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# Capital tax, minimum wage, and labor market outcomes

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## Abstract

Often an increase in the minimum wage is accompanied by a reduction in the capital tax. This paper analyzes the effects of interactions between the minimum wage and the capital tax in the general equilibrium framework. The analysis is conducted in an inter-temporal search model in which firms post wages. A (binding) minimum wage provides a lower support for the distribution of wages. The paper finds that the interaction of these two policy instruments significantly modify labor market outcomes and welfare cost. In the presence of a binding minimum wage, a decrease in the capital tax leads to an increase in wage dispersion. In contrast, when it is not binding, a lower capital tax may reduce the dispersion in wages. A binding minimum wage magnifies the positive effects of a lower capital tax on labor supply, employment, and output. It also enhances the welfare cost of capital tax. A policy change which involves an increase in the minimum wage and a fall in the capital tax such that employment level remains constant increases welfare and output.

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## 1. Introduction

Legal minimum wage is one of the most important labor market institutions. It is widely used as a tool of redistribution in the industrialized countries. The *direct* impact of the minimum wage on dispersion of wages and employment has been extensively studied (Freeman, 1996; OECD, 1998; Brown, 1999; Dolado et al., 2000).<sup>1</sup> In this paper, I study the implications of the interaction between the minimum wage and the capital tax for the labor market outcomes and welfare.

The study is motivated by the fact that these two policy instruments are often used in conjunction. In particular, many times an increase in the minimum wage is accompanied by a decrease in the capital tax. For instance, the 1996–1997 minimum wage increase in the US from \$4.75 per hour in 1996 to \$5.15 per hour was a part of the “Small Business Job Protection Act of 1996” which included tax relief for small businesses. To quote the preamble of the act

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<sup>1</sup> Empirical evidence suggests that changes in minimum wage have substantial effect on the dispersion of wages, but its effect on employment is controversial (Machine and Manning, 1994; DiNardo et al., 1996; Freeman, 1996; OECD, 1998; Katz and Autor, 1999; Lee, 1999; Teuling, 2003; Card and Kruger, 1994). Pries and Rogerson (2005) find that minimum wage is an important factor in explaining the observed difference in the workers’ turnover between the US and the Western Europe.

(it is) “An act to provide tax relief for small businesses, to protect jobs, . . . and to amend the Fair Labor Standard Act of 1938 to increase the minimum wage rate and to prevent job loss by providing flexibility to employers . . .” (Public Law 104–188, August 20, 1996).<sup>2</sup>

In the UK the budget of 1998 included significant corporate tax cuts for “*promoting enterprise through a range of tax incentives to boost investment, small firms and research and development*” (Pocket Budget, HM Treasury).<sup>3</sup> In the same year, the National Minimum Wage Act 1998 was passed which introduced a national minimum wage of 3.60 pound per hour for the first time in the UK for “*encouraging work and making work pay.*”

In Canada where minimum wage is set by provinces, an increase in the minimum wage is frequently accompanied by a decrease in the capital tax. In British Columbia minimum wage was raised from \$6.75 per hour to \$8 per hour effective November 1, 2001. At the same time, the budgets of 2001 and 2002 significantly reduced corporate tax rates. Similarly, in Ontario the minimum wage was raised in 2003 to \$7.15 per hour (effective February 1, 2004) from \$6.85 per hour. The budget of 2003 substantially reduced small business tax rate as well general corporate tax rate.

In this paper, I study the implications of the concurrent use of these two policy instruments. In particular, I examine the following questions:

- (i) Whether the interactions between these two policy instruments are significant for the labor market outcomes and welfare?
- (ii) Whether a reduction in the capital tax mitigates the disemployment effect of an increase in the minimum wage?
- (iii) Whether such a policy change increases or decreases the dispersion of wages?
- (iv) Does such a policy change increase welfare?

I analyze these questions in a general equilibrium inter-temporal search framework. Search models are widely used to analyze labor market issues.<sup>4</sup> The model developed has a Walrasian goods market and a non-Walrasian labor market characterized by search frictions. In the model, households optimally make decisions regarding savings, labor supply, and search strategies. Firms optimally choose investment, vacancies, and the associated wage offers. In the model, unemployment and wage dispersion arise endogenously.

The wage-setting process is modeled using a variant of Burdett and Mortensen (1998) wage posting model. As in the Burdett and Mortensen model, firms post wages and both employed and unemployed workers search among the posted wages. Firms posting higher wages attract larger pool of workers and experience less turnover of workers. Consequently, firms face a trade-off between profit per worker and the size of the work-force. This allows firms to be indifferent among different wages, and non-degenerate distributions of wage offers and earnings are equilibrium outcomes even with *ex ante identical* firms and workers.<sup>5</sup>

Manning (2003) and Mortensen (2003) provide evidence that wage posting is an appropriate characterization of the wage setting process faced by non-unionized workers in the industrialized countries. The variants of Burdett and Mortensen model are widely used to explain wage dispersion (e.g. Bontemps et al., 2000; van den Berg and Ridder, 1998; Vuuren et al., 2000). In addition, the basic mechanism underlying Burdett and Mortensen model has been exploited by many other wage-posting models in order to explain wage dispersion (e.g., Coles, 2001; Postel-Vinay and Robin, 2002).

Turning to the results, I find that the effects of capital tax on the labor market outcomes and welfare crucially depend on the mandated minimum wage. The binding minimum wage amplifies the welfare cost of capital tax. In addition, it magnifies the positive effects of a reduction in the capital tax on labor supply, employment, and output.

<sup>2</sup> At the time of writing, the Fair Labor Standard Act was in the process of being revised. On January 10th, 2007, the House of Representatives passed the bill to raise the minimum wage from \$5.15 per hour to \$7.25 per hour over two years. On January 17th, 2007, the Senate Finance Committee voted to add \$8.30 billion worth of tax breaks for small business to the bill raising the minimum wage.

<sup>3</sup> Available at [www.hm-treasury.gov.uk/budget/budget\\_1998/bud98\\_pocket\\_bud.cfm](http://www.hm-treasury.gov.uk/budget/budget_1998/bud98_pocket_bud.cfm). The budget reduced corporate tax by 1 percent to 30 percent for big companies and to 20 percent for small companies.

<sup>4</sup> Search frictions arise due to imperfect information, mobility costs, heterogeneity etc. These frictions make trading costly and lead to coexistence of unemployed workers and vacancies and have important implications for economic performance and public policy (see Mortensen and Pissarides, 1999 for a review).

<sup>5</sup> Empirical evidence suggests that residual wage dispersion (dispersion of wages among similar workers in similar jobs) accounts for 40–60 percent of total dispersion of wages in industrialized countries (Katz and Autor, 1999).

A lower capital tax increases wage dispersion. But when the minimum wage is not binding, a lower capital tax may raise or lower wage dispersion.

An increase in the minimum wage significantly reduces labor supply, employment, output, the dispersion of wages, and welfare. A concurrent reduction in the capital tax can mitigate the adverse effects of a higher minimum wage on labor supply, employment, output, and welfare. In particular, if an increase in the minimum wage is combined with the reduction in the capital tax such that employment level remains constant, then output, average wage, and welfare rise. The welfare gain from a 5 percent increase in the minimum wage accompanied by a reduction in the capital tax so that there is no disemployment effect is estimated to be 0.77 percent of the steady-state level of the optimal consumption. Such a complementary change in policies also leads to a fall in the dispersion of wages, but by much less compared to the case of no complementary reduction in the capital tax.

Turning to the literature, my paper is related to Wen and Shi (1999), who study the effects of taxes and subsidies on capital accumulation and welfare in a general equilibrium search and matching framework. Their paper does not have wage dispersion and they do not study the interaction between the capital tax and the minimum wage. In addition, in their paper wage is determined through ex post bargaining rather than through wage posting. Manning (2001) studies the effect of wage tax on labor supply in Burdett–Mortensen model in a partial equilibrium framework. In his model, agents are not allowed to invest and save and job-arrival rates are fixed.

The rest of the paper is organized as follows. In the next section, I describe the environment and analyze the optimal behavior of agents. In Section 3, I define and prove the existence of a stationary equilibrium. Section 4 analyzes the effects of the capital tax, when the minimum wage is binding. Section 5 analyzes the efficiency implications of the capital tax and the minimum wage. In Section 6, I provide a numerical example to analyze the effects of the capital tax when the minimum wage is not binding and assess the quantitative significance of policy changes with respect to labor market outcomes and welfare cost. Section 7 concludes the paper. All proofs are in the appendices.

## 2. The economy

Consider an economy comprised of households and firms distributed uniformly on the unit interval. Each household consists of a large number (unit measure) of workers and is assumed to hold a single share in a fully diversified portfolio of claims on the profit earnings by firms.<sup>6</sup> Households face standard consumption-saving choice. But, opportunities to trade in the labor market arise randomly due to search friction. In particular, workers of a household may or may not have job-opportunity, and the probability of getting a job opportunity depends on search effort, availability of jobs, and plain luck. Job opportunity is defined here as an opportunity to work in a firm at a particular job. Given the friction in the labor market, households choose labor force participation and job-acceptance strategies for its workers.<sup>7</sup>

Firms also face a standard profit-maximization problem, except that finding new workers is costly due to friction in the labor market. In order to find new workers, firms have to create vacancies. Firms view their existing employees as a capital asset and use wage as an instrument to attract new workers and retain the existing ones.

Finally, there is a government which imposes capital tax and the level of minimum wage. Every period the government repatriates taxes collected to the households equally in a lump-sum fashion.

### 2.1. Job matching

Let  $N_t$ ,  $U_t$ , and  $L_t$  denote measures of employed workers, unemployed workers, and the labor force in the economy at time  $t$  respectively. Let  $V_t$  be the measure of vacancies at time  $t$ .

