Lecture 1

IS-LM/ AD-AS Analysis

IS-LM and AD-AS models are widely used to analyze macroeconomic issues and policies. In this lecture, we will study these two models. We will see that both models are equivalent. They are based on same assumptions about economic behavior and price adjustments, and they give same answers when used to analyze the effects of various shocks. Using these models, we will analyze two views regarding the effects of monetary shocks on business cycle variables – the Keynesian view and the classical view. We will analyze, in what aspects they are different, why do they have different predictions, and why they have different policy prescriptions.

A. The IS-LM Model

1. The LM Curve

Let us begin with the LM curve. The LM curve gives a relationship between the nominal interest rate \(i\) and real output/income/expenditure \((Y)\), such that the money market is in equilibrium \(i.e.,\) the demand for money is equal to the supply of money. In the derivation of the LM curve, it is generally assumed that money and other financial assets are perfect substitutes.

We can divide financial assets in two categories: monetary assets \((M)\) and non-monetary assets \((NM)\). The demand for monetary and non-monetary assets is part of portfolio-allocation decision (we will study this issue in detail in lecture 3). Agents decide what part of their wealth they should keep in the form of monetary assets and what part as non-monetary assets. This decision is based on the relative desirability of these assets. The most attractive characteristic of money is that it is a medium of exchange and the most liquid asset. By liquidity, we mean that money can be exchanged for goods, services, and other assets quickly and without transaction costs. Non-monetary assets, on the other hand, are relatively illiquid, but they provide some return, which is proxied by
the nominal rate of interest, \( i \). Any agent allocates his wealth among these two assets such that he is indifferent between these two types of assets at margin.

The equilibrium in the asset market requires that the demand for financial assets should be equal to the supply of financial assets.

\[
M^d + NM^d = M + NM. \tag{1.1}
\]

If we define the excess demand for an asset \((E^A)\) as a difference between its demand and supply, then the equilibrium in the asset market requires

\[
M^A + NM^A = 0. \tag{1.2}
\]

Thus if the money market is in equilibrium, \( i.e., \ M^A = 0 \), it must be the case that the market of non-monetary assets is also in equilibrium, \( NM^A = 0 \). This is the application of the \textbf{Walras Law}, which says that if there are \( n \) markets and \( n - 1 \) markets are in equilibrium, then \( n^{th} \) market is also in equilibrium. Thus, more broadly, the LM curve represents the combination of the nominal interest rate, \( i \), and real output, \( Y \), such that the financial market is in equilibrium.

In the derivation of the LM curve, money supply, \( M \), is taken as given. The demand for money is expressed as a function of price level, \( P \), interest rate, \( i \), and real output, \( Y \).

\[
M^d = PL(i, Y), \enspace L_i < 0, \enspace L_Y > 0 \tag{1.3}
\]

where \( L(i, y) \) is the demand function for \textbf{real money balance}, \( M^d_P \), or the liquidity preference. By assumption, the demand for money, \( M^d \), increases one for one with the price level, \( P \). It also an increasing function of real income \( Y \). Idea is that the demand for money increases with these two variables as the number and the size of transactions become larger (the transaction motive for holding money).

The demand for money declines as the nominal rate of interest rises, \( i \), since it represents the opportunity cost of holding money. Idea is that an agent by holding money is forgoing the opportunity to hold other financial assets, which yield return \( i \). A higher \( i \) implies a higher opportunity cost, and thus a lower demand for money, \( M^d \).
The equilibrium in the money market requires that the demand for money, \( M^d \), be equal to the supply of money, \( M \) and thus

\[
\frac{M}{P} = L(i, Y).
\]  

(1.4)

Using (1.4), we can derive the LM curve. Given the properties of liquidity preference, one can easily see that the equilibrium in the money market requires a positive relationship between interest rate, \( i \), and real output, \( Y \), i.e., the LM curve is upward sloping in \((Y, i)\) space. A higher \( Y \) implies a higher \( L(i, Y) \), other thing remaining the same. Then in order to achieve equilibrium, \( i \) should be higher, which pushes down \( L(i, y) \).

In order to derive the slope of the LM curve formally, differentiate both sides of (1.4) by \( Y \).

\[
0 = L_i \frac{di}{dY} + L_Y.
\]  

(1.5)

Rearranging (1.5), we have

\[
\frac{di}{dY} = -\frac{L_Y}{L_i} > 0
\]  

(1.6)

as \( L_i < 0 \) and \( L_y > 0 \). The intuition is as follows: A higher output, \( Y \), leads to a greater demand for money. For a given supply of money and price level, in order to acquire more money agents in the economy try to sell their non-monetary assets, which reduces the demand for these assets and thus their prices. The decline in the prices of non-monetary assets (given their pay-offs) is reflected in the increase in the rate of return of these assets, which is proxied by the interest rate, \( i \).

