Lecture 4
Conduct of Monetary Policy: Goals, Instruments, and Targets; Asset Pricing; Time Inconsistency and Inflation Bias

1. Introduction

In this chapter, we analyze the conduct of monetary policy (or the operating procedure) *i.e.* how is it operationalized, what are its objectives, constraints faced by the central banks etc. The central banks are normally mandated to achieve certain goals such as price stability, high growth, low unemployment. But the central banks do not directly control these variables. Rather they have set of instruments such as open-market operations, setting bank rate etc. which they can use to achieve these objectives.

The problem of a central bank is compounded by the fact that its instruments do not directly affect these goals. These instruments affect variables such as money supply and interest rates, which then affect goal variables with lag. In addition, these lags may be uncertain. Due to above mentioned problems, distinction is made among (i) goals (or objectives), (ii) targets (or intermediate targets), (iii) indicators (or operational targets), and (iv) instruments (or tools) in the conduct of monetary policy.

Target and indicator variables lie between goal and instrument variables. Target variables such as money supply and interest rates have a direct and predictable impact on the goal variables and can be quickly and more easily observed. In the previous chapter, we studied various theories linking target variables (e.g. money supply, interest rate) to goal variables (e.g. output, employment). By observing these variables, a central bank can determine whether its policies are having desired effect or not. However, even these target variables are not directly affected by the central bank instruments. These instruments affect target variables through another set of variables called indicators. These indicators such as monetary base and short run interest rates are more responsive to instruments. The conduct of monetary policy can be represented schematically as follows:

Instruments → Indicators → Targets → Goals

Following is the list of different kinds of variables.

Table 1

Goals or Objectives

1. High Employment
2. Economic Growth
3. Price Stability
4. Interest-Rate Stability
5. Stability of Financial Markets
6. Stability in Foreign Exchange Markets

Targets or Intermediate Targets

1. Monetary Aggregates (M1, M2, M3 etc.)
2. Short Run and Long Run Interest Rates

Indicators or Operational Targets

1. Monetary Base or High-Powered Money
2. Short Run Interest Rate (Rate on Treasury Bill, Overnight Rate)

Instruments or Tools

1. Open Market Operations
2. Reserve Requirements
3. Operating Band for the Overnight Rate
4. Bank Rate

Though we have listed six goals, it does not mean that different countries and regimes give same weight to all these goals. Different goals may get different emphasis in different countries and times. Currently in Canada, a lot of emphasis is put on the goals of price and financial market stability. Also, all the goals may not be compatible with each other. For instance, the goal of price stability may conflict with the goals of high employment and stability of interest rate at least in the short run.

The list of target variables raises the question: how do we choose target variables? Three criteria are suggested: (i) measurability, (ii) controllability, and (iii) predictable effects on goals.

Quick and accurate measurement of target variables are necessary because the target variables will be useful only if it signals rapidly when policy is off track. For a target variable to be useful, a central bank must have a significant influence over it. If the central bank cannot influence a target variable, knowing that it is off-track is of little help. Finally and most importantly, target variables must have a predictable impact on goal variables. If target variables do not have predictable impact on goal variables, the central bank cannot achieve its goal by using target variables. Monetary aggregates and short and long run interest rates satisfy all three criteria.

The same three criteria are used to choose indicators. They must be measurable, the central bank should have effective control over them, and they must have predictable effect on target variables. All the indicators listed above satisfy these criteria.

2. Money Supply Process, Asset Pricing, and Interest Rates

In previous chapter, we extensively analyzed relationships among goal variables such employment, inflation, output and target variables such as money supply and interest rates. Now we turn to analyze relationship among instruments, indicators, and targets. In order to understand relationships among these three types of variables, it is instructive to analyze the money supply process and asset pricing which throw light on relationships among different types of interest rates.

A. Money Supply Process

So far, we have been vague about what determines money supply. We just assumed that it is partly determined by the central bank and partly by non-policy shocks. In this section, we take a closer look at the money supply process. It has important bearing on the conduct of monetary policy.

There are four important actors, whose actions determine the money supply – (i) the central bank, (ii) commercial banks, (iii) depositors, and (iv) borrowers. Of the four players, the central bank is the most important. Its actions largely determine the money supply. Let us first look at its balance sheet.