In the model, both employed and unemployed workers search. Unemployed workers search for suitable jobs. Employed workers work as well as search for better jobs. For analytical simplicity, I assume that both employed and

<sup>6</sup> The market for the shares is suppressed since in the representative household framework shares will not be traded in equilibrium.

<sup>7</sup> I focus on the extensive margin of labor supply. Empirical works suggest that taxes and labor market policies primarily affect labor supply through their effect on the labor force participation rather than number of hours worked, particularly for low-income workers (Heckman, 1993; Kimmel and Kniesner, 1998; Meyer, 2002). Minimum wage is likely to be binding for these workers. Blundell and MaCurdy (1999) emphasize the need to develop models which allow simultaneous choices of savings and labor force participation in order to study the effects of taxes and welfare policies. One can endogenize the number of hours worked by allowing firms to post job offers consisting of wage as well as required hours of work along the lines of Hwang et al. (1998) as well as search intensity. However, allowing intensive as well as extensive margins at the same time makes model very complicated.

unemployed workers search with equal intensity normalized to one.<sup>8</sup> Workers and vacancies are matched through a strictly increasing, concave, and constant returns to scale matching function. It is also assumed that searching employed and unemployed workers are perfect substitutes, i.e., only the aggregate measure of the workers searching matters and not their composition. These assumptions are standard in the search and matching literature. The flow of matches then is given by

$$S(V_t, N_t + U_t) = S(V_t, L_t). \quad (2.1)$$

Let  $q_t \equiv \frac{V_t}{L_t}$  be the labor market tightness. Then, the matching probability of workers is given by

$$\frac{S(V_t, L_t)}{L_t} = s(q_t) \quad \text{with} \quad \lim_{q_t \rightarrow 0} s(q_t) \rightarrow 0 \quad \& \quad \lim_{q_t \rightarrow 0} s_q(q_t) \rightarrow \infty, \quad (2.2)$$

where  $s_q(q_t)$  is the first derivative of the matching probability of workers with respect to  $q$ . The matching probability of vacancies is given by

$$\frac{s(q_t)}{q_t} \quad \text{with} \quad \lim_{q_t \rightarrow 0} \frac{s(q_t)}{q_t} \rightarrow \infty \quad \& \quad \lim_{q_t \rightarrow \infty} \frac{s(q_t)}{q_t} \rightarrow 0 \quad (2.3)$$

where the matching probability of vacancy is a decreasing function of the labor market tightness,  $q_t$ .

### 2.2. Vacancy creation, wage setting, and job destruction

Jobs within a firm can be in two states: active and vacant. An active job is one that is matched with a worker and is currently producing, while a vacant job is seeking a worker. Assume that to create and maintain a vacant job, a firm incurs a recruiting cost  $\xi$  per period in terms of goods. Once a vacancy is matched with a worker, the match starts producing from the next period, which continues until the match is dissolved.

The process of wage determination is modeled using a version of the wage-posting model developed by Burdett and Mortensen (1998). For each vacancy, a firm posts wage offer. A posted wage offer is defined as a wage contract, which fully specifies wages payable to workers for all time to come. While posting wage offers, firms take the distribution of wage offers,  $O_t(w)$ , with support,  $\mathcal{O}_t \in [\underline{w}_t, \bar{w}_t]$ , as given. Here  $\underline{w}_t$  and  $\bar{w}_t$  denote the lower and upper supports respectively. Since, employed workers also search, the distributions of wage offers and earnings (or employees) need not be same. Let  $E_t(w)$  be the distribution of wage earnings.

In order to generate unemployment, I assume that in each period a firm's active job is dissolved with probability,  $\sigma$ . Notice that an existing match can also dissolve, if the employed worker in a match receives a better job offer and leaves the match. But in this case, the employed worker does not become unemployed.

### 2.3. Households

Workers in a household can be in three states – employed, unemployed, and inactive (or out of labor force). Let  $n_t^h$ ,  $u_t^h$ , and  $l_t$  denote measures of employed, unemployed, and active workers ( $n_t^h + u_t^h = l_t \leq 1$ ) in a household. Due to random matching, individual workers face uncertainty in their matching outcomes, which induces a non-degenerate distribution of asset holdings. Heterogeneity of asset holding makes the model analytically intractable and numerically challenging. In order to avoid the problem of heterogeneity of asset holding, I assume that workers pool their resources within the household and they share equally in the utility generated by the household's consumption.

The household makes decision on behalf of workers. The construct of a large household makes the distribution of asset holdings degenerate within the households and allows the analysis of a representative household. This construct of large household is widely used in macro/search literature (Lucas, 1990; Merz, 1995; Wen and Shi, 1999; Head and Kumar, 2005; and Kumar, 2007).

<sup>8</sup> If I allow endogenous search-intensity, then corresponding to the distribution of wage earnings, there will be the distribution of search intensities, which makes the model analytically intractable and numerically challenging. This assumption also implies that the matching probabilities for both the unemployed and the employed workers are identical. Empirical evidence on the matching probabilities is mixed. Some studies find that the matching probability for unemployed workers is higher (Bowlus, 1998; Bontemps et al., 2000), while others find that the matching probabilities of both types workers are similar (van den Berg and Ridder, 1998).

The inter-temporal utility of a typical household is given by

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \mu u_t^h - \phi n_t^h] \quad \text{with} \quad u_c(c_t) > 0, \quad u_{cc}(c_t) < 0 \quad \& \quad \phi > \mu \quad (2.4)$$

where  $\beta \in (0, 1)$  is the rate of discount, and  $c_t$  is consumption at time  $t$ .  $\phi$  and  $\mu$  are disutility incurred by employed and unemployed workers respectively. The assumption of linear and separable disutility costs keeps derivations simple. The assumption of  $\phi - \mu > 0$  ensures that the reservation wage of unemployed workers is strictly positive.<sup>9</sup>  $u_c(c)$  and  $u_{cc}(c)$  are the first and the second derivatives of the utility function respectively.<sup>10</sup> To keep things simple, I assume inactive workers who want to join the labor market first join the pool of unemployed workers.<sup>11</sup>

The budget constraint faced by the representative household is given by

$$a_{t+1} \leq \int_{i \in S_{E_t}} w^i di + (1 + r_t(1 - \tau_d))a_t + (1 - \tau_d)\Pi_t + Tr_t - c_t \quad (2.5)$$

where  $a_t$  denotes the asset level,  $S_{E_t}$  denotes the set of employed workers,  $w^i$  is wage earnings of the  $i$ th employed worker,  $\Pi_t$  is the dividend received at time  $t$ , and  $Tr_t$  is the transfer payment received from the government.  $r_t$  is the rental rate of capital and  $\tau_d$ , the capital tax rate, which is constant over time.

Now, I first characterize the optimal choices of consumption and savings of the households followed by the optimal labor market decisions (labor supply and job-acceptance strategy). While making these decisions, the household takes the labor market tightness,  $q_t$ , the distribution of wage offers,  $O_t(w)$ , the distribution of wage earnings,  $E_t(w)$ , the rate of interest,  $r_t$ , and the strategies of firms and other households as given.

The household chooses the sequence  $\{c_t, a_{t+1}\} \forall t \geq 0$  in order to maximize its inter-temporal utility (2.4) subject to its budget constraint (2.5). The optimal choice of savings is characterized by the following Euler equation:

$$u_c(c_t) = \beta u_c(c_{t+1}) [1 + r_{t+1}(1 - \tau_d)]. \quad (2.6)$$

(2.6) states that at the optimum level of saving, the marginal cost of saving (LHS) equals the marginal benefit from saving (RHS).

Turning to the optimal job-acceptance strategies, it is easy to show that the job acceptance strategies prescribed by the representative household have a *reservation property*. The reservation wage of an unemployed worker,  $\underline{w}_t^h$ , satisfies

$$u_c(c_t) \underline{w}_t^h = \phi - \mu, \quad \forall t. \quad (2.7)$$

(2.7) equates the utility of working at the reservation wage of unemployed workers,  $\underline{w}_t^h$ , to the disutility of working. Thus, it is optimal for an unemployed worker to accept any wage offer  $w \geq \underline{w}_t^h$ .

The reservation wage of an employed worker is the wage he currently earns. The intuition is that the current utility of an employed worker earning higher wage is greater compared to the employed workers earning lower wages. At the same time, in the next period all employed workers face the same matching probability,  $s(q_{t+1})$ , as well as the distribution of wage offers,  $O_{t+1}(w)$ . Therefore, it is optimal for an employed worker to accept any new wage offer higher than his current wage.

In the economy all households are alike. Thus (2.7) characterizes the common reservation wage for all unemployed workers and from now on I suppress superscript  $h$  from the reservation wage. Since posting a wage offer is costly (discussed below), this also implies that no firm will post a wage offer lower than the reservation wage  $\underline{w}_t$  as it will not be able to attract any worker. Thus, an unemployed worker accepts any equilibrium wage offer.

Denote the distribution of employed workers over wages in the household as  $e_t(w)$ . Then the laws of motion for employed and unemployed workers in the household are given by:

<sup>9</sup> One can think of employed workers incurring the labor force participation cost  $\mu$  as well as the disutility from working.

<sup>10</sup> Throughout the paper, for any function  $f(x, y)$ ,  $f_i$ ,  $\forall i = x, y$  will denote the derivative of the function with respect to the argument  $i$  and the cross derivative will be denoted as  $f_{ij}$ ,  $\forall i, j = x, y$ .

