Lecture 4
Conduct of Monetary Policy: Goals, Instruments, and Targets; Time Inconsistency and Targeting Rules

1. Introduction

In this chapter, we analyze the conduct of monetary policy (or the operating procedure) *i.e.* how is it operationalized, what is its objectives, constraints faced by central banks etc. Central banks are normally mandated to achieve certain goals such as price stability, high growth, low unemployment etc. But central banks do not directly control these variables. Rather they have set of instruments such as open-market operations, setting bank rate etc. which they can use to achieve these objectives.

The problem of central bank is compounded by the fact that their instruments do not directly affect these goals. These instruments affect variables such as money supply and interest rates, which then affect goal variables with lag. In addition, these lags may be uncertain. Due to above mentioned problems, in the conduct of monetary policy distinction is made among (i) goals (or objectives), (ii) targets (or intermediate targets), (iii) indicators (or operational targets), and (iv) instruments (or tools).

Target and indicator variables lie between goal and instrument variables. Target variables such as money supply and interest rates have direct and predictable impact on goal variables and can be quickly and more easily observed. In previous chapters, we studied various theories linking target variables to goal variables. By observing these variables, the central bank can determine whether its policies are having desired effect or not. However, even these target variables are not directly affected by central bank instruments. These instruments affect target variables, through another set of variables called indicators. These indicators such as monetary base and short run interest rates are more responsive to instruments. The conduct of monetary policy can be represented schematically as follows:

Instruments → Indicators → Targets → Goals

Following is the list of different kinds of variables.

Table 1

Goals or Objectives

1. High Employment
2. Economic Growth
3. Price Stability
4. Interest-Rate Stability
5. Stability of Financial Markets
6. Stability in Foreign Exchange Markets

Targets or Intermediate Targets

1. Monetary Aggregates (M1, M2, M3 etc.)
2. Short Run and Long Run Interest Rates

Indicators or Operational Targets

1. Monetary Base or High-Powered Money
2. Short Run Interest Rate (Rate on Treasury Bill, Overnight Rate)

Instruments or Tools

1. Open Market Operations
2. Reserve Requirements
3. Operating Band for Overnight Rate
4. Bank Rate

Though we have listed six goals, it does not mean that different countries and regimes give same weight to all these goals. Different goals may get different emphasis in different countries and times. Currently in Canada, a lot of emphasis is put on the goal of price and financial market stability. Also, all the goals may not be compatible with each other. For instance, goal of price stability may conflict with the goals of high employment and stability of interest rate at least in the short run.

The list of target variables raises the question: how do we choose target variables? Three criteria are suggested: (i) measurability, (ii) controllability, and (iii) predictable effects on goals.

Quick and accurate measurement of target variables are necessary because the target will be useful only if it signals rapidly when policy is off track. For a target variable to be useful, a central bank must be able to exercise effective control over it. If the central bank cannot exercise effective control over it, knowing that it is off-track is of little help. Finally and most importantly, target variables must have a predictable impact on goal variables. If target variables do not have predictable impact on goal variables, the central bank cannot achieve its goal by using target variables. Monetary aggregates and short and long run interest rates satisfy all three criteria.

The same three criteria are used to choose indicators. They must be measurable, the central bank should have effective control over them, and they must have predictable effect on target variables. All the indicators listed above satisfy these criteria.

2. Money Supply Process, Asset Pricing, and Interest Rates

In previous chapters, we extensively analyzed relationships among goal variables such employment, inflation, output and target variables such as money supply and interest rates. Now we turn to analyze relationship among instruments, indicators, and targets. In order to understand relationships among these three types of variables, it is instructive to analyze the money supply process and asset pricing which throws light on relationships among different types of interest rates.

A. Money Supply Process

So far, we have been vague about what determines money supply. We just assumed that it is partly determined by the central bank and partly by non-policy shocks. In this section, we take a closer look at the money supply process. It has important bearing on the conduct of monetary policy.

There are four important actors, whose actions determine the money supply – (i) the central bank, (ii) banks, (iii) depositors, and (iv) borrowers. Of the four players, the central bank is the most important. Its actions largely determine the money supply. Let us first look at its balance sheet.