(1.6) shows that a higher income elasticity of money demand, \( L_Y \) and a lower interest elasticity, \( L_i \), make the LM curve steeper. In other words, a relatively small change in income, \( Y \), leads to a larger change in the interest rate, \( i \). Intuitively, with a higher, \( L_y \), a given increase in income leads to relatively large increase in the demand for real money balance, and thus the interest rate rises relatively more. In the case of low interest elasticity, \( L_i \), again for a given increase in, \( Y \), the interest rate rises relatively more in order to restore equilibrium in the financial/money market.
Let us now turn to the relationship between the LM curve, money supply, and price level. Any increase in the money supply for a given price level will require an increase in the money demand for equilibrium to be achieved in the money market. How the money demand will increase? For a given level of output, \( Y \), and price, \( P \), the interest rate, \( i \), has to fall. Thus, a given level of output, \( Y \), is associated with a lower level of interest rate, \( i \), i.e., the LM curve shifts to the right in \((i, Y)\) space. In the case of a decline in money supply, the LM curve shifts to the left.

**Question 1.** What is the mechanism for the adjustment in the interest rate, \( i \), following an increase in the money supply, \( M \), given output, \( Y \), and price, \( P \)?

Given the above reasoning, one can easily work out the relationship between the LM curve and price, for a given money supply and output. A lower price level, \( P \), effectively increases the supply of real money balance, \( \frac{M}{P} \). Thus, the LM curve shifts to the right. A higher price level shifts LM curve to the left.

**Question 2.** What is the mechanism for the adjustment in the interest rate, \( i \), following an increase in price, \( P \), given output, \( Y \), and money supply, \( M \)?

2. **The IS Curve**

The IS curve traces out the combinations of interest rate, \( i \), and income \( Y \), such that the planned expenditure by agents (optimal expenditure given price, interest rate, and income) in the economy is equal to the actual expenditure. Alternatively, the IS curve gives a relationship between the interest rate, \( i \), and income \( Y \), such that the goods market is in equilibrium.

The planned expenditure, \( E \), is assumed to be the function of income, \( Y \), the real rate of interest, \( r \equiv i - \pi^e \), where \( \pi^e \) is the expected inflation rate, the government expenditure, \( G \), and tax, \( T \).

\[
E = E(Y, r, G, T), \quad 0 < E_Y < 1, \quad E_r < 0, \quad E_G > 0, \quad & E_T < 0. \tag{2.1}
\]

A higher real rate of interest, \( r \), and tax, \( T \), reduce investment and consumption. A
higher income, $Y$, and the government expenditure, $G$, lead to a higher planned expenditure.

One important assumption is here is that increase in income, $Y$, leads to less than proportionate increase in the planned expenditure, $E$. This is the basis for the famous Keynesian multiplier. This leads to the conclusion that a given increase in the planned expenditure (autonomous) results in more than proportionate increase in output, in equilibrium.

The equilibrium requires that the planned expenditure, $E$, is equal to the actual expenditure/output/income, $Y$. Thus,

$$Y = E(Y, r, G, T). \quad (2.2)$$

**Question 3** What is the intuition behind the Keynesian multiplier?

From (2.2), one can derive the IS curve, which gives a relationship between the interest rate, $i$, and income, $Y$. One can easily see that the IS curve is downward sloping, or there is a negative relationship between $i$ and $Y$. For a given level of expected inflation, $\pi^e$, a higher interest rate, $i$, implies a higher real rate of interest, $r \equiv i - \pi^e$. A higher real rate of interest, $r$, leads to a lower consumption and investment and thus lower income, $Y$.

The above equilibrium condition can be written in the form of equality between savings and investment. In a closed economy,

$$E \equiv C + I + G. \quad (2.3)$$

By definition, savings, $S$, is

$$S = Y - C - G \quad (2.4)$$

where $C$ is consumption. Thus, equilibrium requires that

$$S = I. \quad (2.5)$$
Using (2.5) as well, one can derive a downward sloping IS curve. Intuitively, higher income, \( Y \), leads to more saving, \( S \). More saving results in the lower real rate of interest, \( r \), and thus lower interest rate, \( i \).

By differentiating (2.2) with respect to \( Y \), we can formally derive the slope of the IS curve.

\[
1 = E_Y + E_r \frac{di}{dY}. \tag{2.6}
\]

Rearranging, we have

\[
\frac{di}{dY} = \frac{1 - E_Y}{E_r} < 0 \tag{2.7}
\]

since \( 0 < E_Y < 1 \) and \( E_r < 0 \). (2.7) shows that higher \( E_Y \) and \( E_r \) imply flatter IS curve, i.e., a given change in output, \( Y \), leads to relatively smaller change in the interest rate, \( i \).

Intuitively, a higher \( E_r \) implies that an increase in \( i \) reduces \( E \) relatively more and thus output \( Y \) must rise relatively more in order to achieve equilibrium. A higher \( E_Y \) makes the planned expenditure curve steeper in \((E,Y)\) space. Thus, an increase in the interest rate, \( i \), reduces equilibrium output, \( Y \), relatively more.