<sup>11</sup> Measured outflows from inactivity show significant direct inflow to employment from inactivity (Blanchard and Diamond, 1990). However, in these cases the unemployed and the inactive are distinguished on the basis their search-intensities. Workers with low search-intensity are categorized as inactive. In my model, only those workers are inactive who do not search at all.

$$n_{t+1}^h e_{t+1}(w) \leq [1 - \sigma - s(q_t)(1 - O_t(w))]n_t^h e_t(w) + s(q_t)u_t^h O_t(w), \quad \forall w \geq \underline{w}_t, \quad (2.8)$$

$$u_{t+1}^h \leq (1 - s(q_t))u_t^h + \sigma n_t^h. \quad (2.9)$$

(2.8) specifies the law of motion of employed workers of the household with wage earnings of  $w$  and less. The term on the left-hand side is the measure of employed workers with wage earnings  $w$  and less at the beginning of period  $t + 1$ . The first term on the right-hand side is the measure of employed workers with wage earnings  $w$  and less at the beginning of the period  $t$  who remain in the same pool at the end of the period  $t$ . An employed worker leaves this pool either due to the exogenous dissolution of the match or because he receives a wage offer higher than  $w$ . The second term is the measure of unemployed workers who receive wage offers of  $w$  and less but higher than and equal to the reservation wage,  $\underline{w}_t$ .

(2.9) describes the law of motion of unemployed workers. The first term on the right-hand side is the measure of unemployed workers in the household at the beginning of time  $t$  who remain unemployed at the end of time  $t$ . The second term is the measure of employed workers who become unemployed due to exogenous dissolutions at time  $t$ .

Turning to the optimal labor supply, let  $\omega_{ut}^h$  and  $\omega_{nt}^h(w)$  be the marginal values of an unemployed worker and an employed worker at wage  $w$  respectively to the household at time  $t$ . Then,  $\omega_{nt}^h(w)$  and  $\omega_{ut}^h$  satisfy:

$$\omega_{nt}^h(w) = u_c(c_t)w - \phi + \beta \left[ \sigma \omega_{ut+1}^h + [1 - \sigma - s(q_t)(1 - O_t(w))] \omega_{nt+1}^h(w) + s(q_t) \int_w^{\bar{w}_t} \omega_{nt+1}^h(x) dO_t(x) \right]. \quad (2.10)$$

(2.10) shows that an employed worker at wage  $w$  has net utility at time  $t$ ,  $u_c(c)w - \phi$ . With probability  $\sigma$ , he becomes unemployed, with probability  $(1 - \sigma - s(q_t)(1 - O_t(w)))$ , he remains at the same wage level, and with probability  $s(q_t)(1 - O_t(w))$ , he receives a wage offer higher than  $w$ .

$$\omega_{ut}^h = -\mu + \beta \left[ (1 - s(q_t)) \omega_{ut+1}^h + s(q_t) \int_{\underline{w}_t}^{\bar{w}_t} \omega_{nt+1}^h(x) dO_t(x) \right]. \quad (2.11)$$

(2.11) shows that for an unemployed worker, the household incurs the participation cost,  $\mu$ . In the labor market, unemployed worker receives a job offer with probability,  $s(q_t)$ , and with probability,  $1 - s(q_t)$ , he remains unemployed. Given that new workers join the pool of unemployed workers, the household would choose the measure of labor force,  $l_t (\equiv n_t^h + u_t^h)$ , such that the marginal value of unemployed workers is<sup>12</sup>

$$\omega_{ut}^h = 0, \quad \forall t. \quad (2.12)$$

#### 2.4. Firms

Suppose that firms have access to strictly increasing, concave, and constant returns to scale production function  $F(k_t, n_t^f)$ , where  $k_t$  and  $n_t^f$  are measures of the capital stock and the employees respectively at time  $t$ . Let  $K_t$  be the aggregate capital-stock and  $\delta$  denote the rate of depreciation. Assume that firms discount their profits at the same discount rate,  $\beta$ , as households. Also assume that there is perfect competition in the goods and the capital markets. The profit of a firm paying wage  $w$  at time  $t$  is given by<sup>13</sup>

$$\pi_t(w) = F(k_t, n_t^f) - wn_t^f - (r_t + \delta)k_t - \xi v_t \quad (2.13)$$

where  $v_t$  is the number of vacancies created by the firm. Assume that a firm repatriates its profit to shareholders (households) each period. The law of motion for the employees at the firm paying wage  $w (\geq \underline{w}_t)$  is given by

$$n_{t+1}^f \leq (1 - \sigma - s(q_t)(1 - O_t(w)))n_t^f + \frac{s(q_t)}{q_t} \left[ \frac{U_t + O_t(w)N_t}{L_t} \right] v_t. \quad (2.14)$$

<sup>12</sup> Here, I am assuming that  $\omega_{nt}^h(w) \geq \omega_{ut}^h \geq 0$ , which is indeed the case in equilibrium. Thus, the household would have no incentive to ask an employed worker either to become unemployed or withdraw from the labor force and the constraints given in (2.8) and (2.9) would be binding.

<sup>13</sup>  $k_t, n_t^f$  and  $v_t$  depend on the wage,  $w$ , paid. To simplify notations, I have suppressed  $w$  as an argument of these variables.

The first term on the right-hand side is the number of employees at the beginning of period  $t$  who are not separated from their matches. An employed worker can leave the match either due to exogenous shock or because he receives a higher wage offer. The second term is the number of new matches formed. A wage offer of  $w$  attracts all the unemployed workers and the employed workers earning wages less than  $w$ .

A firm chooses capital stock,  $k_t$ , next period employment level,  $n_{t+1}^f$ , and the level of vacancies,  $v_t$ , and the associated wage,  $w$ , in order to maximize its inter-temporal profit

$$\max_{k_t, n_{t+1}^f, v_t, w} \sum_{t=0}^{\infty} \beta^t \pi_t(w) \quad (2.15)$$

subject to (2.14). While making these decisions, a firm takes the distributions of wage offers,  $O_t(w)$ , and earnings,  $E_t(w)$ , labor market tightness,  $q_t$ , aggregate capital stock,  $K_t$ , interest rate,  $r_t$ , and the strategies of households and other firms as given.

Denote the capital–labor ratio by  $\kappa \equiv \frac{k}{n}$ . The first order condition for the optimal choice of capital is given by

$$k_t : F_k(\kappa_t) = r_t + \delta \quad (2.16)$$

which equates the marginal product of capital to the marginal cost of hiring capital.

Turning to wage offer and the level of vacancies, let  $\omega_{nt}^f(w)$  and  $\omega_{vt}^f$  be the capital values of an active job paying wage  $w$  and a vacancy respectively. Conditional on  $w \geq \underline{w}_t$ ,  $\omega_{vt}^f$  can be recursively defined as

$$\omega_{vt}^f = \max_w -\xi + \beta \frac{s(q_t)}{q_t} \left[ \frac{U_t + E_t(w)N_t}{L_t} \right] \omega_{nt+1}^f(w) + \beta \left( 1 - \frac{s(q_t)}{q_t} \left[ \frac{U_t + E_t(w)N_t}{L_t} \right] \right) \omega_{vt+1}^f. \quad (2.17)$$

(2.17) states that the capital value of a vacant job offering wage,  $w$ , consists of recruiting cost,  $\xi$ , and the discounted expected future capital value. The vacancy can be filled with probability,  $\frac{s(q_t)}{q_t} \left[ \frac{U_t + E_t(w)N_t}{L_t} \right]$ . The probability depends not only on the matching probability of a vacancy, but also on the measure of workers (employed as well as unemployed), who would accept the wage offer. Greater the wage offer, larger is the pool of workers who would accept the wage offer. The vacancy may remain unfilled with probability  $(1 - \frac{s(q_t)}{q_t} \left[ \frac{U_t + E_t(w)N_t}{L_t} \right])$ .

The capital value of an active job with wage  $w$ ,  $\omega_{nt}^f(w)$ , is given by

$$\begin{aligned} \omega_{nt}^f(w) = & [F_n(\kappa_t) - w] \\ & + \beta [1 - \sigma - s(q_t)(1 - O_t(w))] \omega_{nt+1}^f(w) + \beta [\sigma + s(q_t)(1 - O_t(w))] \omega_{vt+1}^f. \end{aligned} \quad (2.18)$$

(2.18) states that the capital value of an active job paying wage,  $w$ , is equal to the flow value of profit per worker (first term) plus the discounted expected capital value. With probability,  $[1 - \sigma - s(q_t)(1 - O_t(w))]$ , the job will remain active next period and with probability,  $[\sigma + s(q_t)(1 - O_t(w))]$ , the job will become vacant. The probability with which an active job becomes vacant depends on the exogenous separation probability as well as the probability with which the employed worker receives a higher wage offer. Given the setup, firms paying higher wages are likely to retain workers for a longer period. Assume that the parameters are such that  $\omega_{nt}^f(w) > 0$  so that (2.14) is binding which gives next period employment level,  $n_{t+1}^f$ . As shown below this requires that  $F_n(\kappa_t) > w$ .

Finally, a firm will choose the level of vacancy such that

$$\omega_{vt}^f = 0, \quad \forall t. \quad (2.19)$$

## 2.5. The government

The government collects capital tax and repatriates the tax receipts to the households equally in a lump-sum fashion each period.

$$Tr_t = \tau_d(r_t a_t + \Pi_t). \quad (2.20)$$

### 3. Symmetric stationary equilibrium

This paper restricts its attention to an equilibrium in which consumption, output, capital stock, employment, and the distributions of wage offers and earnings are constant over time. Additionally, all households make identical choices. From now on I drop the subscript  $t$  in order to denote the endogenous variables in the stationary state. For notational simplicity, define the after-tax rate of interest,  $r_e \equiv r(1 - \tau_d)$ . The definition of  $r_e \equiv r(1 - \tau_d)$  and Eq. (2.6) imply that  $\beta = \frac{1}{1+r_e}$ .

In the stationary state, (2.7) implies that the reservation wage of unemployed workers,  $\underline{w}$ , satisfies

$$\underline{w}u_c(c) = \phi - \mu. \tag{3.1}$$

(3.1) is one of the key equations of the model. It shows that higher consumption by reducing the marginal utility of consumption increases the reservation wage.