Table 2

Balance Sheet of a Central Bank

Assets	Liabilities
Government Securities	Notes in Circulation
Advances to Banks	Deposits
Foreign Securities & Currencies	

The two liabilities on the balance sheet, notes in circulation and deposits of other financial institutions, are often called **monetary liabilities**. The financial institutions hold deposits with the central bank either because they are required to do so or to settle claims with other financial institutions.

These deposits together with currency physically held by banks make up **bank reserves**. Reserves are assets for the banks but liabilities for the central bank. We will see later that an increase in reserves lead to increase in money supply. Banks hold reserves in order to meet their short-run liquidity requirements. This is called **desired reserve**. Sometimes banks are also required to hold certain fraction of their deposits in terms of currency. These reserves are called **required reserves**.

The three assets of the central bank are important for two reasons. First, changes in the asset items lead to changes in money supply. Second, these assets earn interests (other than the foreign currency), while the liabilities do not. Thus, they are source of revenue for the central bank.

The currency in circulation (C) together with reserves (R) constitute **monetary base or high-powered money** (MB).

$$MB = C + R. \tag{2.1}$$

The central bank controls the monetary base through its purchase or sale of government securities in the open market (**open market operations**), and through its extension of loans to banks. It can also print new currencies. It is through its control over monetary base, the central bank affects money supply. To understand this, let us first look at how monetary base is related to money supply. For illustrative purpose, we will just concentrate on the relationship between monetary base and $M1$ (currency plus chequable deposits).

Money supply $M(\equiv M1)$ is related to monetary base through **money multiplier** (m).

$$M = mMB \tag{2.2}$$

As we can see that money multiplier is simply the ratio of money supply to monetary base. How do we derive the money multiplier? Let D be the deposit and define currency ratio, c, and reserve ratio, r as follows

$$c \equiv \frac{C}{D} \;\&\; r \equiv \frac{R}{D},\; 0 < c, r < 1. \tag{2.3}$$

Using (2.3) and (2.1), we can express MB as

$$MB = (c + r)D. \tag{2.4}$$

Now by definition

$$M = C + D = (1 + c)D. \tag{2.5}$$

Putting (2.4) in (2.5), we have

$$M = \frac{(1 + c)}{r + c} MB. \tag{2.6}$$

The term $\frac{1+c}{r+c} \equiv m$ is the money multiplier and it is strictly greater than unity. Thus, one unit change in monetary base leads to more than one unit change in money supply. Also, higher currency ratio, c, and reserve ratio, r, lead to lower money supply for a given level of monetary base.

From (2.6) it is clear that money supply depends not only on monetary base over which the central bank has lot of control but also on the behavior of banks, depositors, and borrowers which determine currency ratio, c, and reserve ratio, r. c and r depend on rate of return on other assets and their variability, innovations in financial system and cash management, expected deposit outflows etc. In general, broader the measure of money supply, less control the central bank has on its supply.

B. Asset Pricing and Interest Rates

In the analysis so far, we divided financial assets in two categories – monetary and non-monetary assets. We called the rate of return on non-monetary assets as the nominal rate of interest. But we know that there are different types of non-monetary assets with different rates of return. Then the question is : how justifiable is lumping together of different non-monetary assets?

We can lump together different types of non-monetary assets provided there is stable relationship among their rates of return. The rate of return on an asset depends on its pay-off and price. In order to understand, the relationships among rates of return, we need to know how assets are priced. This analysis helps us in establishing relationship among different types of interest rates.

The price of an asset equates the marginal cost (in utility terms) to its expected marginal benefit (in utility terms). Suppose that an asset pays off at time $t+i$ with $i \geq 1$ and its payoff is y_{t+i} (suppose resale value is 0), which is a random variable. Then its price at time t, q_t satisfies

$$q_t u'(c_t) = \beta^i E_t \left[u'(c_{t+i}) y_{t+i} \right]. \tag{2.7}$$

The left hand side is the marginal cost of buying the asset in utility terms and the right hand is the expected marginal benefit. The asset pays off y_{t+i} in period $t+i$, which is converted in utility terms by multiplying it with the marginal utility of consumption at $t+i$. To make it comparable to time t utility, we multiply it by β^i. (2.7) can be rewritten as

$$q_t = \beta^i E_t \left[\frac{u'(c_{t+i}) y_{t+i}}{u'(c_t)} \right]. \tag{2.8}$$

