Econ 203: production theory (ch 9)

(will cover ch. 16.1 later in term)

To this point: have derived demand curve for a market

- started with individual preferences and constraints;
- derived individual demand curves for goods (from price consumption curves);
- derived market demand curves by aggregating demand curves of individual consumers.

Now turn away from consumer side of market, to firms.

Roadmap for this side of market?

1. Production function - technological constraint on firm’s output decision (ch 9);
2. Cost function - translate technological constraint into monetary values, which are of concern to firm (ch. 10);
3. Firm’s output decision: maximize profits; how to do this depends on relation between market demand curve and firm’s demand curve - which depends on market structure (chs 11-13).

Characterize process by which goods are made:

Def'n: *production function*: relationship between inputs (exogenous variables) and outputs (endogenous variables)

- maximum output for a given level of inputs
- written Q=F(L, K) L=labour, K=capital
- describes physical, measurable relationship - significant difference from utility
- given by technology

General properties of production functions?

A. Period of production:

short run (SR): one input fixed (K);

long run (LR): all inputs (L,K) variable.

Why distinguish? Possibility of substitution in LR, not in SR

- different decisions

Example: CD production: Q= $L^{0.5}K^{0.6}$

- in LR, can vary both L and K, and substitution between them is possible

- in SR, K is fixed - a number, say 10, so SR production function is Q = $10^{0.6}L^{0.5} = 10^{0.6}\sqrt{L}$
- shape of this function?

So: first look at SR production function: Q=*f*(L)

- key property? *law of (eventually) diminishing returns:*

if add more and more units of variable factor to given quantity of a fixed factor, ***additional output*** from each ***additional unit*** of the variable factor will eventually decline.

- notice: output does not decline, but increment to output declines-
- why?

Law of diminishing returns gives shape of *total product curve:*
relationship between variable input and output in SR
- depends on level of fixed input

Other important SR functions:

1. *Marginal product:* of a variable input (labour, here)
   $MP_L = \Delta Q/\Delta L$: change in output from unit increase in variable input, *all other factors fixed*
   analogous to marginal utility

2. *Average product*: of labour: $AP_L = Q/L$

Relation between average and marginal product functions?

1. Geometric? See figures 9.6-9.7

2. Algebraic?

Example: back to SR CD function $Q = 10^{0.6} L^{0.5}$

this is total product function.

AP_L = Q/L = $10^{0.6} L^{-0.5}$

MP_L= $\frac{\partial Q}{\partial L} = 0.5 10^{0.6} L^{-0.5}$

Average and marginal products are both increasing in K

Relation between average and marginal product functions?

- both positive;
- average everywhere greater than marginal
- diminishing returns? MP_L is decreasing in L
- AP_L also diminishing in L

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C-D illustrates general relation: if $MP_L < AP_L$, then AP_L is falling as L increases.

More general function: if $MP_L > AP_L$, then AP_L is increasing as L↑.

To see: let Q = TP(L) be short run production function

then $AP_L = TP(L)/L = TP(L) x L^{-1}$

take derivative of AP_L with respect to L:

$$\frac{dAP_L}{dL} = \frac{dTP(L)}{dL} \times L^{-1} - L^{-2} \times TP(L)$$

$$= (MP_L - AP_L) / L$$

so sign of slope of AP_L depends on difference between marginal and average products.

Final point on relation between marginal and average products:

- two are equal when average product is max'd

Why do we care about this?

- marginal product gives effects of one more unit of labour;
- will one more worker increase or decrease average productivity?
- when is it correct to hire one more worker? To stop?

Basis for SR cost functions, which are important in determining firm's output in the SR

And: really, all production occurs in the SR; LR is "planning period"

Properties of LR production function: Q=F(L,K)?

- consider contour lines of function: combinations of capital and labour which yield constant level of output.

- label these contour lines "isoquants"

- shape? Assume:
  1. Negative slope
  2. Higher output associated with isoquants farther from origin
  3. Possibilities for substitution: isoquants are convex to origin: substitution is possible, but more difficult to substitute labour for capital as ratio of K/L becomes very small.
  - measure substitution possibilities by slope of isoquant: define Marginal rate of technical substitution (MRTS) (analogous to MRS)

Can define MRTS in terms of marginal products of inputs:
- at input combination (L^0, K^0),

$$\text{MRTS} = \frac{MP_L(L^0, K^0)}{MP_K(L^0, K^0)}$$

Notice: even though varying both inputs, can still define marginal products.

Ass'm that isoquant is convex = assumption that MRTS decreases as L increases.

Property #4 of LR production function: ***returns to scale (rts)***

- by how much does output increase if vary both inputs *by same proportion*?

- ie, if increase both K and L by 50%, will output go up by 50%? By > 50%? By < 50%?

- algebra: relation between F(tL,tK) and tF(L,K), for t>1

- depends on production function
- labels on isoquants matter (unlike IC's)

Three possibilities:

1. Increasing returns to scale (IRS): F(tL,tK)>tF(L,K)

- scaling up inputs means output increases more than proportionately
- example? Industrial processes where volume impt ie, pipelines, breweries….

2. Constant returns to scale (CRS): F(tL,tK)=tF(L,K)

- scale of production irrelevant
- given plant can always be duplicated

3. Decreasing returns to scale (DRS) F(tL,tK)<tF(L,K)

- scaling up production leads to complications
- organizational problems?

CD functions: rts given by sum of exponents:

example: Q = $L^{0.5}K^{0.6}$

has IRS: if increase both inputs by factor t>1, Q increases by $t^{1.1}>1$

Diagrams? Frequently assume first IRS, then DRS.