From (2.10), I get the expression for the marginal value of employed worker at wage  $w$

$$\omega_n^h(w) = \frac{1}{r_e + \sigma + s(q)(1 - O(w))} \left[ (1 + r_e)(u_c(c)w - \phi) + s(q) \int_w^{\bar{w}} \omega_n^h(x) dO(x) \right]. \tag{3.2}$$

(2.11) implies that the marginal value of unemployed workers is

$$\omega_u^h = \frac{1}{r_e + s(q)} \left[ -(1 + r_e)\mu + s(q) \int_{\underline{w}}^{\bar{w}} \omega_n^h(x) dO(x) \right]. \tag{3.3}$$

(2.12) pins down the labor supply by household and is given by<sup>14</sup>

$$\mu = u_c(c)s(q) \int_{\underline{w}}^{\bar{w}} \frac{1 - O(x)}{r_e + \sigma + s(q)(1 - O(x))} dx. \tag{3.4}$$

By combining (2.6) and (2.16), I have

$$F_k(\kappa) = \left[ \frac{1}{\beta} - 1 \right] \left[ \frac{1}{1 - \tau_d} \right] + \delta. \tag{3.5}$$

Given the form of the production function, (3.5) implies that an increase in the capital tax rate reduces the capital–labor ratio by lowering the return on investment.

<sup>14</sup> Differentiating (3.2) w.r.t.  $w$ , I get

$$\frac{d\omega_n^h(w)}{dw} = \frac{(1 + r_e)u_c(c)}{r_e + \sigma + s(q)(1 - O(w))}.$$

By using the above expression and integration by parts, I get

$$\int_{\underline{w}}^{\bar{w}} \omega_n^h(x) dO(x) = (1 + r_e)u_c(c) \int_{\underline{w}}^{\bar{w}} \frac{1 - O(x)}{r_e + \sigma + s(q)(1 - O(x))} dx.$$

Putting the above expression in (3.3), I derive (3.4).

(2.17)–(2.19) imply that the level of vacancy for a firm offering wage  $w$  is given by<sup>15</sup>

$$\xi = \frac{s(q)}{q} \frac{\sigma}{\sigma + s(q)(1 - O(w))} \frac{F_n(\kappa) - w}{r_e + \sigma + s(q)(1 - O(w))}, \forall w. \quad (3.6)$$

### 3.1. The unemployment rate and the distribution of wage earnings

In the stationary state, the inflow to and the outflow from any employment status are equal. Thus, the measure of unemployed workers satisfies

$$N\sigma \equiv (L - U)\sigma = Us(q). \quad (3.7)$$

The left-hand side of (3.7) is the total inflow to the unemployment pool, and the right-hand side is the total outflow. Solving (3.7), I get the stationary state measure of unemployed workers,

$$U = \frac{\sigma}{\sigma + s(q)} L. \quad (3.8)$$

Similarly, the wage earnings distribution of employed workers,  $E(w)$ , satisfies

$$(L - U)E(w)[\sigma + (1 - O(w))s(q)] = Us(q)O(w). \quad (3.9)$$

The LHS of (3.9) is the outflow from the pool of workers earning wage  $w$  and less. Workers leave this pool either due to the exogenous match dissolution or because they receive wage offers higher than  $w$ . The RHS is the inflow to the pool, which comes from unemployed workers who receive the wage offer of  $w$  and less. (3.8) and (3.9) together imply that the distribution of wage earnings,  $E(w)$ , is given by

$$E(w) = \frac{\sigma O(w)}{\sigma + (1 - O(w))s(q)}. \quad (3.10)$$

### 3.2. Existence of an equilibrium

**Definition.** A symmetric stationary equilibrium (SSE) with dispersed wages is defined as a collection of variables  $A \equiv \{N, U, L, K, q, r\}$  and distributions  $B \equiv \{O(w), E(w)\}$  such that:

- (i) given  $A, B$ , and choices of firms and other households, a household's choice variables  $\{c, a, n^h, l, \underline{w}\}$  and the job-acceptance strategy for employed workers maximize its inter-temporal utility (2.4);
- (ii) given  $A, B$ , and choices of households and other firms, a firm's choice variables  $\{k, n^f, v, w\}$  maximize its inter-temporal profit (2.15);
- (iii) the measure of unemployed workers,  $U$ , is given by (3.8);
- (iv) the wage earnings distribution,  $E(w)$ , is given by (3.10);
- (v) the expected profit of each firm is equal,  $\pi(w) = \Pi, \forall w \in [\underline{w}, \bar{w}]$  where  $\bar{w}$  is the highest wage posted by firms;
- (vi) the equilibrium interest rate,  $r$ , is consistent with clearing of the capital market,  $a = K$ ;
- (vii) the household variables are equal to relevant aggregate variables  $l = L, u^h = U, n^h = N, e(w) = E(w)$ ; and
- (viii) the government balances its budget (2.20) each period.

Since, (3.6) holds for all  $w \in [\underline{w}, \bar{w}]$ , I can use it to derive three variables of interest, the labor market tightness,  $q$ , the wage offer distribution,  $O(w)$ , and the upper wage support,  $\bar{w}$ .

<sup>15</sup> By combining (3.8) and (3.10), I have

$$\frac{U + E(w)N}{L} = \frac{\sigma}{\sigma + s(q)(1 - O(w))}.$$

(2.18) and (2.19) imply that

$$\omega_n^f(w) = \frac{(1 + r_e)(F_n(\kappa) - w)}{r_e + \sigma + s(q)(1 - O(w))}.$$

By putting the above two expressions in (2.19), I derive (3.6).

The labor market tightness,  $q$ , solves<sup>16</sup>

$$\xi = \frac{s(q)}{q} \frac{\sigma}{\sigma + s(q)} \frac{F_n(\kappa) - \underline{w}}{r_e + \sigma + s(q)} \tag{3.11}$$

where reservation wage,  $\underline{w}$ , is given by (3.1) and consumption,  $c$ , is given by<sup>17</sup>

$$c = \frac{s(q)}{\sigma + s(q)} [F(\kappa) - \delta\kappa]l - \xi ql. \tag{3.12}$$

The first term in the right-hand side is the total production net of investment. The second term is the expenditure on creation of vacancies. Combining (3.4) with (3.12), I have an expression for labor supply

$$l = \frac{1}{\frac{s(q)}{\sigma + s(q)} (F(\kappa) - \delta\kappa) - \xi q} \left[ u_c^{-1} \left( \frac{\mu}{s(q) \int_{\underline{w}}^{\bar{w}} \frac{1 - O(x)}{r_e + \sigma + s(q)(1 - O(x))} dx} \right) \right]. \tag{3.13}$$

(3.13) shows that a lower capital–labor ratio,  $\kappa$ , increases labor supply by reducing the level of consumption and increasing the marginal utility of consumption. Stochastic improvement in the distribution of wage offer,  $O(w)$ , by raising the return from working increases labor supply. Similarly, a decline in the disutility cost of labor force participation,  $\mu$ , increases labor supply. An increase in the reservation wage reduces labor supply as households are less willing to work. An increase in the labor market tightness has ambiguous effect on labor supply as it raises the expected return from labor force participation but reduces the marginal utility of consumption.

Since the equilibrium level of capital–labor ratio,  $\kappa$ , is uniquely determined by (3.5), the existence of equilibrium crucially depends on whether (3.11) has strictly positive and finite solution ( $0 < q < \infty$ ). In the proof of Lemma 1 (see below), I show that for any reservation wage,  $\underline{w} \in (0, F_n(\kappa))$ , there exists a unique level of labor market tightness  $q \in (0, \infty)$ , which solves (3.11).

Using the fact that (3.6) holds for all equilibrium wage offer,  $w$ , one can derive the expression for the distribution of wage offers<sup>18</sup>

$$O(w) = \frac{r_e + 2(\sigma + s(q))}{2s(q)} \left[ 1 - \sqrt{\frac{r_e^2 + 4(\sigma + s(q))(r_e + \sigma + s(q)) \frac{F_n(\kappa) - w}{F_n(\kappa) - \underline{w}}}{(r_e + 2(\sigma + s(q)))^2}} \right]. \tag{3.14}$$

Putting  $O(\bar{w}) = 1$  in (3.14), one can solve for the upper support of the wage offer distribution,  $\bar{w}$ .  $\bar{w}$  is given by

$$\bar{w}/\underline{w} = 1 + \left[ 1 - \frac{\sigma(r_e + \sigma)}{(\sigma + s(q))(r_e + \sigma + s(q))} \right] \left( \frac{F_n(\kappa)}{\underline{w}} - 1 \right). \tag{3.15}$$

Now, I turn to the existence of equilibrium. Equations (3.1), (3.5), and (3.11)–(3.15) define a mapping  $\mathcal{T}$  of reservation wage  $\underline{w}$  into itself. For an appropriate choice of the labor force participation cost,  $\mu$ , and the disutility cost of working,  $\phi$ , in Appendix A it is shown that a fixed point of the mapping exists.

**Lemma 1.** *For any reservation wage,  $\underline{w} \in (0, F_n(\kappa))$ , the labor force participation cost,  $\mu$ , and the disutility cost of working,  $\phi$ , can be chosen so that there exists a fixed point of mapping  $\mathcal{T}(\underline{w}) = \underline{w}$ .*

The existence of SSE with dispersed wages follows directly from Lemma 1.

<sup>16</sup> Using equilibrium condition (v), one can derive level of vacancies,  $v$ , created by a firm offering wage  $w$ . (v) and Eq. (2.14) give steady state level of employment at a firm offering wage  $w$ ,  $n^f$ . Finally, using  $n^f$  and the capital–labor ratio given by (3.5), one can derive the capital–stock used by a firm. In this environment, the capital–labor ratio is identical for all firms, but they differ in terms of capital–stock, employment, and vacancies created.