In general, asset price, q_t, satisfies

$$q_t = \beta^i E_t \left[\frac{u'(c_{t+i})(\text{Return on the Asset})}{u'(c_t)} \right] \tag{2.9}$$

The return on the asset is simply the sum of payoff of the asset and its resale value.

$$\text{Return of the Asset} \;=\; \text{Payoff} \;+\; \text{Resale Price} \tag{2.10}$$

$$\text{Rate of Return of the Asset} \;=\; \frac{\text{Return of the Asset}}{\text{Price of the Asset (q)}} \tag{2.11}$$

Let us use (2.9) to price different kinds of assets. Suppose that in the next period, the economy can be in two states – high with probability, p^h and low with probability, p^l.

Riskless Real Bond

Let q_B be the current period price in real terms of a bond which pays 1 unit of good in the next period both in high and low states (discount coupon). We want to know q_B. In order to do so, first we have to specify the return on the riskless bond, which is simply 1 unit of good (payoff = 1 unit of good, resale value = 0). After specifying the return, we can use (2.9) to get its price which is simply

$$q_B = \beta E_1 \left[\frac{u'(c_2) * 1}{u'(c_1)} \right] = \beta E \left[\frac{u'(c_2)}{u'(c_1)} \right]. \tag{2.12}$$

We can also derive the net rate of return of bond, r_B, which is given by

$$r_B \equiv \frac{1}{q_B} - 1 = \frac{u'(c_1)}{\beta E_1(u'(c_2))} - 1. \tag{2.13}$$

Riskless Nominal Bond

Now let Q_B be the price of a one period nominal bond in dollars. Suppose that this bond pays 1 dollar next period regardless of state. Then what is Q_B?

The return from nominal bond next period in real terms is $\frac{1}{p_2}$. The cost of the nominal bond in real terms today is $\frac{Q_B}{p_1}$. Thus, using (2.9), we have

$$\frac{Q_B}{p_1}u'(c_1) = \beta E_1 \frac{u'(c_2)}{p_2}. \tag{2.14}$$

(2.14) implies that

$$Q_B = \beta E_1 \frac{u'(c_2)}{u'(c_1)} \frac{p_1}{p_2}. \tag{2.15}$$

Contingent Claims/ Arrow-Debru Basis Securities

Let us consider an asset with current price q_h in real terms which pays 1 unit of commodity next period in high state and 0 in the low state. Such an asset is known as contingent claim because its payoff depends on the state of the economy. How much is q_h? From (2.9), we know that q_h must satisfy

$$q_h = \beta \left[\frac{p^h u'(c_2^h)}{u'(c_1)} \right] \tag{2.16}$$

Now consider another contingent claim with opposite pay-off. Suppose the current price of an asset, which pays 1 unit of good next period in low state and 0 in high state, is q_l. Then, q_l must satisfy

$$q_l = \beta \left[\frac{p^l u'(c_2^l)}{u'(c_1)} \right] \tag{2.17}$$

Such contingent claims are known as Arrow-Debru basis securities because their return span the space of possible outcomes in period 2. In other words, any outcome in the second period can be represented by a linear combination of return of these securities. Due to this property, knowing the price of AD securities is sufficient to tell us the price of more complicated assets. All one has to do is to construct a portfolio (linear combination) consisting of AD securities which mimics the return of other asset. Then **arbitrage** ensures that the price of other asset is exactly equal to the price of this portfolio. For instance, holding the riskless bond is equivalent to holding both the AD securities and thus

$$q_B = q_h + q_l \tag{2.18}$$

Long and Short Bonds

Let q^L be the price of a bond today, which pays 1 unit in period 3 (long or two-period bond), and q_1^S be the price of one period bond, which pays 1 unit next period. Let $u(c) = \ln c$. Then,

$$q^L = \beta^2 E_1 \left[\frac{c_1}{c_3}\right] \tag{2.19}$$

$$q_1^S = \beta E_1 \left[\frac{c_1}{c_2}\right] \tag{2.20}$$

From (2.19) and (2.20), we can derive rates of interest on long and short bonds. The gross return on long bond satisfies

$$(1 + r^L)^2 = \frac{1}{q^L} = \frac{1}{\beta^2 E_1 \left[\frac{c_1}{c_3}\right]}. \tag{2.21}$$

