > 0$.

Using (20), we can solve for $\kappa(t)$:

$$\kappa(t) \approx \kappa^* + \exp^{-\lambda t}(\kappa(0) - \kappa^*). \tag{22}$$

We can also derive the expression for λ. From (16) we have

$$\lambda \equiv -\frac{d\dot{\kappa}}{d\kappa}\Big|_{\kappa=\kappa^*} = -[sf_\kappa(\kappa^*) - (n + \gamma + \delta)]. \tag{23}$$

Then combining (16) and (23) we have

$$\lambda = [1 - \alpha_\kappa(\kappa^*)](n + \gamma + \delta) \tag{24}$$

where $\alpha_\kappa(\kappa^*) = \frac{\kappa^* f_\kappa(\kappa^*)}{f(\kappa^*)}$ is the share of capital in output. Since, $\lambda > 0$ $\kappa(t)$ would converge to κ^* in the long run.

Golden Rule Level of Capital Stock

Welfare of households does not depend on output, but on consumption. We know that the economy converges to a balanced growth path. It raises the question whether the balanced growth path maximizes household welfare. On the balanced growth path, per-effective labor consumption, c, is given by

$$c = f(\kappa) - (n + \gamma + \delta)\kappa. \tag{25}$$

κ that maximizes c is given by

$$f_\kappa(\kappa) = n + \gamma + \delta. \tag{26}$$

The capital stock that satisfies (26) is known as the **golden rule level of capital stock**.

The steady state κ is given by

$$sf(\kappa) = (n + \gamma + \delta)\kappa. \tag{27}$$

From the comparison of (26) and (27), it is immediately clear that the capital-stock on the balanced growth path can be higher or lower than the golden rule of capital stock. Thus, the balanced growth path may not be welfare maximizing.

Growth Accounting

Many times we are interested in the proximate sources of growth. That is, we want to know how much of a growth over some period in a country is due to the input growth and how much is due to the technical progress. **Growth accounting** is one way to tackle this question. Idea behind growth accounting is quite simple. By differentiating (11), we can derive

$$\frac{\dot{Y}}{Y(t)} = \beta_K(t)\frac{\dot{K}}{K(t)} + \beta_L(t)\frac{\dot{L}}{L(t)} + \beta_E(t)\frac{\dot{E}}{E(t)} \tag{28}$$

where $\beta_J(t) = \frac{J(t)}{Y(t)}\frac{dY(t)}{dJ(t)}$ is the elasticity of output with respect to variable $J(t)$. (28) decomposes growth rate in output (LHS) in three components. The first term in the RHS is the share of growth in capital stock, the second term is the share of growth in labor, and the third term component is the share of growth in technology.

Suppose that we have Cobb-Douglas production function $Y = K^\alpha(EL)^{1-\alpha}$. Then (28) reduces to

$$\frac{\dot{Y}}{Y(t)} = \alpha\frac{\dot{K}}{K(t)} + (1-\alpha)\frac{\dot{L}}{L(t)} + (1-\alpha)\frac{\dot{E}}{E(t)}. \tag{29}$$

3. The Solow Model and Cross-Country Income Differences

The average output per-worker in the developed countries is about 10 times higher than the average output per-worker in the developing countries. Also the average output per-worker in the developed countries today is about 10 times higher than it was 100 years ago. In addition, growth experience of various countries has been quite varied over time. Cross-country variations in output per worker has been quite persistent with no sign of convergence.

Question then is what explains such cross-country differences in growth. Solow model identifies two possible sources of variations in the output per worker across countries: differences in the capital per worker and differences in the effectiveness of labor. One can get an idea about contribution of these two sources.