<sup>17</sup> (3.12) follows from the expression for profit,  $\pi(w)$ , in (2.13), the budget constraints of the households (2.5) and the government (2.20), and the equilibrium conditions (v) and (vi).

<sup>18</sup> The wage offer distribution,  $O(w)$ , solves

$$\frac{\sigma}{\sigma + s(q)} \frac{F_n(\kappa) - \underline{w}}{r_e + \sigma + s(q)} = \frac{\sigma}{\sigma + s(q)(1 - O(w))} \frac{F_n(\kappa) - w}{r_e + \sigma + s(q)(1 - O(w))}, \quad \forall w.$$

This is a quadratic function of  $O(w)$ . The positive root of the above function gives (3.14).

**Proposition 1.** *For any reservation wage,  $\underline{w} \in (0, F_n(\kappa))$ , there exists a symmetric stationary equilibrium with dispersed wages characterized by (3.1), (3.5), (3.8), (3.10), (3.11)–(3.15).*

Before I analyze the general equilibrium effects, it is instructive to discuss the effects of changes in the capital–labor ratio, the labor market tightness, and the reservation wage on the distributions of wage offers and earnings in the partial equilibrium framework.

From (3.10) and (3.14) it is clear that, other things remaining the same, a higher capital–labor ratio leads to stochastic improvement in the distributions of wage offers and earnings, and thus a higher average wage offers and earnings. This happens because a higher capital–labor ratio by increasing the marginal product of labor induces firms to create a larger proportion of high wage jobs. Similarly, a higher labor market tightness, which increases the matching probabilities of workers and thus the turnover of workers, reduces the market power of firms.<sup>19</sup> Consequently, firms are forced to create a larger proportion of high wage jobs. A higher reservation wage, similarly, leads to stochastic improvement in the distribution of wage offers.

Turning to the effects on the support, from (3.15) it is clear that, holding labor market tightness and the reservation wage constant, an increase in the capital–labor ratio widens the support of distributions of wage offers and earnings in the sense that  $\bar{w}/\underline{w}$  rises. This happens because a higher capital–labor ratio raises the marginal productivity of workers, which increases the upper support of the distributions of wage offers and earnings, for a given lower support (reservation wage). An increase in the level of labor market tightness also widens the support. A higher level of labor market tightness increases the matching probability of workers, which reduces the market power of firms and the upper support rises.

An increase in the reservation wage narrows the support, holding other things constant. A higher reservation wage directly raises the upper support, but the increase is not as much as the increase in the reservation wage. This happens because firms posting the highest wage do not face any competition from other firms to retain their workers, while firms posting lower wages do. Consequently, firms posting the highest wage need not increase their wages as much as the firms posting the lowest wage. Thus, the support of the distributions narrows.

In the next section, I introduce the minimum wage and analyze the effects of capital tax when the minimum wage is binding.

#### 4. Minimum wage and capital tax

In this section, I discuss the effects of capital tax when the minimum wage,  $w_{\min}$ , is binding in the sense that it exceeds the equilibrium reservation wage of unemployed workers,  $\underline{w}$ . With the binding minimum wage, the lower support of the distributions of wage offers and earnings is given by the minimum wage and not by the reservation wage of unemployed workers. Consequently, the lower support of the distributions of wage offers and earnings become independent of the capital tax. In this case, the equilibrium labor market tightness,  $q$ , solves

$$\xi = \frac{s(q)}{q} \frac{\sigma}{\sigma + s(q)} \frac{F_n(\kappa) - w_{\min}}{r_e + \sigma + s(q)}. \quad (4.1)$$

Under the assumption that  $F_n(\kappa) > w_{\min}$ , one can easily show that there exists an equilibrium with dispersed wages.<sup>20</sup>

Regarding the effects of the capital tax, it affects equilibrium variables only through the marginal product of labor. Its effects in the presence of binding minimum wage are summarized below.

#### Proposition 2.

(a) *A decline in the capital tax rate increases the labor market tightness and consumption and reduces the unemployment rate.*

<sup>19</sup> By market power of firms, I mean that firms pay wages lower than the marginal product of labor. This terminology is widely used (Mortensen, 2003; Manning, 2003). However, in this environment workers also possess market power in the sense that most of them receive wages in excess of their reservation wage.

<sup>20</sup> Empirical evidence suggests that there exists “spike” in the wage earnings distribution at the minimum wage. The basic Burdett–Mortensen model cannot produce the spike, as there cannot be any mass point in the equilibrium wage distribution. However, one can generate “spike” by introducing heterogeneity in the productivity of firms.

- (b) A fall in the capital tax rate leads to stochastic improvement in the distributions of wage offers,  $O(w)$ , and earnings,  $E(w)$ . Thus, the average wage offer and earnings rise.
- (c) A fall in the capital tax rate widens the support of the distributions of wage offers and earnings in the sense that,  $\bar{w}/w_{\min}$ , rises.

A lower capital tax rate by increasing investment and the marginal productivity of workers raises the return from creating vacancy. Thus the labor market tightness increases and unemployment rate falls. A higher marginal productivity of workers and labor market tightness lead to stochastic improvement in the distributions of wage offers and earnings. Also the upper support of the distributions rises. Since the lower support does not change, the support widens.

The stochastic improvement in the wage offer distribution, a higher labor market tightness, and a wider support increase the return from participating in the labor market for a given level of consumption and the marginal utility of consumption. Thus for equilibrium to be achieved, consumption rises (see Eq. (3.4)). However, labor supply and thus employment may rise or fall. This happens because higher consumption discourages labor force participation, but greater return from participation encourages it. This makes the impact of capital tax on output ambiguous as well.

Now, I discuss the effects of changes in the minimum wage. The effects of change in minimum wage are summarized below.

**Proposition 3.**

- (a) An increase in the minimum wage reduces the labor market tightness and increases the unemployment rate.
- (b) An increase in the minimum wage narrows the support of the distributions of wage offers and earnings in the sense that,  $\bar{w}/w_{\min}$ , falls.

In the model, an increase in the minimum wage lowers the labor market tightness and raises the unemployment rate by reducing the return from creating vacancies. It also reduces the range and the ratio of upper and lower supports of the distributions of wages and earnings. Changes in the minimum wage affects the entire distribution and not only the lower support, i.e., it has substantial spillover effect, which is supported by the empirical evidence (DiNardo et al., 1996; Lee, 1999; Teuling, 2003).

An increase in the minimum wage may raise or lower the average wage offer, earnings, and labor supply. Increase in the minimum wage directly leads to stochastic improvement in the distributions of wage offer and earnings. But, a decline in the labor market tightness has the opposite effect. Since, a higher minimum wage may or may not lead to stochastic improvement in wage offer distribution, it may raise or lower labor supply and thus employment and output.<sup>21</sup>

These results have interesting policy implications with respect to the average wage offer and earnings and the dispersion of wages. A higher minimum wage increases unemployment rate and reduces the dispersion of wages. On the other hand, a lower capital tax reduces unemployment rate and increases wage dispersion. The following proposition summarizes the effects of a policy change which involves a rise in the minimum wage and a decline in the capital tax such that the unemployment rate does not change.

**Proposition 4.** An increase in the minimum wage and a corresponding decrease in the capital tax rate such that the unemployment rate remains unchanged leads to higher average wage offers and earnings. It also narrows the support of the distributions of wage offers and earnings.

Average wage offer and earnings rise both due to increase in the minimum wage and the marginal productivity of workers. Such unemployment rate neutral change in the minimum wage and the capital tax also reduces the ratio of marginal product of labor and the minimum wage leading to narrower support.

In the next section, I analyze the effects of these two policy instruments on efficiency.

<sup>21</sup> In the numerical example presented in Section 6.2, a higher minimum wage reduces labor supply, employment, and output.

## 5. Efficiency

In this section, I derive the social planner allocations and compare them with the market allocations. In deriving the social planner allocations, I assume that the social planner faces labor market frictions similar to households and firms. The social planner chooses the sequence of  $\{c_t, \theta_t, l_t, n_{t+1}^h, k_{t+1}\}$  to maximize

$$\max_{c_t, \theta_t, l_t, n_{t+1}^h, k_{t+1}} \sum_{t=0}^{\infty} \beta^t [u(c_t) - \mu(l_t - n_t^h) - \phi n_t^h]$$

subject to the law of motion of employment

$$n_{t+1}^h \leq (1 - \sigma)n_t^h + s(q_t)(l_t - n_t^h) \quad (5.1)$$

and the law of motion of capital stock.

$$k_{t+1} \leq F(k_t, n_t^h) + (1 - \delta)k_t - c_t - \xi q_t l_t. \quad (5.2)$$

As shown in Appendix B, in the steady state the capital–labor ratio is given by

$$F_k(\kappa) = r + \delta \quad (5.3)$$

where I use the fact that  $\beta = \frac{1}{1+r}$  when the capital tax  $\tau_d = 0$ . The labor market tightness,  $q$ , solves

$$\xi = s_q(q) \frac{\sigma}{\sigma + s(q)} \frac{F_n(\kappa) - \frac{\phi - \mu}{u_c(c)}}{r + \sigma + s(q)}. \quad (5.4)$$

(5.4) equates the social marginal cost of creating vacancy with the expected social marginal benefit. The social planner takes into account the fact that an increase in the labor market tightness reduces the matching probability of workers. The labor supply is given by

$$\mu + \xi q u_c(c) = s(q) u_c(c) \frac{F_n(\kappa) - \frac{\phi - \mu}{u_c(c)}}{r + \sigma + s(q)}. \quad (5.5)$$

(5.5) equates the social marginal cost of labor supply with its expected marginal benefit. The social marginal cost includes not only the labor force participation cost,  $\mu$ , but also the utility cost of creating vacancy,  $\xi q u_c(c)$ . Consumption is given by (3.12).