Similarly, the gross return on short bond satisfies

$$1 + r_1^S = \frac{1}{q_1^S} = \frac{1}{\beta E_1 \left[\frac{c_1}{c_2}\right]}. \tag{2.22}$$

The pattern of returns on long and short bonds are known as **term structure**. The plot of term structure over maturity is also known as **yield curve**. The term structure or yield curve embodies the forecasts of future consumption growth. In general, yield curve slopes up reflecting growth. Downward sloping yield curve often forecasts a recession.

What is the relationship between the prices of short and long bonds? We turn to covariance decomposition ($E(xy) = E(x)E(y) + cov(x, y)$).

$$q^L = \beta^2 E_1 \left[\frac{c_1 c_2}{c_2 c_3}\right] \tag{2.23}$$

which implies

$$q^L = q_1^S E_1 q_2^S + Cov\left(\frac{\beta c_1}{c_2}, \frac{\beta c_2}{c_3}\right) \tag{2.24}$$

where q_2^S is the second period price of one period bond. If we ignore the covariance term, then in terms of returns (2.24) can be written as

$$\left(\frac{1}{1+r^L}\right)^2 = \frac{1}{1+r_1^S} E_1 \frac{1}{1+r_2^S}. \tag{2.25}$$

Taking logarithms, utilizing the fact that $ln(1+r) \approx r$, and ignoring Jensen's inequality we get

$$r^L \approx \frac{r_1^S + E_1 r_2^S}{2}. \tag{2.26}$$

(2.26) suggests that the long run bond yield is approximately equal to the arithmetic mean of the current and expected short bond yields. This is called **expectation hypothesis** of the term structure. (2.24 - 2.27) imply that prices of different types of bonds and thus their return are related to each other. Thus, if one type of rate of interest changes, its effect spreads to other interest rates as well.

Exercise: Find out the prices of short and long term nominal bonds and their relationship.

Exercise: What is the price of bond which pays 1 unit of good in period T in all states?

Forward Prices

Suppose in period 1, you sign a contract, which requires you to pay f in period 2 in exchange for a payoff of 1 in period 3. How do we value this contract? Notice that the price of contract, which is to be paid in period 2, is agreed in period 1. Then the expected marginal cost of the contract in period 1 is $\beta E_1 u'(c_2) f$. The expected benefit of the contract is $\beta^2 E_1 u'(c_3)$. Since the price equates the expected marginal cost with expected marginal benefit of the asset, we have

$$f = \frac{\beta E_1 u'(c_3)}{E_1 u'(c_2)} = \frac{q^L}{q_1^S} \tag{2.27}$$

Exercise: Show that the expectation hypothesis implies that forward rates are equal to the expected future short rates.

Exercise: Find the relationship among forward, long, and short rates of return. Also use covariance decomposition to define risk-premium on forward prices.

Share

Suppose that a share pays a stream of dividend d_i for $i \in [t,\ T]$. The resale value of the share at time T is zero. Then the price of the share at time t is given by

$$q_{ST} = E_t \sum_{i=1}^{T-t} \beta^i \frac{u'(c_{t+i})}{u'(c_t)} d_{t+i}. \tag{2.28}$$

3. Choice of Instruments and Targets

A. Instruments

Having discussed the money supply process and interrelationship among different interest rates, one can analyze how different tools or instruments affect the balance sheet of the central bank and thus money supply and interest rates.

Open market operations refer to buying and selling of government bonds in the open market by the central bank. When the central bank buys government bonds, it increases the amount of currency. Also for a given demand for money, it leads to lower interest rate. Opposite is the case, when central bank buys government bonds.

By changing **reserve requirements** as well the central bank can change money supply and interest rates. Higher reserve requirement leads to higher reserve ratio which in turn leads to lower money supply and higher interest rate. Opposite is the case when the central bank reduces the reserve requirement.