**Question 4** What is the relationship between the Keynesian multiplier and \( E_Y \)?

3. The FE Line

The FE line or full employment line gives a relationship between the interest rate, \( i \), and output, \( Y \), such that the labor market is in equilibrium i.e., the demand for labor is equal to the supply of labor. The demand for labor being equal to supply does not mean that there is no unemployment. Equilibrium in the labor market allows for the possibility of **frictional** and **structural** unemployment.

Frictional unemployment arises as workers search for suitable jobs and firms search for suitable workers. Since workers and jobs are heterogeneous, and there is imperfect information about workers and jobs, there is always some frictional unemployment as matching workers to appropriate jobs in a dynamic economy is a costly and time-consuming
process. Structural unemployment arises when the skill set of a section of workers does not match with skill requirements of jobs. This leads to long term unemployment. In any economy, there is always some frictional and structural unemployment. Frictional and structural unemployment rate together constitute, what is known as the natural rate of unemployment. When we say labor market is in equilibrium, we mean that the unemployment rate is at its natural rate.

Let us now derive FE line. We will assume that there is perfect competition in the labor market. Let the production function be

\[ Y = F(L), \quad F_L > 0, \quad F_{LL} < 0 \]  \hspace{1cm} (3.1)

where \( L \) is labor input. A profit maximizing firm employs workers till the point where the marginal product of labor equals real wage.

\[ F_L(L) = \frac{W}{P} \]  \hspace{1cm} (3.2)

where \( W \) is the nominal wage. From (3.2), we can derive a downward sloping labor demand function, \( L^D(W/P) \). The supply of labor is derived from the utility maximization problem of consumer/workers. Let the utility function be

\[ u(C, L), \quad u_C > 0, \quad u_{CC} < 0, \quad u_L < 0, \quad u_{CL} \geq 0 \]  \hspace{1cm} (3.3)

where \( C \) is consumption. The budget constraint faced by a consumer/worker is

\[ PC = WL. \]  \hspace{1cm} (3.4)

Putting (3.4) in (3.3), and taking the derivative with respect to labor supply, \( L \), we have

\[ -\frac{u_L}{u_C} = \frac{W}{P}. \]  \hspace{1cm} (3.5)

(3.5) equates the marginal rate of substitution between labor and consumption with real wage. From (3.5), one can derive upward sloping labor supply function, \( L^S(W/P) \).

The labor market equilibrium is achieved, when
Let $\bar{L}$ be the equilibrium employment. Then the associated output, $\bar{Y}$, is given by

$$\bar{Y} = F(\bar{L}).$$

$\bar{Y}$ is known as the **potential output** or **full employment output**. This is the profit-maximizing output, which is consistent with the natural rate of unemployment.

Since neither the labor demand nor the labor supply depends on the interest rate, $i$, equilibrium employment also does not depend on it. Thus, the FE line is vertical in $(Y, i)$ space at the potential output, $\bar{Y}$. Anything that shifts labor demand or supply function shifts the FE line as well. Potential candidates are productivity shocks, taxes, change in population, unemployment insurance etc.

The potential output and the natural rate of unemployment also depend on the market structure of goods and labor markets. The potential output and the natural rate of unemployment in the above example are derived under the assumption of competitive labor market. If we assume that the labor market is imperfect, then we will get another level of the potential output and the natural level of unemployment. We will later see that the classical economists assume competitive goods and labor markets, while the Keynesian economists assume imperfect goods and labor markets. Consequently, the classical potential output (natural rate of unemployment) is higher (lower) than the Keynesian potential output (natural rate of unemployment).

**Question 5** Draw the FE line in $(Y, i)$ space and analyze how supply shocks, taxes, change in population etc. affect the FE line.

4. **General Equilibrium**

The general equilibrium is achieved at a point in $(Y, i)$ space at which all the three curves – IS, LM, and FE intersect each other. At this particular combination of interest rate, $i$, and output, $Y$, all the three markets – money/financial, goods, and labor are in equilibrium. The general equilibrium point is also the point of the long run equilibrium.
Any shift in any of the curves takes the economy out of general equilibrium. The possibility that an economy at a point in time can be out of equilibrium raises three questions:

1. When the economy is out of general equilibrium, what determines output, interest rate, employment, price and other variables, or what determines the short-run equilibrium?
2. When the economy is out of general equilibrium or the long-run and the short-run equilibrium points do not coincide, what adjustment processes take the economy back to the general equilibrium point?
3. How fast is the adjustment process or how much time does it take the economy to go back to the general equilibrium point?

There is no controversy regarding the answers to first two questions. When the economy is out of general equilibrium, output, interest rate, employment, price and other variables are given by the intersection of the IS and LM curves. In other words, the short-run equilibrium is given by the intersection of the IS and the LM curves. Thus, employment can be higher or lower than the full employment level temporarily. The long run equilibrium occurs at the point of the intersection between the IS curve and the FE line.

When the economy is out of general equilibrium, prices in all three markets adjust, which brings the economy back to the general equilibrium, such that short-run and long-run equilibrium coincide. If for instance, the intersection of the IS and the LM curve occurs at output level $Y > \overline{Y}$, then price, $P$, rises which shifts the LM curve up. Price rises because firms are producing more than their long term profit-maximizing level of output. This process goes on till all the three curves intersect each other, and the economy goes back to the general equilibrium point.