For convenience, I reproduce here the corresponding equations for market allocations for these variables in the absence of capital tax and minimum wage. With no capital tax,  $r_e = r$ . The market capital–labor ratio is given by (5.3), i.e., in the absence of capital tax the social planner's and the market choices of capital–labor ratio coincide.

The market labor market tightness is given by

$$\xi = \frac{s(q)}{q} \frac{\sigma}{\sigma + s(q)} \frac{F_n(\kappa) - \frac{\phi - \mu}{u_c(c)}}{r + \sigma + s(q)}. \quad (5.6)$$

The market labor supply is given by

$$\mu = u_c(c) s(q) \int_{\underline{w}}^{\bar{w}} \frac{1 - O(x)}{r + \sigma + s(q)(1 - O(x))} dx. \quad (5.7)$$

Comparing (5.4) and (5.6), it is immediately clear that for a given labor supply,  $l$ , the social planner's choice of labor market tightness coincides with the market labor market tightness only in a special case when  $s_q(q) = \frac{s(q)}{q}$  or the elasticity of matching probability with respect to labor market tightness,  $\eta(q) \equiv \frac{q s_q(q)}{s(q)} = 1$ . However, given the assumption that  $\frac{s(q)}{q}$  is decreasing in  $q$ ,  $\eta(q) < 1$ . This implies that for a given  $l$ , the market labor market tightness is higher than the socially optimal labor market tightness, i.e., firms create too many vacancies.

A comparison of (5.5) and (5.7) shows that for a given labor market tightness,  $q$ , the socially optimal labor supply can be higher or lower than the market labor supply. This happens because both the social marginal cost and the

social marginal benefit of labor supply are higher than the market marginal cost and benefit. The social marginal cost is higher because the social planner takes into account the cost of creating vacancies. The social marginal benefit is higher due to the market power of firms which reduces the expected benefit from working in the market economy.<sup>22</sup>

In the case, the utility cost of creating vacancy is small,  $\xi qu_c(c) \approx 0$ , then the market labor supply is too low relative to the social optimal level for a given level of labor market tightness. This happens because the market return from working is too low.

Turning to policy, the above discussion suggests that there can be inefficiency at two margins, vacancy creation and labor supply. Also in the absence of capital tax, market and social optimal levels of capital–labor ratio coincide. From this it immediately follows that in order to achieve social optimal allocation, the capital tax should be zero, i.e.  $\tau_d = 0$ .

Now the question is whether the minimum wage can achieve socially optimal labor market tightness and labor supply. Minimum wage cannot simultaneously achieve both socially optimal labor market tightness and labor supply. It can potentially achieve either socially optimal labor market tightness or labor supply.

From (5.4) and (5.6), one can derive the minimum wage which achieves socially optimal labor market tightness for a given labor supply,  $l$ . Such a minimum wage is given by

$$w_{\min} = (1 - \eta(q))F_n(\kappa) + \eta(q)\frac{\phi - \mu}{u_c(c)}. \quad (5.8)$$

The socially optimal level of minimum wage is the weighted average of the marginal product of labor and the reservation wage. The weights are determined by the elasticity of the matching probability of workers with respect to labor market tightness.

A comparison of (5.5) and (5.7) suggests that for a small utility cost of creating vacancy,  $\xi qu_c(c) \approx 0$ , the minimum wage cannot achieve socially optimal level of labor supply for a given labor market tightness,  $q$ . This follows from the fact that expected return from working in the market is too low (see footnote 22). When  $\xi qu_c(c)$  is not small then the minimum wage may be used to achieve socially optimal level of labor supply for a given level of labor market tightness. Such an optimal level of the minimum wage implicitly solves

$$\int_{w_{\min}}^{\bar{w}} \frac{1 - O(x)}{r + \sigma + s(q)(1 - O(x))} dx = \frac{F_n(\kappa) - \frac{\phi - \mu}{u_c(c)}}{r + \sigma + s(q)} - \frac{\xi q}{s(q)}. \quad (5.9)$$

Note that though the minimum wage can potentially achieve socially optimal level of labor market tightness or labor supply, it does not mean that on its own it necessarily improves welfare. For example, a minimum wage that achieves the socially optimal labor market tightness can further move away the economy from the socially optimal labor supply. The welfare gain from the socially optimal labor market tightness may be more than offset by the increased sub-optimality of labor supply.

In the next section, I provide a numerical example to further examine the interactions between the minimum wage and the capital tax and the welfare implications. I also analyze the effects of capital tax when the minimum wage is not binding. This exercise provides additional insight on the interactions between the minimum wage and the capital tax.

With a non-binding minimum wage, due to algebraic complexity one cannot derive results analytically. This happens because of the endogeneity of the reservation wage,  $\underline{w} = \frac{\phi - \mu}{u_c(c)}$ . If the effect of capital tax on the reservation wage

<sup>22</sup> This follows from the fact that

$$\int_{\underline{w}}^{\bar{w}} \frac{1 - O(x)}{r + \sigma + s(q)(1 - O(x))} dx < \frac{F_n(\kappa) - \frac{\phi - \mu}{u_c(c)}}{r + \sigma + s(q)}.$$

The value of integrand is bounded above by  $\frac{1}{r + \sigma + s(q)}$ . Thus the value of the integral on the left-hand side is bounded by

$$\frac{\bar{w} - \underline{w}}{r + \sigma + s(q)} < \frac{F_n(\kappa) - \frac{\phi - \mu}{u_c(c)}}{r + \sigma + s(q)}.$$

The inequality follows from (3.15).

is small,  $\frac{dw}{d\tau_d} \approx 0$  (or if one considers the limiting case of risk-neutral households), then the results summarized in Proposition 2 regarding the effects of changes in the capital tax case go through. For the analysis of the general case, I solve the model numerically.

## 6. A numerical example

In order to solve the model numerically, I need to specify functional forms and parameter values. Choices of functional forms and parameter values are discussed below. I take time period to be a quarter and assume that the period utility function is logarithmic.

$$u(c) = \ln c. \tag{6.1}$$

I assume Cobb–Douglas forms of the production function and the matching function, which are quite common in the literature.

$$F(k, n) = k^\alpha n^{1-\alpha}, \tag{6.2}$$

$$s(q) = q^\gamma. \tag{6.3}$$

The values of parameters are given in Table 1. There are two sets of parameters: parameters for which estimates are available and parameters for which estimates are not available. To pin down values of the second set of parameters, I use the estimates of the statistics of the labor market experience of low-skilled young adult workers during 1980s in the United States provided by Bowlus (1998). This is done for two reasons. Firstly, minimum wage is likely to be binding for this group of workers (Abowd et al., 2000).<sup>23</sup> Secondly, inflows and outflows from labor force is very important for this group of workers (Blundell and MaCurdy, 1999; Abowd et al., 2000).

Carey and Tchilinguirian (2000) provide the estimate of the average capital tax rate for the United States for the 1980s. In line with their estimate, I set the capital tax rate to be 0.292. The risk-free rate of interest in the United States is estimated to be about 1% per quarter (Kydland and Prescott, 1982). I set the rate of discount,  $\beta$ , to be 0.9934 to match this rate. The value of depreciation rate,  $\delta$ , is chosen so that the capital-output ratio is equal to 10 per quarter, a value reported for the US economy in Kydland and Prescott (1982). Similar to Kydland and Prescott (1982), I set the elasticity of output with respect to capital,  $\alpha$ , to be equal to 0.36. It must be noted that unlike the economy analyzed in the Kydland and Prescott (1982), in a search economy  $1 - \alpha$  does not correspond to the share of labor income in the output. However, in the example considered here the total recruiting cost is small and the share of labor income in the output is about 61 percent.

To set the values of other parameters (mainly related to labor market), I use the estimates of the labor market statistics provided by Bowlus (1998). Bowlus (1998) using NLSY panel data estimates the structural parameters of a variant of Burdett and Mortensen (1998) model. In her model, the matching probabilities of unemployed and employed workers as well as the size of labor force are fixed. She studies employment histories of young adult white

Table 1

Parameter values

Time period	1 Quarter
Elasticity of output w.r.t. capital, $\alpha$	0.36
Capital tax, $\tau_d$	0.292
Rate of discount, $\beta$	0.9934
Rate of depreciation, $\delta$	0.026
Exogenous separation rate, $\sigma$	0.05
Recruiting cost, $\xi$	0.8
Labor force participation cost, $\mu$	0.571
Disutility from working, $\phi$ ,	0.99
Elasticity of vacancy w.r.t. No. of matches, $\gamma$	0.36
Minimum wage, $w_{\min}$	0.95

<sup>23</sup> Mellor (1987) provides evidence that for those workers earning minimum wage and less in the US in 1986, about 23 percent were 20 to 24 years old.

Table 2  
Estimated statistics and parameter values (Bowlus, 1998)

Unemployment rate	0.0912
$w_{\min}$ /Average wage earnings	0.4272
Exogenous separation rate, $\sigma$	0.05
C.V. of wage earnings	0.3995

male workers between the ages 20 and 25 years over the years 1986 and 1987. Sample is restricted to full-time workers who have completed at least nine years of education but no more than a high school degree. The means for the sample and the estimated search parameters are in Table 2.<sup>24</sup>

I set the exogenous separation rate,  $\sigma$ , equal to the rate estimated by Bowlus (1998). I choose the minimum wage,  $w_{\min}$ , such that the ratio of the minimum wage to the average wage earnings is equal to 0.43 as reported by Bowlus (1998). Shimer (2005) estimates the recruitment cost,  $\xi$ , to be 21 percent of the average labor productivity. I set the recruitment cost,  $\xi$ , equal to 0.8 to match this number. The elasticity of matching function with respect to labor market tightness,  $\gamma$ , together with the recruitment cost and the minimum wage determines the unemployment rate. I set  $\gamma$  to be 0.36 so that the model generates unemployment rate of 9.12 percent.