The **overnight interest rate** refers to the rate at which financial institutions borrow and lend overnight funds. This rate is the shortest-term rate available and forms the base of term structure of interest rates relation. Many central banks including Bank of Canada implement their monetary policy by announcing **target overnight rate**. Idea is to keep the actual overnight rate within a narrow band (usually about 50 basis point or 0.5% wide).

This band is also known as **channel or corridor or operating band**. The upper limit of this band is known as **bank rate**. This is the rate at which the central bank is willing to lend to financial institutions for overnight. The lower limit of the band is the rate, which the central bank pays to the overnight depositors. One can immediately see that these operating bands put limit on the actual overnight rate. No financial institution will borrow overnight fund for more than the bank rate because they can borrow as much as they require from the central bank at the bank rate. Similarly no lender will lend overnight fund at the rate below the lower limit of the operating band, because they can always deposit their overnight fund at the central bank at that rate.

B. Choice of Instruments or Targets

Table 1 shows that the central bank has two sets of instruments (as well as indicators and targets) – monetary aggregates and interest rates. However, these two sets of instruments are not independent of each other. If the central bank chooses monetary aggregate, then it will have to leave interest rate to be determined by the market forces (through money market). If it chooses interest rate, then monetary aggregate is determined by the market forces. Same is true for two sets of indicators and targets.

Now the question is: which set of instrument the central bank should choose? Answer is: if the central bank's target variable is money supply then use monetary aggregate tools and if the target variable is interest rate, then choose interest rate as instrument.

But again it raises the question, which set of target variables to choose? The choice of target variables and thus instruments depends on the stochastic structure of the economy *i.e* the nature and relative importance of different types of disturbances. The general conclusion is that if the main source of disturbance in the economy is shocks to IS curve or goods market, then targeting money supply (or using money supply tool) is optimal. On the other hand, if the main source of disturbance is shocks to demand for money or financial market, then targeting interest rate is optimal.

To understand the intuition behind this conclusion, let us consider an economy where the objective of the central bank is to stabilize output. Suppose that the central bank must set policy before observing the current disturbances to the goods and money markets, and assume that information on interest rate, but not on output is immediately available. Suppose that the IS curve is given by the following equation

$$y_t = -\alpha i_t + u_t \tag{3.1}$$

and the LM curve by

$$m_t = y_t - c i_t + v_t. \tag{3.2}$$

Here price level is assumed to be constant and thus analysis pertains to short-term (or choices of instruments and indicators). Both u_t and v_t are mean zero *i.i.d* exogenous shocks with variance σ_u^2 and σ_v^2 respectively. The objective of the central bank is to minimize the the variance of output deviations from potential output set to zero:

$$\min E(y_t)^2. \tag{3.3}$$

The timing is as follows: the central bank sets either interest rate, i_t or money supply m_t at the start of the period; then stochastic shocks are realized, which determine the value of output, y_t. The question is which policy rule minimizes (3.3). In other words, whether the central bank should try to hold market rate of interest constant or should hold money supply constant while allowing interest rate to move.

Let us first consider money target rule. Here, the central bank optimally chooses m_t letting i_t determined by IS and LM curves. Substituting (3.2) in (3.1), we get

$$y_t = u_t + \alpha \left[\frac{m_t - y_t - v_t}{c} \right] \tag{3.4}$$

which implies

$$y_t = \frac{\alpha m_t + c u_t - \alpha v_t}{\alpha + c}. \tag{3.5}$$

Putting (3.5) in (3.1), the optimization problem reduces to

$$\min_{m_t} E \left(\frac{\alpha m_t + c u_t - \alpha v_t}{\alpha + c} \right)^2. \tag{3.6}$$

The first order condition is

$$2E\left(\frac{\alpha m_t + cu_t - \alpha v_t}{\alpha + c}\right)\frac{\alpha}{\alpha + c} = 0. \tag{3.7}$$

From (3.7), we get optimal money supply rule as

$$m_t = 0. \tag{3.8}$$

With this policy rule, the value of objective function is

$$E_m(y_t)^2 = E_m\left(\frac{cu_t - \alpha v_t}{\alpha + c}\right)^2 = \frac{c^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + c)^2}. \tag{3.9}$$