Table 2

Balance Sheet of a Central Bank

Assets	Liabilities
Government Securities	Notes in Circulation
Advances to Banks	Deposits
Foreign Securities & Currencies	

The two liabilities on the balance sheet, notes in circulation and deposits of other financial institutions, are often called **monetary liabilities**. The financial institutions hold deposits with the central bank either because they are required to do so or to settle claims with other financial institutions.

These deposits together with currency physically held by commercial banks make up **bank reserves**. Reserves are assets for the commercial banks but liabilities for the central bank. We will see later that an increase in reserves lead to increase in money supply. Commercial banks hold reserves in order to meet their short-run liquidity requirements. This is called **desired reserve**. Sometimes commercial banks are also required to hold certain fraction of their deposits in terms of currency. These reserves are called **required reserves**.

The three assets of the central bank are important for two reasons. First, changes in the asset items lead to changes in the money supply. Second, these assets earn interests (other than the foreign currency), while the liabilities do not. Thus, they are source of revenue for the central bank.

The currency in circulation (C) together with reserves (R) constitute **monetary base or high-powered money** (MB).

$$MB = C + R. \tag{2.1}$$

The central bank controls the monetary base through its purchase or sale of government securities in the open market (**open market operations**), and through its extension of loans to commercial banks. It can also print new currencies. It is through its control over monetary base, the central bank affects money supply. To understand this, let us first look at how monetary base is related to the money supply. For illustrative purpose, we will just concentrate on the relationship between monetary base and $M1$ (currency plus chequable deposits).

Money supply $M(\equiv M1)$ is related to monetary base through **money multiplier** (m).

$$M = mMB. \tag{2.2}$$

As we can see that money multiplier is simply the ratio of money supply to monetary base. How do we derive the money multiplier? Let D be the deposit and define currency-deposit ratio, c, and reserve-deposit ratio, r as follows

$$c \equiv \frac{C}{D} \ \& \ r \equiv \frac{R}{D}, \ 0 < c, r < 1. \tag{2.3}$$

Using (2.3) and (2.1), we can express MB as

$$MB = (c + r)D. \tag{2.4}$$

Now by definition

$$M = C + D = (1 + c)D. \tag{2.5}$$

Putting (2.4) in (2.5), we have

$$M = \frac{(1 + c)}{r + c} MB. \tag{2.6}$$

The term $\frac{1+c}{r+c} \equiv m$ is the money multiplier and it is strictly greater than unity. Thus, one unit change in the monetary base leads to more than one unit change in the money supply. Also, a higher currency-deposit ratio, c, and reserve-deposit ratio, r, lead to lower money supply for a given level of monetary base.

From (2.6) it is clear that money supply depends not only on the monetary base over which the central bank has lot of control but also on the behavior of commercial banks, depositors, and borrowers which determine currency-deposit ratio, c, and reserve-deposit ratio, r. c and r depend on the rate of return on other assets and their variability, innovations in the financial system and cash management, expected deposit outflows etc. In general, broader the measure of money supply, less control the central bank has on its supply.

B. Asset Pricing and Interest Rates

In the previous chapter, we divided financial assets in two categories – monetary and non-monetary assets. We called the rate of return on non-monetary assets as the nominal rate of interest. But we know that there are different types of non-monetary assets with different rates of return. Then the question is : how justifiable it is to lump together different non-monetary assets?

We can lump together different types of non-monetary assets provided there is a stable relationship among their rates of return. The rate of return on an asset depends on its pay-off and price. In order to understand, the relationships among rates of return, we need to know how assets are priced.

In lecture 2, we developed DSGE model. We can use this model to price various types of assets and establish relationship among their rates of return. Recall that the optimal choices of agents in the economy are characterized by Euler equations (eq. 4.17 pp. 15). We will use these equations to price various types of assets.

Actually, (4.17) in lecture 2 implicitly gives price of the investment good, which is an asset. Since, we dealt with one-good economy the price of investment good was simply normalized to one. This price satisfied

$$1 = \beta E\left[\frac{u'(c_2)Af'(k)}{u'(c_1)}\right]. \tag{2.7}$$

Basically we have rewritten (4.17). In general, asset price, q, satisfies

$$q = \beta E \left[\frac{u'(c_2)(\text{Return on the Asset})}{u'(c_1)} \right] \tag{2.8}$$

where q is the price of an asset. q is the price which equates the marginal cost ($qu'(c_1)$) of holding the asset to its expected marginal benefit ($\beta E u'(c_2)$(Return on the Asset)).