Finally, I need to choose parameter values of the disutility cost of labor force participation,  $\mu$ , and the disutility from working,  $\phi$ . The labor force participation rate for this group of workers is estimated to be 85 percent. I set,  $\mu$ , to 0.571 in order to match the observed labor force participation rate. The final parameter, I need to set is the disutility from working,  $\phi$ , which determines the reservation wage,  $\underline{w}$ . Shimer (2005) estimates the reservation wage to be 40 percent of the average wage earnings. I set  $\phi$  to 0.99 so that the model in the absence of the binding minimum wage generates the ratio of the reservation wage to the average wage earnings equal to 0.4. Note that the reservation wage generated is lower than the minimum wage so that in the benchmark model the minimum wage is binding.

With these functional forms and parameter values the benchmark model (with the binding minimum wage) generates the coefficient of variation of wage earnings of 0.11 which is roughly twenty-eight percent of the estimated coefficient of variation. Low wage dispersion in my economy is due to the fact that there is no productivity difference across firms. By introducing heterogeneity across firms, one can generate realistic value of dispersion of wage earnings (Bowlus, 1998; Mortensen, 2003). Bowlus (1998) requires seven types of firms in order to generate realistic shape of wage distribution.

In what follows, first I consider the effects of changes in the capital tax with and without binding minimum wage. Then, I analyze the effects of changes in the minimum wage. Finally, I discuss the welfare implications of various policy changes.

### 6.1. The effects of changes in the capital tax

To analyze the effects of changes in the capital tax, I reduce the capital tax rate by five percent and then calculate the elasticities of various important variables. Table 3 presents these elasticities.

The table shows that a reduction in the capital tax rate increases labor supply, employment, and output. However, elasticities of these variables are much larger, when the minimum wage is binding. The table suggests that a 5-percent fall in the capital tax rate increases labor supply and employment by 0.43 percent and output by 0.75 percent, when the minimum wage is binding. Output rises both because of increase in the capital–labor ratio and employment. When the minimum wage is not binding, a 5-percent fall in the capital tax rate increases labor supply and employment by 0.2 percent and output by 0.5 percent. A lower capital tax rate reduces unemployment rate in both cases marginally.

A lower capital tax increases total wage. The elasticity of total wage is higher when the minimum wage is binding because of the higher elasticity of employment. A lower capital tax rate increases average wage earnings in both cases marginally. The reduction in the capital tax rate increases the dispersion of wages. In contrast, when the minimum wage is not binding, the dispersion of wages falls marginally. Numerical experiments show that the changes in the capital tax has marginal effect on the dispersion of wages when the minimum wage is not binding.

<sup>24</sup> Bowlus (1998) estimates parameter values at weekly frequency. I converted the estimates to quarterly frequency.

Table 3  
Effects of reducing the capital tax

	Binding	Non-binding
<i>Elasticities with binding and non-binding minimum wage</i>		
Output	0.1487	0.1029
Labor supply	0.0854	0.0407
Employment	0.0879	0.0422
Unemployment rate	−0.0256	−0.0150
Lower support	0	0.0627
<i>Wage earnings</i>		
Total wage	0.1479	0.1039
Average wage	0.0597	0.0615
Upper support/Lower support	0.0606	−0.0020
Coeff. of variation	0.0313	−0.0083

The intuition for the differential impact of the reduction in the capital tax rate under two cases is quite simple. The capital tax affects equilibrium variables directly through investment and indirectly through reservation wage. A lower capital tax rate by increasing investment and the marginal product of labor raises the return from creating vacancies. Labor demand rises. Greater competition for workers induces firms to create a larger proportion high wage jobs as well as raise the highest wage offered. All these factors lead to higher labor supply, output, employment, and lower unemployment rate. The average wage offers and earnings as well as the dispersion of wages rise.

A lower capital tax rate by increasing the demand for labor as well as higher average wage offer also increases the reservation wage. A higher reservation wage partly offsets the effects of capital tax operating through higher investment. However, when the minimum wage is binding, the capital tax does not affect equilibrium variables through the reservation wage. The result is that the effects of capital tax on labor supply, output, and total wage are magnified. The effect on wage dispersion may be reversed.

## 6.2. The effects of changes in the minimum wage

To analyze the effects of changes in the minimum wage, I increase the minimum wage by five percent. I consider two cases. In the first case, only the minimum wage is increased. In the second case, I reduce the capital tax as well so that the increase in the minimum wage has no disemployment effect. In order to have no disemployment effect, the capital tax rate must fall to 0.155 from 0.292. Table 4 presents the elasticities of various important variables.

The table shows that the effects of an increase in the minimum wage are very different when accompanied by the offsetting change in the capital tax. When there is no offsetting reduction in the capital tax, a 5-percent increase in the minimum wage reduces output, employment, and the labor supply by about 3.65 percent. The estimated elasticity for disemployment is  $-0.7351$ , which is in the middle range of the elasticities estimated for the low-wage workers in the United States. Neumark and Wascher (2003) estimate this elasticity to be  $-0.40$ , while Pries and Rogerson (2005) estimate it to be between  $-1.5$  to  $-2$ .

Table 4  
Effects of increasing the minimum wage

Capital tax	No change	Offsetting change
<i>Elasticities with and without offsetting changes in the capital tax</i>		
Output	−0.7351	0.4984
Labor supply	−0.7182	−0.0047
Employment	−0.7351	0.0
Unemployment rate	0.1745	−0.0407
<i>Wage earnings</i>		
Total wage	−0.7066	0.5213
Average wage	0.0296	0.5213
Upper support/Lower support	−0.9506	−0.4749
Coeff. of variation	−0.6356	−0.3667

An increase in the minimum wage increases the unemployment rate. The average wage earnings increases marginally. The total wage falls significantly, which is largely due to fall in employment. A higher minimum wage reduces the dispersion of wage earnings significantly. A 5-percent increase in the minimum wage reduces the ratio of the upper and the lower supports by about 4.75 percent and the coefficient of variation by about 3.20 percent.

An increase in the minimum wage when accompanied by the offsetting decrease in the capital tax increases output and total wage. The average wage earnings rises significantly. A 5-percent increase in the minimum wage in this case increases output, total wage, and the average wage by about 2.5 percent. The unemployment rate declines marginally. The dispersion of wage earnings still falls but by much less. The estimated elasticities are roughly half of the case in which there is no offsetting change in the capital tax.

### 6.3. Welfare

Now I consider the welfare costs of various policies. Let  $\Delta c$  denote the quantity of consumption required to give a representative household the same period utility as it would receive at the optimum. The quantity satisfies:  $\ln(c + \Delta c) - \mu u - \phi n^h = \ln(c^*) - \mu u^* - \phi n^{h*}$ , where  $c^*$ ,  $u^*$ , and  $n^{h*}$  are consumption, unemployment, and employment levels chosen by the social planner. The welfare cost of various policies,  $wc$ , expresses this amount as a percentage of  $c^*$

$$wc = \frac{\Delta c}{c^*} * 100. \tag{6.4}$$

The welfare costs are presented in Table 5.

The first panel of Table 5 shows that in the absence of policy intervention (no capital tax and the minimum wage) the welfare cost of market allocations is 1.07 percent of the optimal consumption. The welfare cost rises to 1.25 percent, when the minimum wage is set at the benchmark level but there is no capital tax. Thus the additional welfare cost (relative to the welfare cost of no policy intervention) is 0.18 percent. The welfare cost of the capital tax is very high. The additional cost of capital tax set at the benchmark level is 1.74 percent (2.81% – 1.07%), when there is no binding minimum wage. The combination of these two policies magnifies the welfare cost. The additional welfare cost of the combined policies (the benchmark case) is 2.32 percent (3.39% – 1.07%).

The capital tax is welfare reducing for the reasons discussed in Section 5. In the numerical example, the minimum wage is also welfare reducing. This happens because without the binding minimum wage the labor market tightness is too high and the labor supply and output (consumption) are too low relative to their respective optimal levels for the given set of parameter values. The introduction of the binding minimum wage reduces the labor market tightness and improves welfare. However, it also reduces the labor supply and output (consumption) which more than offsets the welfare gain arising from the lower labor market tightness.

The second panel of the table shows that a reduction in the capital tax is welfare improving both when the minimum wage is binding and when it is not. This happens largely due to an increase in the labor supply and output (consumption).

The third panel shows that an increase in the minimum wage without an offsetting reduction in the capital tax is welfare reducing. Welfare increases when an increase in the minimum wage is accompanied by an offsetting decrease

Table 5  
Welfare cost

	<i>wc</i> (in %)
Benchmark case	3.39
Capital tax and non-binding minimum wage	2.81
Zero capital tax and binding minimum wage	1.25
No policy intervention	1.07
<i>Reduction in the Capital Tax</i>	
Binding minimum wage	3.24
Non-binding minimum wage	2.69
<i>Increase in the Minimum Wage</i>	
No offsetting change in the capital tax	3.89
Offsetting change in the capital tax	2.62

in the capital tax as output (consumption) rises and the total recruitment cost falls without much impact on the labor supply. In this case, the welfare gain is estimated to be 0.77 percent (3.39%–2.62%) of the optimal consumption relative to the benchmark case.