Let us now consider interest rate rule. Under this rule the central bank optimally chooses i_t and allows money supply to adjust. In order to derive, optimal interest rate, i_t, put (3.2), in (3.1). The optimization problem is now

$$\min_{i_t} E(-\alpha i_t + u_t)^2. \tag{3.10}$$

From the first order condition, we get

$$i_t = 0. \tag{3.11}$$

Putting (3.11) in the objective function, we have

$$E_i(y_t)^2 = \sigma_u^2. \tag{3.12}$$

In order to find out optimal policy rule, we just have to compare (3.9) and (3.12). We can immediately see that interest rate rule is preferred iff

$$E_i(y_t)^2 < E_m(y_t)^2 \tag{3.13}$$

which is equivalent to

$$\sigma_v^2 > \left(1 + \frac{2c}{\alpha}\right)\sigma_u^2. \tag{3.14}$$

From (3.14) it is clear that if the only source of disturbance in the economy is money market, $\sigma_v > 0$ & $\sigma_u = 0$, then the interest rule is preferred. In the case, the only source of disturbance is goods market, $\sigma_u > 0$ & $\sigma_v = 0$, then the money supply rule is preferred.

If only good market shocks are present, a money rule leads to smaller variance in output. Under interest rule, a positive IS shock leads to higher interest rate. This acts to reduce aggregate spending, thereby partially the original shock. Since, the adjustment of i automatically stabilizes output, preventing this interest rate adjustment by fixing i leads to larger output fluctuations. If only money-demand shocks are present, output can be stabilized perfectly by interest rate rule. Under a money rule, monetary shocks cause the interest rate to move to maintain money market equilibrium, which causes output fluctuations.

In the case, there is disturbances in both the markets, then the optimal policy rule depends on size of variances as well as relative steepness of IS and LM curves. The interest rate rule is more likely to be preferred when the variance of money market disturbances is larger, the LM curve is steeper (lower c) and the IS curve is flatter (bigger α). Conversely, the money supply rule is preferred if the variance of goods market shocks is large, the LM curve is flat, and the IS curve is steeper.

Currently, Bank of Canada uses interest rate tool. It conducts its monetary policy by announcing bank rate or operating band of overnight rate periodically. During 70's and 80's Bank of Canada used to target money supply. However, during 80's the demand for money function became highly unstable due to various financial innovations and Bank of Canada abandoned monetary targeting and moved to interest rate targeting.

C. Taylor Rule

Many central banks including Bank of Canada and Federal Reserve conduct their monetary policy through announcing bank rate or setting operating band for the overnight rate. It raises the question, how central banks set the bank rate?

John Taylor showed that the behavior of the federal funds interest rate in the U.S. from the mid-1980's to 1992 could be fairly matched by a simple rule of the form

$$i_t = \pi_t + 0.5(y_t - \overline{y}_t) + 0.5(\pi_t - \pi^T) + r^* \tag{3.15}$$

where π^T was the target level of average inflation (assumed to be 2% per annum) and r^* was the equilibrium level of real rate of interest (again assumed to be 2% per annum). In the equation, the nominal interest rate deviates from the level consistent with the economy's equilibrium real rate and the target inflation rate if the output gap is nonzero or if inflation deviates from target. A positive output gap leads to rise in nominal interest rate as does actual inflation higher than the target level.

The Taylor rule for general coefficients is often written as

$$i_t = r^* + \pi^T + \alpha(y_t - \overline{y}_t) + \beta(\pi_t - \pi^T). \tag{3.16}$$

A large literature has developed that has estimated Taylor rule for different countries and time-periods. The rule does quite well to match the actual behavior of overnight rates, when supplemented by the addition of lagged nominal interest rate.

D. Uncertainty About the Impact of Policy Instruments or Model Uncertainty

So far we have assumed that the central bank knows the true model of the economy with certainty or knows the true impact of its policy. Fluctuations in output and inflation arose from disturbances that took the form of additive errors. But suppose that the central bank does not know the true model with certainty or measures parameter values with error. In other words, the error terms enter multiplicatively. In this case, it may be optimal for the central bank to respond to shocks more slowly or cautiously.