The return on the asset is simply the sum of payoff of the asset and its resale value.

$$\text{Return of the Asset} \ = \ \text{Payoff} \ + \ \text{Resale Price} \tag{2.9}$$

$$\text{Rate of Return of the Asset} \ = \ \frac{\text{Return of the Asset}}{\text{Price of the Asset (q)}} \tag{2.10}$$

Going back to our previous example the return on one unit of investment is $Af'(k)$ (Payoff = MPK, Resale Price = 0, since $\delta = 1$). Normalizing the price of investment good to one we get (2.7). Let us use (2.8) to price other kinds of assets.

An Example

Riskless Bond

Consider a two-period economy. Suppose that economy can be in two states: high or low. We want to price a one period risk-less bond: a bond which pays 1 unit of good in the second period regardless of what state occurs.

Let q_B be the current period price of a bond which pays 1 unit in the next period both in high and low states (discount coupon). We want to know q_B. In order to do so, first we have to specify the return on the riskless bond, which is simply 1 unit of good (payoff = 1 unit of good, resale value = 0). After specifying the return, we can use Euler equation to get its price which is simply

$$q_B = \beta E \left[\frac{u'(c_2) * 1}{u'(c_1)} \right] = \beta E \left[\frac{u'(c_2)}{u'(c_1)} \right]. \tag{2.11}$$

We can also derive the net rate of return of bond, r_B, which is given by

$$r_B \equiv \frac{1}{q_B} - 1 = \frac{u'(c_1)}{\beta E(u'(c_2))} - 1. \tag{2.12}$$

Let us verify that indeed this is the case. Consider example 11 in lecture 2 with one modification. Now the agents in this economy can save both in terms of investment good as well as one period riskless bond. Let us suppose that the representative agent in this economy buys k units of investment good and B units of riskless bond. Then the maximization problem for the representative agent is

$$\max_{c_1, c_2^h, c_2^l, k, B} U = u(c_1) + \beta[p^h u(c_2^h) + p^l u(c_2^l)] \equiv u(c_1) + \beta E(u(c_2)) \tag{2.13}$$

subject to

$$c_1 + k + q_B B = y \tag{2.14}$$

$$c_2^h = A^h f(k) + B \tag{2.15}$$

$$c_2^l = A^l f(k) + B. \tag{2.16}$$

Plugging the constraints in the objective function we get,

$$\max_{k,B} U = u(y - k - q_B B) + \beta[p^h u(A^h f(k) + B) + p^l u(A^l f(k) + B)]. \tag{2.17}$$

The first order conditions are

$$k: \ u'(c_1) = \beta E\left[u'(c_2) A f'(k)\right] \tag{2.18}$$

$$B: \ q_B u'(c_1) = \beta E u'(c_2). \tag{2.19}$$

As is evident, (2.18) and (2.19) correspond to (2.7) and (2.11) respectively.

Long and Short Bonds

We can use the same approach to price multi-period bonds. Suppose that there is also a two-period riskless bond. This bond pays one unit of good after two-period, regardless of what state occurs.

Let q^L be the price of a bond today, which pays 1 unit in period 3 (long or two-period bond), and q_1^S be the price of one period bond, which pays 1 unit next period. Then

$$q^L = \beta^2 E_1 \left[\frac{u'(c_3)}{u'(c_1)} \right] \tag{2.20}$$

where E_1 is expectation operator conditional on information available at period 1. q^S will be given by (2.19).

Taking a specific example, let $u(c) = \ln c$. Then,

$$q^L = \beta^2 E_1 \left[\frac{c_1}{c_3} \right] \tag{2.21}$$

$$q_1^S = \beta E_1 \left[\frac{c_1}{c_2} \right] \tag{2.22}$$

From (2.21) and (2.22), we can derive rates of interest on long and short bonds. The gross return on long bond satisfies

$$(1 + r^L)^2 = \frac{1}{q^L} = \frac{1}{\beta^2 E_1 \left[\frac{c_1}{c_3} \right]}. \tag{2.23}$$

Similarly, the gross return on short bond satisfies

$$1 + r_1^S = \frac{1}{q_1^S} = \frac{1}{\beta E_1 \left[\frac{c_1}{c_2} \right]}. \tag{2.24}$$

The pattern of returns on long and short bonds are known as **term structure**. The plot of term structure over maturity is known as **yield curve**. The term structure or yield curve embodies the forecasts of future consumption growth. In general, yield curve slopes up reflecting growth. Downward sloping yield curve often forecasts a recession.