**Question 6** Draw the IS, the LM, and the FE curves and show a general equilibrium point. What happens if there are shocks to these three curves?

**Question 7** How changes in the money supply and the expected inflation rate affect equilibrium $Y$ and $i$?
The third question is highly controversial in macroeconomics. It is with respect to this question the Classical and the Keynesian economists greatly differ in their viewpoints. The classical economists, generally, believe that the process of adjustment is very fast. The economy through its price mechanism adjusts to any shock very rapidly, and thus for all practical purpose one can assume that economy is always in the general equilibrium.

The Keynesian economists believe that the adjustment process is slow due to various types of nominal and real rigidities in the markets, and the economy can be out of general equilibrium for a long periods of time. The other major difference between the Keynesian and the Classical economists is with respect to their assumption about the market structure. The Keynesians believe that markets are characterized by the monopolistic competition. Such market structure along with the nominal and real rigidities slow down the process of adjustment. The Classicals believe that perfect competition is better characterization of the real world markets.

We will see that these two contrasting viewpoints have strong policy implications. According to the Classicals, since economy always remains at the general equilibrium point, the government intervention is unnecessary from the point of view of enhancing social welfare and improving the efficiency of allocations. Many believe that the government interventions actually make economy worse off. The Keynesians, on the other hand, believe that the government interventions may improve welfare and the efficiency of allocations.

The speed of adjustment has implications regarding the effects of changes in the money supply on real variables. Money is said to be neutral if changes in the money supply has no real effects. Both the Keynesians and the Classicals believe that money is neutral in the long run. But the Keynesians also believe that money is not neutral in the short run, i.e., changes in money supply have real effects in the short run. The Classicals on the other hand believe that money is neutral even in the short run, since the process of adjustment is very fast.

**Question 8** Show the effects of changes in the money supply in the IS-LM-FE framework?
B. The AD-AS Model

Having analyzed the IS-LM model, let us now turn to the AD-AS model. We will begin with the derivation of the AD curve.

5. The AD (Aggregate Demand) Curve

AD curve gives relationship between the price level, $P$, and the desired real expenditure, $Y$, such that both the goods and the money markets are in equilibrium. In other words, it is given by the intersection of the IS and the LM curves. In order to derive the AD curve all we have to do is to change the level of price $P$ and using the IS and the LM curves find out the equilibrium $Y$.

Suppose, price level is increased. From (2.2), it is clear that the IS curve is not affected, as it does not affect the planned expenditure. However, it shifts the LM curve to the left (1.4) as the supply of real money balance falls, $\frac{M}{P}$. The result is that output, $Y$, falls and the interest rate, $i$, rises. Thus there is a negative relationship between price and output.

The intuition is that a higher price reduces supply of real money balance, which increases the interest rate. A higher interest rate reduces consumption and investment and thus output falls.

To formally derive the slope of the AD curve, take the derivative of (1.4) and (2.2) with respect to price, $P$. The derivative of (1.4) gives

$$\frac{dY}{dP} = -\frac{M}{P^2} = L_i \frac{di}{dP} + L_y \frac{dY}{dP}. \quad (5.1)$$

The derivative of (2.2) gives

$$\frac{dY}{dP} = E_y \frac{dY}{dP} + E_r \frac{di}{dP}. \quad (5.2)$$

Combining (5.1) and (5.2), we have

$$\frac{dY}{dP} = -\frac{M/P^2}{L_y + \frac{(1-E_Y)L_i}{E_r}} < 0. \quad (5.3)$$
The AD curve is derived under the assumption that the money supply, $M$, the government expenditure, $G$, and tax, $T$, are constant. Any change in these variables shifts the AD curve. For example, an increase in the money supply, $M$, reduces the interest rate, $i$, which in turn increases consumption and investment for any given level of price, $P$. Thus the AD curve shifts up to the right, implying that a given price level is associated with a higher output. In addition, the AD curve becomes steeper (see equation 5.3). Any increase in the expected inflation also shifts the AD curve up and to the right.

**Question 9** How do changes in the government expenditure and tax affect the AD curve?

6. The AS (Aggregate Supply) Curve

As discussed earlier, the AD curve gives relationship between price, $P$, and output, $Y$. In order to pin down the equilibrium level of price and output, we need an additional equation in price and output. This additional equation is provided by the AS curve. The AS curve gives the relation between price level, $P$, and the aggregate amount of output, $Y$, which firms wish to supply.

There are two types of aggregate supply curve – **Long Run Aggregate Supply Curve (LRAS)** and **Short Run Aggregate Supply Curve (SRAS)**. As names suggest LRAS is long run relationship between price, $P$, and the aggregate supply, $Y$, while SRAS is the short run relationship between the two.