To concretize this idea, suppose that the central bank's objective function is

$$L = \frac{1}{2}E_t(\pi_t^2 + \lambda y_t^2). \tag{3.17}$$

Here for simplicity, I have assumed that social welfare maximizing output, y_t^* and inflation, π_t^* are zero. Now suppose that aggregate demand evolves as follows

$$y_t = \beta_t \pi_t + e_t \tag{3.18}$$

where e_t is mean zero *i.i.d.* shock. Also assume that the central bank does not know the true β_t, but has to rely on estimated β_t. The true β is related to estimated β as follows

$$\beta_t = \overline{\beta} + v_t \tag{3.19}$$

where v_t is mean zero *i.i.d.* shock with variance σ_v^2 and $\overline{\beta}$ is the true parameter. Now suppose that the central bank observes demand shock e_t but not v_t before choosing π_t. Now the question is: what is the optimal π_t?

To derive optimal π_t, put (3.18) in (3.17), then we have

$$\min_{\pi_t} = \frac{1}{2} E_t \left[\pi_t^2 + \lambda(\beta_t \pi_t + e_t)^2 \right]. \tag{3.20}$$

The first order condition is

$$E_t(\pi_t + \lambda(\beta_t \pi_t + e_t)\beta_t) = 0. \tag{3.21}$$

Simplifying, we have

$$\pi_t = -\frac{\lambda \overline{\beta}}{1 + \overline{\beta}^2 + \sigma_v^2} e_t. \tag{3.22}$$

As one can see that the coefficient of demand shock e_t is declining in σ_v^2. This basically says that in the presence of multiplicative disturbances, it is optimal for the central bank to respond less (or more cautiously) to e_t.

4. Time Inconsistency and Targeting Rules

Empirical literature suggests that inflation is mainly accounted for by the increase in money supply at least in medium and long run. Given that inflation is costly, it raises the question, why the governments follow inflationary policy or expansionary monetary policy? One reason can be that the increase in money supply is a source of revenue for the government (seniorage). However, this explanation does not seem to very appropriate for the developed countries, where government revenue from money creation is not very important.

The other explanation is that output-inflation trade-off faced by the central banks induces them to pursue expansionary policy. When output is low, they may be tempted to increase inflation. On the other hand, when inflation is high, they may be reluctant to reduce it for the fear of reducing output. However, this explanation as stated also falls short because there is no long run trade-off between output and inflation. If there is no long-run tarde-off, why do we observe long run inflation?

Kydland and Presscott (1977) in a famous paper showed that when the central banks have discretion to set inflation and if they only face short-run output-inflation trade-off, then it gives rise to excessively expansionary policy. Intuitively, when expected inflation is low, the marginal cost of additional inflation is low. This induces central banks to increase inflation (for a given expected inflation), in order to increase output. However, the public while forming their expectation take into account the incentives of the central bank and thus do not expect low inflation. In other words, the promise of central bank to follow low inflation is not credible. Consequently, central banks' discretion results in inflation without any increase in output.

A. Time Inconsistency

Lack of credibility of central bank's low inflation policy gives rise to the problem of dynamic inconsistency of low inflation monetary policy. Idea is that the central bank would like public to believe that it will follow low inflation policy *i.e.* it will announce low inflation target. However, once the public has formed their expectation based on central bank's announcement, the central bank has incentive to increase inflation as by doing so it can increase output. Since, the central bank does not comply with its announcement, its announcement is not time-consistent. In other words, at the time of choosing actual inflation, the central bank deviates from its inflation target. Let us now formalize these ideas.

Let the objective function of the central bank be

$$L = \frac{1}{2}\lambda(y_t - \overline{y} - k)^2 + \frac{1}{2}(\pi_t - \pi^*)^2 \tag{4.1}$$

where $\overline{y}$ is the potential output, k is some constant, and π^* is socially optimal inflation rate. Here $\overline{y} + k$ stands for socially optimal output level. The deviation in the socially optimum level of output and potential output can be due to distortionary taxes or imperfections in markets.

Let the trade-off between inflation and output be given by

$$y_t = \overline{y} + a(\pi_t - \pi_t^e). \tag{4.2}$$

(4.2) is the Lucas supply curve which we discussed in chapter 1. The central chooses actual inflation, π_t, in order to minimize (4.1) subject to (4.2).

