Cost curves: ch 10

- moving from production to cost
- look at costs of various input bundles
- translate this to cost of output
  (in SR and LR)

Why do we care about properties of prod'n f'n?

Assume: producers goal:
max profit = revenues - costs.

To max profits, must first minimize costs.

How unit costs vary with level of output depends on properties of production function.

Def'n: *cost function*: maps quantity of output to monetary values: gives cheapest way of producing any given level of output

Obviously, costs depend on input quantities, prices:

Def'n: *optimal combination of inputs for given output level*: input combination producing that output at minimum cost.

Costs of production= costs of inputs used to produce output

- here, two inputs: K has unit cost of *r* (interest rate)
  L has unit cost of w (wage rate)
- total cost of input bundle $(L^0, K^0) = wL^0 + rK^0$

1. SR : capital fixed at $K = K_0$, labour variable

- two types of cost: fixed (FC) and variable(VC)
- fixed cost: $FC = rK_0$ = cost of fixed factor
- variable cost: VC = cost of variable input (L)

- how to calculate variable cost?
  1. SR production function Q=F(L) gives max Q for given labour input (with fixed K);
  2. Can invert this function to solve for $L=L(Q)=F^{-1}(Q)$ gives minimum labour requirement for given output level
  3. Then $VC(Q)=wL(Q)$

Example: the CD production function $Q=L^{0.5}K^{0.6}$

- in SR, when K=10, $Q=10^{0.6}L^{0.5}$
- can invert SR production function to solve for *minimum* labour input required for given Q:
- have $L=L(Q) = (Q/10^{0.6})^2$
- then variable cost is $VC(Q) = wL(Q) = w(Q/10^{0.6})^2$

- what if K = 20 in SR?

SR total cost: STC(Q)= FC + VC(Q) = rK + wL(Q)

SR unit costs:

1. Marginal cost: addition to total cost of producing one more unit of output;
2. Average cost: cost per unit of output
   - Average variable cost
   - Average fixed cost
   - Average total cost

1. Marginal cost: $SMC(Q) = dTC(Q)/dQ = dVC(Q)/dQ$

- addition to total cost if produce one more unit
- $SMC(Q)=w(dL(Q)/dQ) = w/MP_L(Q)$

- shape of SMC(Q)? Depends on shape of MP_L curve:
  1. If MP_L always decreasing, SMC(Q) always increasing;
  2. If MP_L first increasing, then decreasing, SMC(Q) u-shaped. Let Q* be output produced when input L at the level which max's MP_L. Then,
     i) for $Q<Q^*$, SMC(Q) ↓ as Q↑;
     ii) for $Q>Q^*$, SMC(Q) ↑ as Q↑;
     iii) SMC(Q) is minimized at $Q=Q^*$.

2. Average variable cost: $AVC(Q) = VC(Q)/Q = wL(Q)/Q$
   - bears same relationship to AP_L as SMC(Q) does to MP_L .

Sketch the two in one diagram?

3. Average fixed cost? AFC(Q) = FC/Q = rK/Q. Shape?

4. Average total cost? SATC(Q) = AFC(Q) + AVC(Q)
- shape?

impt feature: SRMC intersects AVC and SRAC, from below, at the minimum point of these curves (*not* min'm of SRMC)
-why?

Summary: SR cost curves give minimum cost of producing each level of output, *given a level of the fixed factor.*

- one family of SR cost curves for **each** level of fixed factor

- relationship between SR cost curves for different levels of fixed factor? Depends on RTS (returns to scale) - LR costs.

Long run cost function

- isoquant map
- choose an isoquant - output
  - cheapest input combo?

- input prices -
  - for now: producer takes as given
  - choose an input combination on isoquant.
    - » What is total cost of this input bundle?

Other input combos costing same amount as (L_0,K_0) satisfy

- $TC_0 = wL + rK$ *Isocost line*
- rearrange to solve for $K= \frac{TC_0}{r} - \frac{w}{r}L$
- negatively sloped line, slope = -(w/r)
  - slope gives opportunity cost of a unit of L in terms of K
  - intercepts?
- isocost lines for $TC > TC_0$?

So: back to isoquant for output level :

- each input combo has isocost line through it
- cheapest input combo to produce $Q = Q_0$?

Ans: *Tangency between isocost and isoquant*

– Two conditions for optimal input combo (L*.K*):

– Rearranging: slopes’ condition becomes
  -)on isoquant: $F(L^*,K^*)=Q_0$
  -)slope isocost = slope isoquant

» w/r = MRTS or
» $w/r = MP_L/MP_K$

Intuition of tangency condition?

- Rearrange to obtain $$\frac{MP_L}{w} = \frac{MP_K}{r}$$
- last $ spent on each input yields same increase in output.
- (Exactly the same intuition as for MRS = ratio of goods' prices in consumer theory)

Long run cost: optimal input bundle for given level of output
- on isoquant for that output level, where isocost tangent
- so market rate of tradeoff between inputs, given by market prices, = technological rate of tradeoff (MRTS)

For each level of output, one optimal input bundle (given assumptions on production function)
- can derive long run cost curve. (Sketch)

Given LR total cost, can derive
LRAC (average cost)=total cost/output
LRMC (marginal cost)= $dLTC(Q)/dQ$

Shapes? - depends on RTS

If CRS - LRAC horizontal
If IRS - LRAC is decreasing in output ("economies of scale")
If DRS - LRAC is increasing in output ("diseconomies of scale")