## 7. Conclusion

The paper studied the effects of capital tax on the dispersion of wages, output, labor supply, and welfare in the presence of mandated minimum wage in a general equilibrium inter-temporal search framework. I find that a binding minimum wage significantly modifies the effects of capital tax on labor market outcomes and welfare. The presence of the minimum wage increases the welfare cost of capital tax. In addition, the effects of capital tax on labor supply, output, and total wage earnings are magnified. A lower capital tax increases the dispersion of wages. When the minimum wage is not binding, a lower capital tax may raise or lower the dispersion of wages. An increase in the minimum wage accompanied by a reduction in the capital tax can increase output and welfare.

In the paper, all jobs and workers are identical in productivity. Wage dispersion is *within-skill* or *residual* wage dispersion (dispersion of wages among similar workers in similar jobs). Though the residual wage dispersion accounts for one-third to two-third of total wage dispersion in the industrialized countries, it leaves out many important aspects of wage dispersion such as skill-premium and mismatch of skills. In future work, I would like to analyze the general equilibrium effects of taxes on these aspects of wage dispersion. I would also like to study the effects of taxes in the presence of other labor market institutions such as unemployment insurance and unions.

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## Appendix A

**Proof of Lemma 1.** The capital–labor ratio,  $\kappa$ , is uniquely determined by (3.5). Now choose a reservation wage,  $w^* \in (0, F_n(\kappa))$ . (3.11) can be rewritten as

$$\xi(\sigma + s(q))(r_e + \sigma + s(q)) = \sigma \frac{s(q)}{q} [F_n(\kappa) - w^*]. \quad (\text{A.1})$$

Given the assumptions that  $s_q(q) > 0$ , the LHS of (A.1) is strictly increasing in  $q$ . Further  $\lim_{q \rightarrow 0} s(q) = 0$ , which implies  $\lim_{q \rightarrow 0} LHS = \xi\sigma(r_e + \sigma)$ . Under the assumption that  $\frac{s(q)}{q}$  is decreasing in  $q$ , the RHS of (A.1) is strictly decreasing in  $q$ . In addition, given  $\lim_{q \rightarrow 0} \frac{s(q)}{q} = \infty$  and  $\lim_{q \rightarrow \infty} \frac{s(q)}{q} = 0$ ,  $\lim_{q \rightarrow 0} RHS = \infty$  and  $\lim_{q \rightarrow \infty} RHS = 0$ . Thus for a given  $w^*$  there exists a unique labor market tightness,  $q(w^*)$ .

Combining (3.1) and (3.4), I get

$$\underline{w} = \frac{\phi - \mu}{\mu} s(q) \int_{\underline{w}}^{\bar{w}} \frac{1 - O(x)}{r_e + \sigma + s(q)(1 - O(x))} dx. \quad (\text{A.2})$$

Given  $\kappa$ ,  $w^*$ , and  $q(w^*)$ , (3.14) and (3.15) determine the distribution of wage offer,  $O(\cdot; w^*)$ , and upper wage support,  $\bar{w}(w^*)$ , respectively. Now denote

$$\mathcal{T}(w^*) = \frac{\phi - \mu}{\mu} s(q(w^*)) \int_{w^*}^{\bar{w}(w^*)} \frac{1 - O(x; w^*)}{r_e + \sigma + s(q(w^*))(1 - O(x; w^*))} dx. \quad (\text{A.3})$$

Then by choosing  $\phi$  and  $\mu$  such that

$$\frac{\mu}{\phi - \mu} = \frac{s(q(w^*))}{w^*} \int_{w^*}^{\bar{w}(w^*)} \frac{1 - O(x; w^*)}{r_e + \sigma + s(q(w^*))(1 - O(x; w^*))} dx \quad (\text{A.4})$$

and putting (A.4) in (A.2), I have

$$w^* = \mathcal{T}(w^*). \quad \square \quad (\text{A.5})$$

**Proof of Proposition 1.** Follows from Lemma 1.  $\square$

**Proof of Proposition 2.**

(a) Differentiation of (3.5) with respect to tax,  $\tau_d$ , shows that  $d\kappa/d\tau_d < 0$ . Differentiation of the RHS of (4.1) with respect to  $\tau_d$ , then implies that  $dq/d\tau_d < 0$  since  $s_q(q) > 0$ ,  $s(q)/q$  is decreasing in  $q$ , and the LHS is independent of the capital tax. Thus a lower capital tax reduces unemployment rate.

Above two results imply that  $d\bar{w}/d\tau_d < 0$  (see (3.15)). In addition, differentiation of the distribution of wage offers,  $O(w)$ , with respect to tax,  $\tau_d$ , implies  $dO(w)/d\tau_d > 0$  (see (3.14)).

$dq/d\tau_d < 0$ ,  $d\bar{w}/d\tau_d < 0$ , and  $dO(w)/d\tau_d > 0$  imply that the RHS of (3.4) is decreasing in the capital tax,  $\tau_d$ , for a given consumption level ( $\underline{w}$  is replaced by  $w_{\min}$  in (3.4)). Since,  $u_{cc}(c) < 0$ , and the LHS of (3.4) is independent of the capital tax, it implies that  $dc/d\tau_d < 0$ .

(b) Since,  $dO(w)/d\tau_d > 0$  and  $d\bar{w}/d\tau_d < 0$ , a lower tax leads to stochastic improvement in the wage offer distribution and a higher average wage offer.  $dO(w)/d\tau_d > 0$  and  $dq/d\tau_d < 0$  imply that  $dE(w)/d\tau_d > 0$ . Thus, a lower tax leads to stochastic improvement in the wage earnings distribution and a higher average wage earnings.

(c) Since,  $d\bar{w}/d\tau_d < 0$  and  $w_{\min}$  is unaffected,  $d\frac{\bar{w}}{w_{\min}}/d\tau_d < 0$ .  $\square$

**Proof of Proposition 3.**

(a) Follows from differentiation of RHS of (4.1) with respect to  $w_{\min}$ .

(b) Follows from Proposition 3(a) and differentiation of (3.15) (with  $\underline{w}$  replaced by  $w_{\min}$ ) with respect to  $w_{\min}$ .  $\square$

**Proof of Proposition 4.** Unemployment rate neutral changes in the minimum wage and the capital tax:

Total differentiation of (4.1) and the condition that the unemployment rate and thus the labor market tightness,  $q$ , does not change imply that

$$\frac{dw_{\min}}{d\tau_d} = F_{nk}(\kappa) \frac{d\kappa}{d\tau_d} < 0. \quad (\text{A.6})$$

Now differentiating (3.15) with respect to  $\tau_d$  shows that

$$\frac{d\frac{\bar{w}}{w_{\min}}}{d\tau_d} = \frac{1}{w_{\min}^2} \left[ 1 - \frac{\sigma(r_e + \sigma)}{(\sigma + s(q))(r_e + \sigma + s(q))} \right] [w_{\min} - F_n(\kappa)] \frac{dw_{\min}}{d\tau_d} > 0. \quad (\text{A.7})$$

Thus an unemployment rate neutral increase in the minimum wage and a reduction in the capital tax rate leads to narrower support.

Given that  $q$  does not change and (A.7)

$$\frac{d\left(\frac{F_n(\kappa) - w}{F_n(\kappa) - w_{\min}}\right)}{d\tau_d} = \frac{F_{nk}(\kappa)}{F_n(\kappa) - w_{\min}} \frac{d\kappa}{d\tau_d} < 0. \quad (\text{A.8})$$

Then the differentiation of (3.14) implies that

$$\frac{dO(w)}{d\tau_d} > 0. \quad (\text{A.9})$$

The average wage offer,  $M(O(w))$  is given by

$$\int_{w_{\min}}^{\bar{w}} x dO(x) = \bar{w} - \int_{w_{\min}}^{\bar{w}} O(x) dx. \quad (\text{A.10})$$

Using the Leibnitz rule on the RHS of (A.10), I have

$$\frac{dM(O(w))}{d\tau_d} = - \int_{w_{\min}}^{\bar{w}} O_{\tau_d}(x) dx < 0. \quad (\text{A.11})$$

Since the distribution of wage earnings,  $E(w)$ , is an increasing function (Eq. (3.10)) of  $O(w)$ , a decrease in the capital tax rate also raises the average wage earnings.  $\square$

## Appendix B. The social planner problem

The social planner chooses the sequence of  $\{c_t, \theta_t, l_t, n_{t+1}^h, k_{t+1}\}$  to maximize

$$\max_{c_t, \theta_t, l_t, n_{t+1}^h, k_{t+1}} \sum_{t=0}^{\infty} \beta^t [u(c_t) - \mu(l_t - n_t^h) - \phi n_t^h]$$

subject to the law of motion of employment

$$n_{t+1}^h \leq (1 - \sigma)n_t^h + s(q_t)(l_t - n_t^h) \quad (\text{B.1})$$

and the law of motion of capital stock

$$k_{t+1} \leq F(k_t, n_t^h) + (1 - \delta)k_t - c_t - \xi q_t l_t. \quad (\text{B.2})$$

Denote the Lagrangian multipliers associated with (B.1) and (B.2) as  $\lambda_{nt}$  and  $\lambda_{kt}$  respectively. Then the first order conditions yield

$$u_c(c_t) = \lambda_{kt}, \quad (\text{B.3})$$

$$s_q(q_t)(l_t - n_t^h)\lambda_{nt} = \xi l_t \lambda_{kt}, \quad (\text{B.4})$$

$$s(q_t)\lambda_{nt} = \mu + \xi q_t \lambda_{kt}, \quad (\text{B.5})$$

$$\lambda_{nt} = \beta [F_n(k_{t+1})\lambda_{kt+1} - (\phi - \mu) + (1 - \sigma - s(q_{t+1}))\lambda_{nt+1}], \quad (\text{B.6})$$

$$\lambda_{kt} = \beta [F_k(k_{t+1}) + 1 - \delta]\lambda_{kt+1}. \quad (\text{B.7})$$

The above equations and the relation  $\beta = \frac{1}{1+r}$  yield (5.3), (5.4), and (5.5) in the steady state.

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