Now suppose the timing of events are as follows. The central bank first announces its target inflation rate. After the announcement of the central bank, public form their expectation about inflation rationally. Once public has formed its expectation, the central bank chooses actual inflation. The key here is that the central bank chooses actual inflation after public have formed their expectation.

Given the environment, we need to answer two questions: (i) what is the actual inflation chosen by the central bank? (ii) what is the expected inflation? We will answer these two questions under two policies – (i) full commitment and (ii) discretion. By full commitment, we mean that the central bank adheres to its announcement. By discretion, we mean that the central bank can choose actual inflation different from the announced one.

Now under the full commitment, the socially optimal policy is

$$\pi_t = \pi^* = \pi_t^e. \tag{4.3}$$

The value of objective function is

$$L_c = \frac{1}{2}\lambda k^2. \tag{4.4}$$

Under discretion, the optimal, π_t, can be derived as follows. Putting (4.2) in (4.1), we have

$$\min_{\pi_t} \frac{1}{2}\lambda(a(\pi_t - \pi_t^e) - k)^2 + \frac{1}{2}(\pi_t - \pi^*)^2. \tag{4.5}$$

The first order condition yields,

$$\lambda(a(\pi_t - \pi_t^e) - k)a = (\pi^* - \pi_t). \tag{4.6}$$

Under rational expectation and no uncertainty, $\pi_t^e = \pi_t$ and thus (4.6) becomes

$$-\lambda ak = \pi^* - \pi_t \tag{4.7}$$

which simplifies to

$$\pi_t = \pi^* + \lambda ak. \tag{4.8}$$

Time-consistent inflation, π_t, is higher than the socially optimum inflation rate, π^*, and the size of inflation bias is λak. The value of objective function under time-consistent policy is

$$L_d = \frac{1}{2}\lambda k^2 + \frac{1}{2}(\lambda ak)^2 \tag{4.9}$$

which is higher than value of the objective function under full commitment. In other words, the economy does worse-off under discretion.

Many solutions have been proposed to address the problem of time-inconsistency. One set of solution is to target some nominal variable – money supply, exchange rate, nominal income, price level, inflation etc. Next we turn to analyze inflation targeting.

B. Inflation-Targeting

Inflation targeting basically involves announcing an inflation target and increasing the weight of deviation of actual inflation from targeted inflation in the social welfare function. The idea of inflation targeting can be captured as follows.

Suppose that the target inflation rate is equal to the optimal inflation rate. Let the objective function of the central bank be

$$V = \frac{1}{2}\lambda(y_t - \overline{y} - k)^2 + \frac{1}{2}(\pi_t - \pi^*)^2 + \frac{1}{2}h(\pi_t - \pi^*)^2. \tag{4.10}$$

The last term in (4.10) is the additional penalty on the central bank. If $h = 0$, we go back to the original case. The problem of the central bank is to choose inflation rate π_t in order to minimize (4.10) subject to (4.2). Now under the full commitment, the socially optimal policy is still

$$\pi_t = \pi^*. \tag{4.11}$$

Under inflation-targeting regime, the optimal, π_t, can be derived as follows. Putting (4.2) in (4.10), we have

$$\min_{\pi_t} \frac{1}{2}\lambda(a(\pi_t - \pi^e) - k)^2 + \frac{1}{2}(\pi_t - \pi^*)^2 + \frac{1}{2}h(\pi_t - \pi^*)^2. \tag{4.12}$$

The first order condition yields,

$$\lambda(a(\pi_t - \pi_e) - k)a = (\pi^* - \pi_t) - h(\pi_t - \pi_t^*). \tag{4.13}$$

Under rational expectation, $\pi_t = \pi_t^e$ and thus (4.13) becomes

$$-\lambda ak = (1+h)\pi^* - (1+h)\pi_t \tag{4.14}$$

which simplifies to

$$\pi_t = \pi^* + \frac{\lambda ak}{1+h}. \tag{4.15}$$

By comparing (4.15) with (4.8), we can immediately see that the size of inflation bias is smaller under inflation targeting.