Risks and Human Capital Investment

- Human Capital Investment Risk and Human Capital Investment
- Parental Income Risk and Human Capital Investment
Levhari and Weiss (1974) Model

Basic Question: What is the effect of labor market risks on an individual’s choice of human capital investment?

- Two-period model
- In the first-period, an individual chooses amount of human capital investment, $s$, and saving, $k$.
- Gross rate of interest $R$ is certain (risk-less capital investment).
- Human capital investment in the first period increases earnings/human capital in the second period.
- The human capital function is given by $\phi h(s)$ with $h_s(s) > 0$ and $h_{ss}(s) < 0$.
- $\phi$ is a random variable with mean $\overline{\phi}$ and variance, $\sigma^2_\phi$. 
Optimization Problem

The risk-averse individual’s problem is

$$\max_{c_1, c_2, s, k} \ U(c_1) + \beta EU(c_2)$$

subject to

$$c_1 + s + k = y_1 \ \& \ (1)$$

$$c_2 = \phi h(s) + Rs + y_2 \ \ (2)$$

where $y_1$ and $y_2$ are endowment incomes in period 1 and 2 respectively. $\beta$ is the discount factor and $E$ is the expectation operator.
The first order conditions are:

\[ s : \ U_c(c_1) = \beta EU_c(c_2) \phi h_s(s) \ \& \quad (3) \]

\[ k : \ U_c(c_1) = \beta REU_c(c_2). \quad (4) \]

Combining (3) and (4), we have

\[ EU_c(c_2)[\phi h_s(s) - R] = 0. \quad (5) \]
Certainty Case

Suppose that there is no uncertainty and $\phi = \bar{\phi}$. Then (5) implies that the optimal $s$ will be given by

$$\bar{\phi}h_s(s) = R.$$  \hspace{1cm} (6)

An individual will equate the marginal return from human capital investment to the rate of interest. This also characterizes efficient level of human capital investment.
Risky Human Capital Investment

Now (5) can be written as

\[ EU_c(c_2)[\phi h_s(s) - R] = [\phi h_s(s) - R]EU_c(c_2) + \text{Cov}(U_c(c_2), \phi h_s(s)) = 0. \]  

(7)

Now we have

\[ [\phi h_s(s) - R]EU_c(c_2) = -\text{Cov}(U_c(c_2), \phi h_s(s)). \]  

(8)

Since, individual is risk-averse \(\text{Cov}\) will be negative. Thus, with risk

\[ \phi h_s(s) > R. \]  

(9)

The human capital investment will be lower than the certainty case and also below the efficient level.
Kumar (2015) Model

Basic Question: What is the effect of parental income risk on the parental investment in the human capital of their children, when the human capital investment is risky?

- Two-period model
- A family consists of a parent and a child.
- Parent is altruistic and cares about the utility of child.
- In the first-period, the parent chooses amount of its saving, \( k \), and human capital investment for its child, \( s \).
- In the second period, the parent chooses amount of bequest, \( b \), for its child.
Kumar (2015) Model

- Let $y_1$ and $y_2$ be the parental endowment income in period 1 and 2 respectively.
- Gross rate of interest $R$ is certain (risk-less capital investment).
- Human capital investment in the first period increases earnings/human capital of the child in the second period.
- The human capital function of child is given by $\phi h(s)$ with $h_s(s) > 0$ and $h_{ss}(s) < 0$.
- Suppose both $y_2$ and $\phi$ are random variables with means, $\overline{y}_2$ and $\overline{\phi}$ and variances, $\sigma^2_{y_2}$ and $\sigma^2_{\phi}$ respectively.
- Denote the covariance by $\sigma_{\phi,y_2}$. 
Parental Optimization Problem

The parental problem is

$$\max_{c_1, c_2, s, k, b} U(c_1) + \beta E[U(c_2) + \delta U(c)]$$

subject to

$$c_1 + s + k = y_1$$ (10)

$$c_2 = Rs + y_2 - b \&$$ (11)

$$c = \phi h(s) + b$$ (12)

where $\delta \in (0, 1)$ is the degree of parental altruism and $c$ is consumption of child in the second period.
Optimal Choices

The first order conditions are:

\[ s : \quad U_c(c_1) = \beta \delta EU_c(c) \phi h_s(s); \]  \hspace{1cm} (13)

\[ b : \quad U_c(c_2) = \delta EU_c(c) \& \] \hspace{1cm} (14)

\[ k : \quad U_c(c_1) = \beta REU_c(c_2). \] \hspace{1cm} (15)
From (14), we have

\[ \frac{db}{dy_2} \& \frac{db}{dR} > 0 \& \frac{db}{d\phi} < 0. \]  \hspace{1cm} (16)

A higher second period parental endowment income and the rate of return on saving increases and a higher productivity of the human capital investment reduces bequest from the parent for a given level of human capital investment. The reason is that a higher endowment income and return on saving reduces the marginal cost of bequest, while a higher productivity of the human capital investment reduces the marginal benefit of bequest.
Combining (13)-(15), we have

\[ EU_c(c_2)[\phi h_s(s) - R] = 0. \]  \hspace{1cm} (17)

**Certainty Case**

Then (17) implies that the optimal \( s \) will be given by

\[ \overline{\phi} h_s(s) = R. \]  \hspace{1cm} (18)

The parent will equate the marginal return from human capital investment to the rate of interest as before.
Using the co-variance decomposition, (17) can be written as

\[ [\bar{\phi}_s(s) - R]EU_c(c_2) = -Cov(U_c(c_2), \phi h_s(s)). \] (19)

Suppose that the period utility function is homothetic, then

\[ c = (1 - M)(y_2 + Rk + \phi h(s)) \quad \text{&} \quad c_2 = M((y_2 + Rk + \phi h(s)). \] (20)

Using second order Taylor series approximation around \((\bar{y}_2 \ & \ \bar{\phi})\), we can derive

\[ Cov(U_c(c_2), \phi h_s(s)) \approx MU_{cc}(c_2^p)[h(s)\sigma_{\phi}^2 + \sigma_{y_2,\phi}]. \] (21)
(19) and (21) imply that

\[
[\phi_h(s) - R] \cdot EU_c(c_2) \approx -MU_{cc}(c_2^p)[h(s)\sigma^2_\phi + \sigma_{y_2,\phi}].
\] (22)

Now suppose that only human capital investment is risky, \(\sigma^2_{y_2} = 0\). Then (22) implies that

\[
\phi_h(s) > R.
\] (23)

Now suppose that only parental income is risky, \(\sigma^2_\phi = 0\). Then,

\[
\phi_h(s) = R.
\] (24)
Both risks

Suppose now that both the parental income and the human capital investment are risky, $\sigma^2_{y_2}$ & $\sigma^2_\phi > 0$. Then, the effect will depend on the covariance term, $\sigma_{y_2,\phi}$.

Now suppose that $\sigma_{y_2,\phi} \geq 0$. Then (22) implies that

$$\bar{\phi} h_s(s) > R.$$  \hspace{1cm} (25)

However, if $\sigma_{y_2,\phi} < 0$, it is possible that

$$\bar{\phi} h_s(s) < R.$$  \hspace{1cm} (26)

The human capital investment can be inefficiently high.