Suppose that production function in country i is $Y_i = K_i^{\alpha}(E_i L_i)^{1-\alpha}$. Let lower case letter denote log of per worker variable e.g. $y = \ln(Y/L)$. Let d denote developed countries and u denote underdeveloped countries. Then the log difference of output per worker between these two groups of countries can be written as

$$y_d - y_u = \alpha(k_d - k_u) + (1-\alpha)(e_d - e_u). \tag{30}$$

We can also use variance-decomposition method to get an estimate of the contribution of various factors in explaining the cross-country productivity differences. Using the production function, real income per worker at time t, y_{it}, can be written as

$$y_{it} = \alpha k_{it} + (1-\alpha)e_i. \tag{31}$$

Then, the variance of real income per-worker can be written as

$$var(y_{it}) = \alpha^2 Var(k_{it}) + (1-\alpha)^2 Var(e_{it}) + 2cov(\alpha k_{it}, (1-\alpha)e_{it}). \tag{32}$$

Using (32) the fraction of the variance in y_i attributable to the productivity differentials can be estimated as the sum of $var(e_i)$ along with two times the covariance terms involving e_i divided by $var(y_i)$. Similarly, we can calculate the fraction of variance in y_i attributable to the physical capital.

Using either (31) or (32), the economists have found that the observed differences in the capital per worker explains just 15-20 % of the differences in the per worker output across these two groups of countries. Most of the differences in the output per worker is accounted for the differences in the effective labor. It begs the question why effective labor differs across countries. Many researchers have sought to explain it by differences in the human capital – education and health across countries. This has led to the development of Augmented Solow model, in which human capital directly figures as an input in the production function.

Augmented Solow Growth Model

Assume that the production function is given by

$$Y = K^\alpha H^\beta (EL)^{1-\alpha-\beta} \tag{33}$$

where H is the level of education of the labor-force.

(33) includes education as a factor of production. One can also use health as a factor of production. In this case, (33) becomes

$$Y = K^\alpha H^\beta I^\mu (EL)^{1-\alpha-\beta-\mu} \tag{34}$$

where I is the indicator of health.

Empirical evidence suggests that the differences in education and health are important determinants of cross-country income differences. However, variations in different types of capital (physical and human) account for only about half of the cross-country differences in output per-worker. Rest are accounted for by the differences in total factor productivity.

Given that TFP and its growth play such a fundamental role in the growth process, the process of technological progress, its diffusion and determinants have received considerable attention from researchers in the last thirty years. This has led to the emergence of the **endogenous growth models**. In this course, we will not cover these models. Interested students should read chapter 3 of Romer.

Convergence Debate:

The issue that whether poor countries tend to grow faster than the rich countries has received a great deal of attention from researchers. There are number of reasons for such convergence. Firstly, Solow model predicts that an economy converges to its steady state. In addition, the speed of convergence is higher further an economy is away from its steady state (see equation 18). Thus if we assume that poorer countries are further away from their steady state than richer countries, growth rate in poor countries should be higher. Secondly, the marginal product of capital and thus the return on capital is higher in poorer countries. Thus, capital would flow from rich countries to poorer countries. Thirdly, if per-capita income differences are due to technological differences, such differences should decline over time.

Many researchers have tested this prediction of the Solow model by examining the convergence pattern in per-capita income across countries. Researchers differentiate between two notions of convergence – unconditional convergence and conditional convergence. Unconditional convergence assumes that all countries have identical steady state. On the other hand, conditional convergence assume that different economies may have different steady states (s and n may differ).

The notion of unconditional convergence can be tested by estimating the following regression model:

$$y_{iT} - y_{i0} = a + by_{i0} + \xi_i \tag{35}$$

where y_{iT} and y_{i0} are the per-worker income in the terminal period and the initial period respectively and ξ_i is the error term. If $b < 0$, then it indicates unconditional convergence.

For conditional convergence, we estimate the following regression model:

$$y_{iT} - y_{i0} = a + by_{i0} + \textit{ Other Explanatory variables} + \xi_i \tag{36}$$

Other explanatory variables determine country-specific steady state. They may include variables such as savings rate, labor force growth rate etc. Again if $b < 0$, then it indicates conditional convergence. Empirical evidence suggests that there is no evidence of unconditional convergence. However, there is a strong evidence of conditional convergence.