What is the relationship between the prices of short and long bonds? We turn to covariance decomposition ($E(xy) = E(x)E(y) + cov(x, y)$).

$$q^L = \beta^2 E_1 \left[\frac{c_1 c_2}{c_2 c_3}\right] \tag{2.25}$$

which implies

$$q^L = q_1^S E_1 q_2^S + Cov\left(\frac{\beta c_1}{c_2}, \frac{\beta c_2}{c_3}\right) \tag{2.26}$$

where q_2^S is the second period price of one period bond. If we ignore the covariance term, then in terms of returns (2.26) can be written as

$$\left(\frac{1}{1+r^L}\right)^2 = \frac{1}{1+r_1^S} E_1 \frac{1}{1+r_2^S}. \tag{2.27}$$

Taking logarithms, utilizing the fact that $ln(1+r) \approx r$, and ignoring Jensen's inequality we get

$$r^L \approx \frac{r_1^S + E_1 r_2^S}{2}. \tag{2.28}$$

(2.28) suggests that the long run bond yield is approximately equal to the arithmetic mean of the current and expected short bond yields. This is called the **expectation hypothesis** of the term structure. (2.26 - 2.28) imply that prices of different types of bonds and thus their return are related to each other. Thus, if one type of rate of interest changes, its effect spreads to other interest rates as well.

Nominal Bond

Suppose q_{NB} is the price of a nominal bond in current dollar which pays 1 dollar next period in both the states. Then how much is the q_{NB}? Let P_1 and P_2 be the price levels in period 1 and 2 respectively, then q_{NB} satisfies

$$\frac{q_{NB}}{P_1} = \beta E \frac{u'(c_2)}{u'(c_1)} \frac{\$1}{P_2} \tag{2.29}$$

Here the nominal price of bond and its nominal return have been converted in real terms using price levels. If we define inflation rate as $\frac{P_2}{P_1} \equiv 1 + \pi$, then (2.29) can be written as

$$q_{NB} = \beta E \frac{u'(c_2)}{u'(c_1)} \frac{1}{1+\pi} \tag{2.30}$$

Using covariance decomposition we have

$$q_{NB} = q_B E \frac{1}{1+\pi} + \beta Cov\left(\frac{u'(c_2)}{u'(c_1)}, \frac{1}{1+\pi}\right) \tag{2.31}$$

(2.31) can be written as

$$\frac{1}{1+r_{NB}} = \frac{1}{1+r_B} E \frac{1}{1+\pi} + \beta Cov\left(\frac{u'(c_2)}{u'(c_1)}, \frac{1}{1+\pi}\right) \tag{2.32}$$

(2.32) is called **the Fisher Relation** which relates return on nominal bond to return on real bond and expected inflation.

Forward Prices

Suppose in period 1, you sign a contract, which requires you to pay f in period 2 in exchange for a payoff of 1 in period 3. How do we value this contract? Notice that the price of the contract, which is to be paid in period 2, is agreed in period 1. Then the expected marginal cost of the contract in period 1 is $\beta E_1 u'(c_2) f$. The expected benefit of the contract is $\beta^2 E_1 u'(c_3)$. Since the price equates the expected marginal cost with expected marginal benefit of the asset, we have

$$f = \frac{\beta E_1 u'(c_3)}{E_1 u'(c_2)} = \frac{q^L}{q_1^S} \tag{2.33}$$

Share

Suppose that a share pays dividend d in period 2 where d is a random variable. Assume that $E(d) = 1$. The resale value of the share is zero. Then the price of the share in period is given by

$$q_{Sh} = \beta E \frac{u'(c_2)}{u'(c_1)} d. \quad (2.34)$$

Using the co-variance decomposition, (2.34) can be written as

$$q_{Sh} = q_1^s + \beta Cov(\frac{u'(c_2)}{u'(c_1)}, d) \quad (2.35)$$

where $\beta Cov(\frac{u'(c_2)}{u'(c_1)}, d)$ gives you a measure of risk-premium.

3. Choice of Instruments and Targets

A. Instruments

Having discussed the money supply process and interrelationship among different interest rates, one can analyze how different tools or instruments affect the balance sheet of the central bank and thus the money supply and the interest rates.

