Lecture III: Special preferences

Quasi-linear preferences

- goods *x* and *y*

$U(x,y) = B(x) + y$ $\qquad B'(\cdot) > 0 > B''(\cdot)$ for x>0

- separable in goods

- non-linear in one good (here,x), linear in other

- Ex: $U(x,y) = \alpha \ln x + y$

- Campbell's notation:

Properties of demand?

A. Individual optimum:

Given (p_x, p_y, w), consumer problem is

$$\max_x \; B(x) + \frac{w}{P_y} - \frac{p_x}{P_y} x$$

a) interior solution to consumer's problem:

FOC for interior solution: $B'(x) = \frac{p_x}{p_y}$

- optimal amount independent of w (no income effect)

- remainder of w spent on other good

b) corner solutions to consumer problem:

1. only second good (linear) consumed:

- if $B'(0) < \frac{p_x}{p_y}$, then $x^* = 0$ and $y^* = \frac{w}{p_y}$

2. only first good (concave) consumed:

- if $B'(\frac{w}{p_x}) > \frac{p_x}{p_y}$, then $y^* = 0$ and $x^* = \frac{w}{p_x}$

Pictures? IC's are vertically parallel

(Horizontally parallel if U linear in x, and concave in y)

B. fficient allocations?

Simplify - let $p_y = 1$, so $y = w - p_x x$ is spending on all other goods.

Consider population of N individuals, with preferences $U_i(x^i, y^i) = f_i(x^i, x^{-i}) + y^i$, i=1,…,N

Note: $x^{-i} = (x^1, x^2, \ldots x^{i-1}, x^{i+1}, \ldots, x^N)$

Thus this utility function allows individual i's utility to depend on the distribution of good x across all individuals, rather than just own consumption

Private vs public goods?

- purely public good: $U_i(x^i, y^i) = f_i(\sum_{i=1}^{N} x^i) + y^i$
- purely private good: $U_i(x^i, y^i) = f_i(x^i) + y^i$

- How to characterize efficient allocations?

  Assume: everyone consumes some x,y

  Then: with *quasi-linear preferences*, an allocation is efficient iff it

maximizes $\sum_{i=1}^{N} U_i(x^i, y^i)$ over feasible outcomes

C. Consumer surplus and welfare:

- CS is area under demand curve, above price
- general preferences, CS approximates change in welfare
- Quasi-linear preferences, no income effects on good 1, so CS gives accurate measure of consumer welfare

D. if > 2 goods? $U(x,y,z) = f(x) + g(y) + z$

- then good z "soaks up" all income effects
- there are cross-price effects between goods

Finally:

Log-linear preferences: $U(c) = \alpha \ln c$

Under uncertainty, $EU(L) = p_1 \ln(c_1) + p_2 \ln(c_2)$

and slope IC = $-\frac{p_1}{p_2}\frac{c_2}{c_1}$

IC's in state space look like those from Cobb-Douglas utility function