The LRAS is vertical in $(Y, P)$ space at the level of potential output, $\bar{Y}$, *i.e.*, changes in price has no effect on the aggregate supply in the long run. This is because the potential output is the profit maximizing output in the long run. Unless the profit maximizing employment changes or the FE line shifts there is not going to be any change in the potential output.

The general equilibrium is achieved at the point where the AD curve intersects the LRAS curve. At this combination of $(Y, P)$, all the three markets are in equilibrium and the aggregate planned expenditure is equal to the LRAS. Given that the LRAS curve is vertical at the potential output, one can immediately see that changes in the aggregate
demand has no effect on output, employment, and any other real variables in the long run. Any change in the aggregate demand curve is reflected simply in price or nominal variables.

The shape of the SRAS curve and reasons behind it are subject of controversy between the Keynesians and the Classical economists. This controversy can be traced to their views regarding the speed of adjustment process and the market structure. The shape of the SRAS curve is quite important from both the positive and the normative point of view. For example, if the SRAS curve is vertical just as the LRAS curve, it follows immediately that any shift in the AD curve will not have any effect on real variables. The variations in the aggregate demand will just affect nominal variables. One immediate implication is that money is neutral in the short-run. However, if the SRAS curve is non-vertical, then any change in the aggregate demand has real effects in the short-run.

Below, we discuss different models which generate different types of SRAS curve. We begin with the case in which markets are competitive and there is no nominal rigidities.

7. Perfect Information Walrasian Markets and the SRAS Curve

Suppose that there is no nominal or real rigidity in the markets and there is perfect information regarding prices. Suppose that there are many types of goods. Each good is produced by a large number of identical consumer/producer individuals. The individual’s production function is

$$Y_i = L_i \quad \text{(7.1)}$$

where $L_i$ is the amount that the individual works and $Y_i$ the amount he produces. The utility function is

$$U_i = C_i - \frac{1}{\gamma} L_i^\gamma, \quad \gamma > 1. \quad \text{(7.2)}$$

The budget constraint faced by the consumer is simply

$$C_i = \frac{Y_i P_i}{P} \quad \text{(7.3)}$$
where $P$ is the average price level and $P_i$ is the price of good $i$. (7.3) equates consumption to real income. Putting (7.1) and (7.3) in (7.2), and taking derivative with respect to $L_i$, we can derive the labor supply function, which is

$$L_i = \left(\frac{P_i}{P}\right)^{\frac{1}{1-\gamma}}$$

(7.4) shows that the labor supply is increasing in the relative price of his own good. Taking log of (7.4) and using lower case letters to denote logarithm of the corresponding upper case letters, we have

$$l_i = \frac{1}{\gamma - 1}(p_i - p).$$

(7.5)

Given the production function, (7.5) also specifies the supply function for good $i$. (7.5) shows that what matters for the production decision is relative price, i.e., the gap between ones own price and general price. Higher the gap or relative price, more profitable it is to produce.

Let us suppose that demand for any good $i$ depends on: real income, the good’s relative price, and a random disturbance to preferences. Assume that demand is log-linear. Specifically, the demand for good, $i$, is

$$y_i = y + z_i - \eta(p_i - p), \quad \eta > 0$$

(7.6)

where $y$ is log of real income, $z_i$ is i.i.d. mean zero shock to demand for good, $i$, and $\eta$ is the elasticity of demand for each good. $y_i$ is the demand per producer of good $i$. $y$ is assumed to be equal to the average demand across all goods, and $p$ is the average of $p_i$’s:

$$y = \overline{y}_i$$

(7.7)

and

$$p = \overline{p}_i.$$
\[ y = m - p. \] 

(7.9) says that the aggregate demand is equal to the aggregate real money balance. It captures the idea that the aggregate demand is negatively related to price and positively related to the money supply.

With this set-up, the equilibrium in the market for good \( i \) requires equality between its demand and supply. From (7.5) and (7.6), we have

\[ \frac{1}{\gamma - 1} (p_i - p) = y + z_i - \eta (p_i - p). \] 

(7.10)

Solving this expression for \( p_i \) yields

\[ p_i = \frac{\gamma - 1}{1 + \eta \gamma - \eta} (y + z_i) + p. \] 

(7.11)

Averaging \( p_i \)'s and using the fact that \( z_i \) is mean zero shock, we obtain

\[ p = \frac{\gamma - 1}{1 + \eta \gamma - \eta} y + p. \] 

(7.12)

(7.12) is satisfied in equilibrium only when

\[ y = 0. \] 

(7.13)

(7.13) shows that equilibrium \( y \) is completely independent. Monetary shock has no effect on real output. (7.9) and (7.13) imply that

\[ m = p. \] 

(7.14)

(7.14) shows that any change in money supply only affects price, and thus money is neutral.

The intuition is simply that the aggregate supply curve is independent of price. To see this, just look at the individual good supply curve (7.5). If we average the individual supply curve, we get the aggregate supply curve

\[ \bar{y}_i = 0 = y \] 

(7.15)
which is independent of price. Thus the SRAS curve is vertical at the potential output.