Open market operations refer to buying and selling of government bonds in the open market by the central bank. When the central bank buys government bonds, it increases the amount of currency. Also for a given demand for money, it leads to lower interest rate. Opposite is the case, when central bank buys government bonds.

By changing **reserve requirements** as well the central bank can change money supply and interest rates. A higher reserve requirement leads to a higher reserve-deposit ratio which in turn leads to lower money supply and higher interest rate. Opposite is the case when the central bank reduces the reserve requirement.

The **overnight interest rate** refers to the rate at which financial institutions borrow and lend overnight funds. This rate is the shortest-term rate available and forms the base of term structure of interest rates relation. Many central banks including the Bank of Canada implement their monetary policy by announcing the **target overnight rate**. The idea is to keep the actual overnight rate within a narrow band (usually about 50 basis point or 0.5% wide).

This band is also known as the **channel or corridor or operating band**. The upper limit of this band is known as the **bank rate**. This is the rate at which the central bank is willing to lend to financial institutions for overnight. The lower limit of the band is the rate, which the central bank pays to the overnight depositors. One can immediately see that these operating bands put limit on the actual overnight rate. No financial institution will borrow overnight fund for more than the bank rate because they can borrow as much as they require from the central bank at the bank rate. Similarly no lender will lend overnight fund at the rate below the lower limit of the operating band, because they can always deposit their overnight fund at the central bank at that rate.

B. Choice of Instruments or Targets

Table 1 shows that the central bank has two sets of instruments (as well as indicators and targets) – monetary aggregates and interest rates. However, these two sets of instruments are not independent of each other. If the central bank chooses monetary aggregate, then it will have to leave interest rate to be determined by the market forces (through the money market). If it chooses interest rate, then monetary aggregate is determined by the market forces. Same is true for the two sets of indicators and targets.

Now the question is: which set of instruments the central bank should choose? Answer is: if the central bank's target variable is the money supply then use the monetary aggregate tools and if the target variable is the interest rate, then choose the interest rate as instrument.

But again it raises the question, which set of target variables to choose? The choice of target variables and thus instruments depends on the stochastic structure of the economy *i.e* the nature and the relative importance of different types of disturbances. The general conclusion is that if the main source of disturbance in the economy is shocks to the IS curve or the goods market, then targeting money supply (or using money supply tool) is optimal. On the other hand, if the main source of disturbance is shocks to the demand for money or the financial markets or the LM curve, then targeting interest rate is optimal.

To understand the intuition behind this conclusion, let us consider an economy where the objective of the central bank is to stabilize output. Suppose that the central bank must set policy before observing the current disturbances to the goods and the money markets, and assume that information on the interest rate, but not on output is immediately available. Suppose that the IS curve is given by the following equation

$$y_t = -\alpha i_t + u_t \tag{3.1}$$

and the LM curve by

$$m_t = y_t - \beta i_t + v_t \tag{3.2}$$

where $y_t = \ln Y_t$ and $m_t = \ln M_t$. Here price level is assumed to be constant and thus the analysis pertains to short-term (or choices of instruments and indicators). Both u_t and v_t are mean zero $i.i.d$ exogenous shocks with variance σ_u^2 and σ_v^2 respectively. The objective of the central bank is to minimize the variance of output deviations from potential output set to zero:

$$\min E(y_t)^2. \tag{3.3}$$

The timing is as follows: the central bank sets either interest rate, i_t, or money supply, m_t, at the start of the period; then stochastic shocks are realized, which determine the value of output, y_t. The question is which policy rule minimizes (3.3). In other words, whether the central bank should try to hold the market rate of interest constant or should hold the money supply constant while allowing the interest rate to move.

Let us first consider the money target rule. Here, the central bank optimally chooses m_t letting y_t and i_t to be determined by the IS and the LM curves. Substituting (3.2) in (3.1), we get

$$y_t = u_t + \alpha \left[\frac{m_t - y_t - v_t}{\beta} \right] \tag{3.4}$$

which implies

$$y_t = \frac{\alpha m_t + \beta u_t - \alpha v_t}{\alpha + \beta}. \tag{3.5}$$

Putting (3.5) in (3.1), the optimization problem reduces to

$$\min_{m_t} E\left(\frac{\alpha m_t + \beta u_t - \alpha v_t}{\alpha + \beta}\right)^2. \tag{3.6}$$