The Solow model takes the structure of an economy as given and then examines the growth process within that structure. This begs the question, what determines the structure of an economy? Is the structure of an economy immutable or that can be changed? How do geographical, political, and social factors affect the growth process? Questions like these has led to emergence of a new literature which focuses on what is known as the **deep determinants of growth**.

Deep Determinants of Growth and TFP

Geography

There is a large literature which suggests that the geographical factors are important determinants of per-capita income, TFP, and health capital. Geographical factors affect TFP both directly and indirectly through their effects on quality of institutions and human capital. Among the geographical factors, the effects of three variables latitude, mean temperature, and access to coastal areas on TFP and per-capita income have received a great deal of attention. It has been argued that the extreme heat and humidity in tropical countries contribute to low soil fertility and agricultural productivity. On the other hand, temperate zones have higher soil fertility and agriculture productivity.

Tropical countries also have higher disease burden leading to higher mortality rate, morbidity rate, and lower health capital. Higher disease burden not only directly reduces TFP, but also reduces TFP indirectly. It has been argued that higher mortality rate of European settlers in tropical countries induced them to develop exploitative institutions in these countries. Latitude also affects diffusion of technology. It is also argued that technological diffusion works most effectively within ecological zones and therefore in an east-west direction along a common latitude rather than in a north-south direction.

Easy access to coasts enhances the extent of market (both internal and external) and thereby increases the opportunity of specialization. In addition, it is argued that coastal areas are conducive for urban growth and thus countries with access to ocean are more likely to reap the benefit of agglomeration economies. Transport cost has historically played an important role in the diffusion of technology, ideas, and new products. Coastal areas with lower transportation costs compared to the land-locked countries are likely to be more exposed to newer products, ideas and technical advancements. In addition, a coastal economy faces a higher elasticity of output response with respect to trade taxes, whereas an inland economy does not. This may induce governments in inland economies to impose harsh taxes on trade.

Finally, some countries may be endowed with high value natural resources such as hydro-carbons. These countries are likely to have higher per-capita income and TFP.

Legal Origin

There is a large literature which suggests that legal origin of a country have significant effect on per-capita income and productivity. Legal system of a country determines the security and enforcement of private property rights, rights of the states, and also quality of governance. All these factors are crucial for technological innovation and adoption. Other things remaining the same, a society in which private property rights are more secure and the government is less intrusive is likely to be more friendly to private innovations. It has been argued that common law countries with an English legal origin are more supportive of private outcomes, whereas civil law (French origin) seeks to replace such outcomes with state desired allocations. The socialist system is of course an extreme form of state intervention which completely supplants the market.

Ethnic Heterogeneity

Recently the effects of ethnic diversity on investment, growth, quality of government, civil wars, political instability etc. have received a great deal of attention. Ethnic diversity can affect income and TFP in many ways. Firstly, some authors have argued that ethnically diverse societies have tendency of ethnic conflicts, civil wars, and political instability. Such conflicts and instabilities have a negative impact on investment. Also in heterogeneous societies the adoption and the diffusion of technological innovations are more difficult, particularly, when there is ethnic conflict among groups in a country. Ethnic conflicts and political instability may generate a high level of corruption, private property may not be secure, and in general lead to lower quality of governance.

Religion and Culture

Religion has been used as a proxy for work ethic, tolerance, trust, openness to new ideas. One strand of argument emphasizes the historical importance of the protestant ethic in the spread of capitalism, arguing that Protestantism places an emphasis on honesty and taking personal responsibility of one's own fate. It has been suggested that protestants have better work ethics and more open to new learning and ideas. It is also argued that Catholic and Muslim religions have been historically hostile to new ideas, learning and institutional development. These societies enormously increased power of religious organizations and states to maintain their political and religious influence.

Among various deep determinants, the empirical evidence suggests that geography, legal origin, and ethnic heterogeneity are important determinants of per-capita income and TFP. There is no empirical evidence that religion matters for growth or TFP.