The view that there is no difference between the SRAS and the LRAS curves and monetary shocks have no real effects is attributed to the **Real Business Cycle economists**. These economists believe that monetary shocks are not important in explaining business cycle facts since the economy is always at the general equilibrium point.

**Question 10** What happens when there is only one type of good and one type of consumer/producers?

### 8. Imperfect Information, Walrasian Markets, and the SRAS Curve

There is another strand of the Classical economists (Friedman, Lucas, Sargent), who believe that monetary shocks have real effects in the short run. They posit that producers have imperfect information about prices at the time of making production decision, since they only observe a small set of prices including their own. Based on these observed prices, they have to make forecast about general prices. Why do producers need to make forecast? It is because general price determines the relative price, which in turn determines profit maximizing production. The assumption of imperfect information about prices has important implication for the shape of SRAS curve.

To keep things simple, let us assume that producers can observe the price of their own good, $P_i$, but not the general price level, $P$. You can think of general public comes to know about the price level with a lag, once Stat Canada publishes price level/inflation rate. Alternatively, one can assume that full-information regarding monetary shock becomes available after the producers have made their production decisions. In this case, at the time of production, producers have to make forecast about the general price level.

The utility function in this case is

$$U_i = E \left( C_i - \frac{1}{\gamma} L^\gamma_i \right), \quad \gamma > 1.$$  \hfill (8.1)

where $E$ is the expectation operator. Now the producer chooses the labor supply, $l_i$, in order to maximize (8.1) subject to budget constraint (7.3). At the time of making this decision, producer knows, $P_i$, and thus makes the forecast of, $P$, conditional on $P_i$. 

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The first order condition is

\[ L_i = \left( E_{P_i} \left( \frac{P_i}{P} \right) \right)^{\frac{1}{\gamma-1}}. \]  

(8.2)

where \( E_{P_i} \) is expectation operator conditional on price, \( P_i \). Taking log of both sides, we have

\[ l_i = \frac{1}{\gamma-1} \ln \left( E_{P_i} \left( \frac{P_i}{P} \right) \right). \]  

(8.3)

To simplify matters, assume that

\[ \ln \left( E_{P_i} \left( \frac{P_i}{P} \right) \right) = E_{P_i} \left( \ln \frac{P_i}{P} \right). \]  

(8.4)

This is called the certainty-equivalence principle. This is true only when we have linear functions. It is not true in the non-linear case due to the Jensen’s inequality. Using (8.4), (8.3) can be written as

\[ l_i = \frac{1}{\gamma-1} E_{P_i} (p_i - p). \]  

(8.5)

The individual’s problem is to find expectation of \( E_{P_i} (p_i - p) \) given \( p_i \). Under the assumption that \( z_i \) and money supply \( m \) are normally distributed, one can show that

\[ l_i = b(p_i - E(p)) \]  

(8.6)

where \( b \) is some constant. (8.6) is very similar to (7.5) except for the expectation term and has similar interpretation. Any increase in the relative price, relative to the expected price level, increases supply of good \( i \). Summing (8.6) over all the goods, we have

\[ y = b(p - E(p)). \]  

(8.7)

(8.7) is known as the **Lucas supply curve**. It shows that any unexpected increase in the price level increases supply. In other words, we have **upward sloping short run aggregate supply curve** for a given level of expected price.
Equilibrium is achieved when the aggregate demand (7.9) is equal to the aggregate supply (8.7). This implies

\[ m - p = b(p - E(p)). \]  

(8.8)

This gives us expression for price, \( p \)

\[ p = \frac{1}{1+b} m + \frac{b}{1+b} E(p). \]  

(8.9)

If we take expectation of both sides, then we have

\[ E(p) = \frac{1}{1+b} E(m) + \frac{b}{1+b} E(p). \]  

(8.10)

(8.10) implies that

\[ E(p) = E(m). \]  

(8.11)

(8.11) has strong implication that any expected (anticipated) increase in money supply only increases price and has no effect on output. Putting (8.9) and (8.11) in (8.7), we get an expression for output, \( y \),

\[ y = \frac{b}{1+b} (m - E(m)). \]  

(8.12)

(8.12) shows that any unexpected (unanticipated) increase in money supply increases output. In other words, in this model expected change in money supply affects only nominal variables, and unexpected changes in money supply have real effects.

We have so far discussed two strands of the classical view on the SRAS curve. Now we turn to the Keynesian view on the SRAS curve.