The first order condition is

$$2E\left(\frac{\alpha m_t + \beta u_t - \alpha v_t}{\alpha + \beta}\right)\frac{\alpha}{\alpha + \beta} = 0. \tag{3.7}$$

From (3.7), we get optimal money supply rule as

$$m_t = 0. \tag{3.8}$$

With this policy rule, the value of objective function is

$$E_m(y_t)^2 = E_m\left(\frac{\beta u_t - \alpha v_t}{\alpha + \beta}\right)^2 = \frac{\beta^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + \beta)^2}. \tag{3.9}$$

Let us now consider the interest rate rule. Under this rule the central bank optimally chooses i_t and allows the money supply to adjust. In order to derive the optimal interest rate, i_t, put (3.2), in (3.1). The optimization problem is now

$$\min_{i_t} E(-\alpha i_t + u_t)^2. \tag{3.10}$$

From the first order condition, we get

$$i_t = 0. \tag{3.11}$$

Putting (3.11) in the objective function, we have

$$E_i(y_t)^2 = \sigma_u^2. \tag{3.12}$$

In order to find out optimal policy rule, we just have to compare (3.9) and (3.12). We can immediately see that interest rate rule is preferred iff

$$E_i(y_t)^2 < E_m(y_t)^2 \tag{3.13}$$

which is equivalent to

$$\sigma_v^2 > \left(1 + \frac{2\beta}{\alpha}\right)\sigma_u^2. \tag{3.14}$$

From (3.14) it is clear that if the only source of disturbance in the economy is the money market, $\sigma_v > 0$ & $\sigma_u = 0$, then the interest rule is preferred. In the case, the only source of disturbance is the goods market, $\sigma_u > 0$ & $\sigma_v = 0$, then the money supply rule is preferred.

If only the good market shocks are present, a money rule leads to a smaller variance in output. Under the money rule, a positive IS shock leads to a higher interest rate. This acts to reduce the aggregate spending, thereby partially offsetting the effects of the original shock. Since, the adjustment of i automatically stabilizes output, preventing this interest rate adjustment by fixing i leads to larger output fluctuations. If only the money-demand shocks are present, output can be stabilized perfectly by the interest rate rule. Under the interest rate rule, monetary authorities adjust the money supply in response to monetary shocks to maintain the interest rate, which completely offsets the output fluctuations caused by the monetary shocks.

In the case, there is disturbances in both the markets, then the optimal policy rule depends on the size of variances as well as the relative steepness of the IS and the LM curves. The interest rate rule is more likely to be preferred when the variance of the money market disturbances is larger and both the LM ans the IS curves are steeper (lower β and bigger α). Conversely, the money supply rule is preferred if the variance of the goods market shocks is large and both the LM and IS curves are flatter.

Currently, the Bank of Canada uses interest rate tool. It conducts its monetary policy by announcing the bank rate or the operating band of overnight rate periodically. During 70's and 80's, the Bank of Canada used to target money supply. However, during 80's the demand for money function became highly unstable due to various financial innovations and the Bank of Canada abandoned the monetary targeting and moved to the interest rate targeting.

C. Taylor Rule

Many central banks including the Bank of Canada and the Federal Reserve conduct their monetary policy through announcing the bank rate or setting the operating band for the overnight rate. It raises the question, how do the central banks set the bank rate?

John Taylor showed that the behavior of the federal funds interest rate in the U.S. from the mid-1980's to 1992 could be fairly matched by a simple rule of the form

$$i_t = \pi_t + 0.5(y_t - \overline{y}_t) + 0.5(\pi_t - \pi^T) + r^* \tag{3.15}$$

where π^T was the target level of average inflation (assumed to be 2% per annum) and r^* was the equilibrium level of real rate of interest (again assumed to be 2% per annum). In the equation, the nominal interest rate deviates from the level consistent with the economy's equilibrium real rate and the target inflation rate if the output gap is nonzero or if inflation deviates from target. A positive output gap leads to rise in the nominal interest rate as does the actual inflation higher than the target level.

The Taylor rule for general coefficients is often written as

$$i_t = r^* + \pi_t + \alpha(y_t - \overline{y}_t) + \beta(\pi_t - \pi^T). \tag{3.16}$$

A large literature has developed that has estimated Taylor rule for different countries and time-periods. The rule does quite well to match the actual behavior of the overnight rates, when supplemented by the addition of the lagged nominal interest rates.

