Economics 452/551, Spring 2007
Assignment 1(b): for 551

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Total value: TBA
Due: January 9, 2007
4:00 pm.

1. Consider an Expected Utility maximizing consumer with a utility of wealth function $u(w) = kw^{\alpha} + K$ where *a*, *k* and *K* are constants.

(i) Is this consumer Risk averse? Does your answer depend on any of the constants *a*, *k*, or *K*?

Ans: with this utility function, $u'(w) = \alpha k w^{\alpha-1}$ and $u''(w) = \alpha(\alpha-1)kw^{\alpha-2}$. Risk aversion is indicated by the sign of the second derivative, which depends on the values of *k* and α.

(ii) Can any of the constants *a*, *k*, or *K* be changed, and the resulting function still represent the same preferences?

Ans: For the utility function above, the coefficient of absolute risk aversion is $-u''(w)/u'(w)$=(α-1)/w. Since this coefficient is independent of *k* and *K*, either of these can be changed without changing risk aversion. However, the first and second derivatives will change sign if the sign of *k* is changed. Hence, the function is unique up to a positive affine transformation.

(iii) Let $C = \{c_1, c_2, c_3, c_4\} = \{10, 14, 16, 20\}$ .Which of the following lotteries would this consumer prefer? $L = (\frac{1}{2}, 0, 0, \frac{1}{2})$ or $L' = (0, \frac{1}{2}, \frac{1}{2}, 0)$?

Ans: this individual will prefer the latter: same expected outcome, but smaller variance – to check, calculate the expected utility for each lottery and compare.

2. Suppose that a consumer's utility of wealth function can be written as a quadratic function $u(w) = \alpha + \beta w - \gamma w^2$.

(i) Show that the expected utility of a lottery has the convenient property that it depends only on the mean and variance of the lottery.

Ans: consider a lottery with two possible outcomes, $\{w_1, w_2\}$, where prob$\{ w = w_1\} = p_1$ and $prob\{w = w_2\} = p_2 = 1 - p_1$. Then

$$\begin{aligned} EU &= p_1 u(w_1) + (1-p_1)u(w_2) \\ &= p_1(\alpha + \beta w_1 - \gamma w_1{}^2) + (1-p_1)(\alpha + \beta w_2 - \gamma w_2{}^2) \\ &= \alpha + \beta(p_1 w_1 + p_2 w_2) - \gamma(p_1 w_1^2 + p_2 w_2^2) \end{aligned}$$

Now: the mean of the lottery, the expected value, is $\mu = p_1 w_1 + p_2 w_2$, while the variance is $\sigma^2 = p_1(w_1 - \mu)^2 + p_2(w_2 - \mu)^2 = (p_1 w_1^2 + p_2 w_2^2) - \mu^2$. Using these variables, the expected utility can be rewritten as $EU = \alpha + \beta\mu - \gamma(\sigma^2 + \mu^2)$: thus EU depends only on the mean and the variance of the lottery.

(ii) Show that the marginal utility of wealth must eventually become negative.

Ans: $\frac{du(w)}{dw} = \beta - 2\gamma w < 0$ iff $w > \frac{\beta}{2\gamma}$, which is finite value if u(w) is well defined.