9. The Keynesian View and the SRAS Curve

We will derive the Keynesian SRAS curve under two different assumptions. First, we will assume that nominal wage is fixed. This model generates an upward sloping SRAS
curve. Then we will assume that price is fixed. This model generates a horizontal SRAS curve. In both models, expected and unexpected monetary shocks affect real output.

i. Nominally Rigid Wages, Flexible Prices, And SRAS Curve

Suppose that nominal wage is fixed at a certain level

\[ W = \bar{W}. \]  

(9.1)

Also suppose that \( \bar{W} \) is higher than the nominal wage that will prevail in the case of no nominal rigidity i.e., there is involuntary unemployment. Suppose that production function is

\[ Y = F(l), \quad F_l > 0 \text{ } & \text{ } F_{ll} < 0. \]  

(9.2)

Profit maximizing firm will employ workers such that

\[ F_l(l) = \frac{\bar{W}}{P}. \]  

(9.3)

(9.3) equates the marginal product of labor to the real wage. In this case, any increase in the aggregate demand increases price and lowers the real wage. This induces firms to hire more workers and output goes up. This implies upward sloping short-run aggregate supply curve. Thus, employment and output are pro-cyclical. However, the model predicts anti-cyclical real wage, which is not supported by data. Data suggests that real wage is either acyclical or mildly pro-cyclical.

ii. Rigid Prices, Flexible Wages, And SRAS Curve

Suppose instead that price is rigid over certain range of supply but wage is flexible.

\[ P = \bar{P}, \quad \forall \ Y \leq Y_{max}. \]  

(9.4)

(9.4) basically assumes that firms are willing to supply as much output as demanded as long as the aggregate demand is less than \( Y_{max} \). In other words, the short-run aggregate supply
curve is horizontal over range of output $Y \in [0, Y_{max}]$. We will study a micro-foundation for this assumption later in lecture 2.

$Y_{max}$ is the level of output at which price, $\bar{P}$, equals the marginal cost of production, $mc(W)$. The marginal cost of production depends on wage, $W$. So for any $Y < Y_{max}$, price, $\bar{P}$, exceeds the marginal cost. Thus it is profitable for the firm to meet all the demand as long as $Y < Y_{max}$. Notice that the level of $Y_{max}$ generally depends on price, $P$, and wage, $W$. A lower price and a higher wage will reduce the level of $Y_{max}$ as the first reduces the marginal revenue and the second increases the marginal cost of production.

These assumptions have implication for the shape of the labor demand curve. It is no longer just given by (9.3). As long as $Y < Y_{max}$, the demand for labor is given by

$$l = F^{-1}(Y), \tag{9.5}$$

since the firm meets all the demand and employs just enough workers to produce, $Y$. If demand exceeds $Y > Y_{max}$, then the demand for labor is given by (9.3). Such labor demand is called the **effective labor demand**. Labor demand depends on the quantity of goods firm is able to sell. The equilibrium real wage is given by the intersection of the labor supply curve and the effective labor demand.

In this model, any decline in the aggregate demand leads to lower output, and thus lower employment, and lower real wage. This model implies a pro-cyclical real wage and an anti-cyclical mark-up (the ratio of price to the marginal cost). A fall in demand reduces the marginal cost both because real wage falls and the marginal product of labor rises.

Having discussed the aggregate supply curves, we now turn to the discussion of the Phillips curve. The shape of Phillips curve is closely related to the shape of aggregate supply curve.

10. The Phillips Curve

The Phillips curve refers to the relationship between output/unemployment rate and the inflation rate. The exact shape of the Phillips curve is subject to controversy. Analogous to the two types of aggregate supply curves, two types of Phillips curves are distinguished – long run and short run. The general consensus is that the long run Phillips
curve is vertical in \((Y, \pi)\) space. In other words, there is no long run trade-off between output/unemployment rate and the inflation rate. It simply reflects the fact that the LRAS curve is vertical at the potential output and changes in money supply do not affect it.

The short run Phillips curve is either vertical or upward sloping depending on assumptions regarding the market structure (perfect or imperfect competition), the presence or absence of rigidities, perfect or imperfect information regarding prices. The shape of short-run Phillips curve depends on the shape of the SRAS curve. If the SRAS curve is vertical, then the short run Phillips curve is vertical too. If the SRAS curve is upward sloping or horizontal, then the short run Phillips curve is upward sloping in the \((Y, \pi)\) space or downward sloping in the unemployment-inflation space.

\section*{i. The Traditional Phillips Curve}

We will work with the model of nominally rigid wage. Suppose that

\[ Y = F(L) = \frac{1}{\alpha} L^\alpha, \quad 0 < \alpha < 1. \]  
(10.1)

Wage is rigid at \(W = \overline{W}\). Labor employed is given by

\[ l = \left(\frac{W}{P}\right)^{\frac{1}{\alpha-1}}. \]  
(10.2)

Putting it in (10.1), we have

\[ Y = \left(\frac{W}{P}\right)^{\frac{\alpha}{\alpha-1}}. \]  
(10.3)

Taking log, we have

\[ y = \frac{\alpha}{(\alpha - 1)} (\overline{w} - p). \]  
(10.4)

Now suppose that \(\overline{W}\) is equal to last period price, \(P_{t-1}\). Then (10.4) can be written as

\[ y_t = \frac{\alpha}{(1 - \alpha)} (p_t - p_{t-1}). \]  
(10.5)

which gives an expression for the traditional Phillips curve.
\[ y_t = \frac{\alpha}{(1 - \alpha)} \pi_t \]  

where \( \pi_t \) is the inflation rate. (10.6) suggests that there is a permanent trade-off between output and inflation. By increasing inflation, policy makers can permanently raise output level.