D. Uncertainty About the Impact of Policy or Model Uncertainty

So far we have assumed that the central bank knows the true model of the economy with certainty or knows the true impact of its policy. Fluctuations in output and inflation arose from disturbances that took the form of additive errors. But suppose that the central bank does not know the true model with certainty or measures parameter values with error. In other words, the error terms enter multiplicatively. In this case, it may be optimal for the central bank to respond to shocks more slowly or cautiously.

To concretize this idea, suppose that the central bank's objective function is

$$L = \frac{1}{2}E_t(\pi_t^2 + \lambda y_t^2). \tag{3.17}$$

Here for simplicity, I have assumed that social welfare maximizing output, y_t^*, and inflation, π_t^*, are zero. Now suppose that aggregate demand evolves as follows

$$y_t = \beta_t \pi_t + e_t \tag{3.18}$$

where e_t is mean zero *i.i.d.* shock. Also assume that the central bank does not know the true β_t, but has to rely on the estimated β_t. The true β is related to the estimated β as follows

$$\beta_t = \overline{\beta} + v_t \tag{3.19}$$

where v_t is mean zero *i.i.d.* shock with variance σ_v^2 and $\overline{\beta}$ is the true parameter. Now suppose that the central bank observes demand shock e_t but not v_t before choosing π_t. Now the question is: what is the optimal π_t?

To derive the optimal π_t put (3.18) in (3.17), then we have

$$\min_{\pi_t} = \frac{1}{2} E_t \left[\pi_t^2 + \lambda(\beta_t \pi_t + e_t)^2 \right]. \tag{3.20}$$

The first order condition is

$$E_t(\pi_t + \lambda(\beta_t \pi_t + e_t)\beta_t) = 0. \tag{3.21}$$

Simplifying, we have

$$\pi_t = -\frac{\lambda \overline{\beta}}{1 + \lambda \overline{\beta}^2 + \lambda \sigma_v^2} e_t. \tag{3.22}$$

As one can see that the coefficient of demand shock e_t is declining in σ_v^2. This basically says that in the presence of multiplicative disturbances, it is optimal for the central bank to respond less (or more cautiously) to e_t.

4. Time Inconsistency and Inflation Bias

In the last hundred years, in almost all the countries prices have increased over time. Empirical literature suggests that inflation is mainly accounted for by the increase in money supply in the medium and the long run. It raises the question, why the governments follow inflationary policy or expansionary monetary policy? One reason can be that the increase in money supply is a source of revenue for the government (seniorage). However, this explanation does not seem to very appropriate for the developed countries, where government revenue from money creation is not very important.

The other explanation is that output-inflation trade-off faced by the central banks induces them to pursue expansionary policy. When output is low, they may be tempted to increase inflation. On the other hand, when inflation is high, they may be reluctant to reduce it for the fear of reducing output. However, this explanation as stated also falls short because there is no long run trade-off between output and inflation. If there is no long-run tarde-off, why do we observe long run inflation?

Kydland and Presscott (1977) in a famous paper showed that when the central banks have discretion to set inflation and if they only face short-run output-inflation trade-off, then it gives rise to excessively expansionary policy. Intuitively, when expected inflation is low, the marginal cost of additional inflation is low. This induces central banks to increase inflation (for a given expected inflation), in order to increase output. However, the public while forming their expectation take into account the incentives of the central bank and thus do not expect low inflation. In other words, the promise of the central bank to follow low inflation is not credible. Consequently, the central bank's discretion results in inflation without any increase in output.

A. Time Inconsistency

The lack of credibility of the central bank's low inflation policy gives rise to the problem of dynamic inconsistency of low inflation monetary policy. Idea is that the central bank would like public to believe that it will follow low inflation policy *i.e.* it will announce low inflation target. However, once the public has formed their expectation based on the central bank's announcement, the central bank has incentive to increase inflation as by doing so it can increase output. Since, the central bank does not comply with its announcement, its announcement is not time-consistent. In other words, at the time of choosing the actual inflation, the central bank deviates from its inflation target. Let us now formalize these ideas.

Let the objective function of the central bank be

$$L = \frac{1}{2}\lambda(y_t - \overline{y} - k)^2 + \frac{1}{2}(\pi_t - \pi^*)^2 \tag{4.1}$$

where $\overline{y}$ is the potential output, k is some constant, and π^* is socially optimal inflation rate. Here $\overline{y} + k$ stands for socially optimal output level. The deviation in the socially optimum level of output and potential output can be due to distortionary taxes or imperfections in markets.