This conclusion crucially depends on the assumption regarding how firms/workers adjust the nominal wage. Here by assumption, current period wage is indexed to the last period price level. Workers/firms do not take into account the fact that the government is following a permanent expansionary policy, which leads to lower real wage. However, such behavior is not rational. For instance, in this example if wage is indexed to the expected current price, \( P_t \), then from (10.6), one can immediately see that \( y_t = 0 \) and is independent of the inflation rate (or output is at its natural rate).

ii. The Expectation Augmented Phillips Curve

The idea that firms/workers take into account any permanent or expected inflation rate while fixing wages and prices, leads to the concept of the expectation-augmented Phillips curve. Let \( \pi_t^* \) be the core or underlying inflation (or inflation which is consistent with the natural rate of unemployment). A typical modern Phillips curve is

\[ \pi_t = \pi_t^* + \lambda(y_t - \overline{y}_t) + \xi_t^S \]  

where \( \overline{y}_t \) is the potential output and \( \xi_t^S \) is supply shock. (10.7) suggests that there is a trade-off only when the actual inflation deviates from the core inflation.

What should be the core inflation, \( \pi_t^* \)? This is subject of controversy and has strong policy implications. In the case of the classical economists, the core inflation is simply equal to the expected inflation \( \pi_t^e \). In this case, (10.7) becomes

\[ \pi_t - \pi_t^e = \lambda(y_t - \overline{y}_t) + \xi_t^S. \]  

Only a higher unexpected inflation leads to a higher output. In this case, there is a trade-off between the unexpected inflation and output, but that is something which the policy
makers cannot exploit. This model can be derived using the Lucas supply curve. Of course, when \( \pi_t - \pi_t^e \), then the deviation of the actual output from the potential output occurs only due to supply shocks. The real business cycle theorists believe that for all practical purpose this is the case.

Modern Keynesian economists believe that the core inflation depends on the expected inflation as well as the past inflation, i.e.,

\[
\pi_t^* = \phi \pi_t^e + (1 - \phi) \pi_{t-1}.
\]  

(10.9)

Idea is that there is some inertia in wage and price inflation. There is some link between the past and the future inflation beyond effects operating through expectation. Putting (10.9) in (10.7), we have

\[
\pi_t = \phi \pi_t^e + (1 - \phi) \pi_{t-1} + \lambda(y_t - \bar{y}_t) + \xi_t^S.
\]  

(10.10)

(10.10) can be written as

\[
\pi_t - \pi_{t-1} = \phi(\pi_t^e - \pi_{t-1}) + \lambda(y_t - \bar{y}_t) + \xi_t^S.
\]  

(10.11)

From (10.11), one can immediately see that for a given past inflation rate, any increase in the current inflation rate raises current output. Thus, there is a exploitable trade-off between inflation and output.

A special case arises when \( \phi = 0 \). Then we have the case of adaptive expectation. Individuals form expectation about the future inflation by extrapolating the past trend. In this case,

\[
\pi_t - \pi_{t-1} = \lambda(y_t - \bar{y}_t) + \xi_t^S.
\]  

(10.12)

Now the government can raise output above the natural rate, if it is willing to tolerate ever increasing inflation rate. This is the famous Friedman-Phelps Phillips curve. In this case, there is a trade-off between change in inflation and output.
iii. The New Keynesian Phillips Curve

Till early nineties, the standard Keynesian models used for most monetary analysis combined the assumption of nominal rigidity with a simple structure linking the quantity of money to the aggregate spending. The quantity of money is linked to the aggregate spending either through the quantity theory of money or the traditional text-book IS-LM model, which we discussed in the previous sections.

Now the standard approach is to build a dynamic, stochastic, general equilibrium model based on the optimizing behavior of agents (which we will study in lecture 3), combined with some kind of nominal wage and/or price rigidity (which we will study in lecture 2). Also it is assumed that there is monopolistic competition in the product market and/or monopsonistic competition in the labor market and expectations are rational. These assumptions typically yield relationship between the current inflation rate, the expected future inflation rate, and the real marginal cost, which is known as the New Keynesian Phillips Curve.

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa mc_t \]  \hspace{1cm} (10.13)

where \( mc_t \) is the deviation of real marginal cost from its steady state value and \( \kappa \) is a constant. (10.13) is an example of the stochastic difference equation. In the next lecture, we will study how to solve such equations.

The new Keynesian Phillips curve implies that the real marginal cost is the driving variable for the inflation process and also it is forward-looking, with current inflation a function of the expected future inflation. In empirical work, \( mc_t \) is generally proxied by the deviation of output from its potential level.

\[ mc_t = \gamma (y_t - \overline{y}_t) \]  \hspace{1cm} (10.14)

With this modification, the New Keynesian Phillips curve becomes

\[ \pi_t = \beta E_t \pi_{t+1} + A(y_t - \overline{y}_t) \]  \hspace{1cm} (10.15)
where $A$ is some constant. The estimation of the New Keynesian Phillips curve is still in early stages, with no consensus regarding whether it is a good characterization of inflation process.