Let the trade-off between inflation and output be given by

$$y_t = \overline{y} + a(\pi_t - \pi_t^e). \tag{4.2}$$

The central chooses actual inflation, π_t, in order to minimize (4.1) subject to (4.2).

Now suppose the timing of events are as follows. The central bank first announces its target inflation rate. After the announcement of the central bank, public form their expectation about inflation rationally. Once public have formed their expectations, the central bank chooses actual inflation. The key here is that the central bank chooses actual inflation after public have formed their expectation.

Given the environment, we need to answer two questions: (i) what is the actual inflation chosen by the central bank? (ii) what is the expected inflation? We will answer these two questions under two policies – (i) full commitment and (ii) discretion. By full commitment, we mean that the central bank adheres to its announcement. By discretion, we mean that the central bank can choose actual inflation different from the announced one.

Under the full commitment, the socially optimal policy is

$$\pi_t = \pi^* = \pi_t^e. \tag{4.3}$$

The value of objective function is

$$L_c = \frac{1}{2}\lambda k^2. \tag{4.4}$$

Under discretion, the optimal, π_t, can be derived as follows. Putting (4.2) in (4.1), we have

$$\min_{\pi_t} \frac{1}{2}\lambda(a(\pi_t - \pi_t^e) - k)^2 + \frac{1}{2}(\pi_t - \pi^*)^2. \tag{4.5}$$

The first order condition yields,

$$\lambda(a(\pi_t - \pi_t^e) - k)a = (\pi^* - \pi_t). \tag{4.6}$$

Under rational expectation and no uncertainty, $\pi_t^e = \pi_t$ and thus (4.6) becomes

$$-\lambda ak = \pi^* - \pi_t \tag{4.7}$$

which simplifies to

$$\pi_t = \pi^* + \lambda ak. \tag{4.8}$$

Time-consistent inflation, π_t, is higher than the socially optimum inflation rate, π^*, and the size of inflation bias is λak. The value of objective function under time-consistent policy is

$$L_d = \frac{1}{2}\lambda k^2 + \frac{1}{2}(\lambda ak)^2 \tag{4.9}$$

which is higher than the value of the objective function under full commitment. In other words, the economy does worse-off under discretion.

Many solutions have been proposed to address the problem of time-inconsistency, such as appointing the central banker who is inflation-hawk, changing the mandate of the central bank including inflation targeting.

B. Solution to Inflation Bias

Inflation targeting basically involves announcing an inflation target and increasing the weight of deviation of actual inflation from targeted inflation in the social welfare function. The idea of inflation targeting can be captured as follows.

Suppose that the target inflation rate is equal to the optimal inflation rate. Let the objective function of the central bank be

$$V = \frac{1}{2}\lambda(y_t - \overline{y} - k)^2 + \frac{1}{2}(\pi_t - \pi^*)^2 + \frac{1}{2}h(\pi_t - \pi^*)^2. \tag{4.10}$$

The last term in (4.10) is the additional penalty on the central bank. If $h = 0$, we go back to the original case. The problem of the central bank is to choose inflation rate π_t in order to minimize (4.10) subject to (4.2). Now under the full commitment, the socially optimal policy is still

$$\pi_t = \pi^*. \tag{4.11}$$

Under inflation-targeting regime, the optimal, π_t, can be derived as follows. Putting (4.2) in (4.10), we have

$$\min_{\pi_t} \frac{1}{2}\lambda(a(\pi_t - \pi^e) - k)^2 + \frac{1}{2}(\pi_t - \pi^*)^2 + \frac{1}{2}h(\pi_t - \pi^*)^2. \tag{4.12}$$

The first order condition yields,

$$\lambda(a(\pi_t - \pi_e) - k)a = (\pi^* - \pi_t) - h(\pi_t - \pi_t^*). \tag{4.13}$$

Under rational expectation, $\pi_t = \pi_t^e$ and thus (4.13) becomes

$$-\lambda ak = (1 + h)\pi^* - (1 + h)\pi_t \tag{4.14}$$

which simplifies to

$$\pi_t = \pi^* + \frac{\lambda ak}{1 + h}. \tag{4.15}$$

By comparing (4.15) with (4.8), we can immediately see that the size of inflation bias is smaller under inflation targeting.