Economics 452
Assignment 2

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1. (14) Following up on the model from class: let the principal's payoff be S(q)=$8q^{0.5}$ and the possible agent types be $\{\underline{\theta}, \overline{\theta}\} = \{2, 4\}$. For the moment, retain the assumptions that F and each type's reservation utility is equal to zero.

   a) (4) Assuming an agent's type is costlessly observable, derive the first best contract for each type of agent.

Ans: 1st best contract has the quantity which max's social surplus = $S(q) - \theta q$, and payment *t* which just covers agent's costs.

For efficient agent, this is max $(8\sqrt{q} - 2q);$ FOC for q yields q=4, and *t*=8.

For inefficient agent, this is max $(8\sqrt{q} - 4q);$ FOC yields *q* = 1, *t* = 4.

   b) (2) Which type of agent does the principal prefer? Why?

Ans: which agent leaves P with higher surplus? From eff. agent, P obtains $8\sqrt{4} - 8 = 8$; from inefficient agent, P obtains $8\sqrt{1} - 4 = 4$. Therefore P prefers efficient agent.

   c) (3) Illustrate the first best solution in a diagram (using at least one-half page), and identify the first best contracts.

Now suppose that an agent's type is private information, unobservable by the principal.

   d) (1) Write down the self-selection constraints which must be satisfied by an incentive feasible contract menu.

Ans: SS for efficient agent: $\underline{t} - 2\underline{q} \geq \overline{t} - 2\overline{q}$

SS for inefficient agent; $\overline{t} - 4\overline{q} \geq \underline{t} - 4\underline{q}$

   e) (2) On your diagram in (c), identify a pair of contracts which satisfy these constraints, as well as a pair which violate them. Explain why, in each case.

   f) (2) How would your results be affected if F>0?

Ans: If F>0, cost of production of each type of agent increases, for every quantity. Thus, to satisfy the participation constraint of each type of agent, the principal must make a higher payment, for each level of quantity. Optimal quantities will be unaffected, so long as the principal is still willing to hire either type of agent.

2. (16 marks) Jason is seeking a producer for his new line of glow-in-the-dark nunchucks. The nunchucks have a value to him of S(n)=10ln(n), where n is the number of nunchucks. There are

two possible types of producers in town, who differ in their marginal cost of production; assume that each producer knows their own marginal cost. Efficient producers can provide nunchucks at a constant marginal cost of 1, whereas inefficient producers can provide them at marginal cost equal to 2; there is no fixed cost of production. All producers have a reservation utility of zero.

a) (3) If Jason were to contract with someone he knew to be an efficient producer, how many nunchucks would he order, and what would he pay for them?

Ans: If Jason has all the bargaining power, he would choose *n* to max the social surplus = $10\ln(n) - n$, which yields *n* = 10. Price will be equal to marginal cost = 1.

Suppose now that Jason cannot costlessly identify the efficiency of any given producer.

b) (2) In this situation, should Jason be more suspicious of someone who claims to be efficient, or inefficient? Explain briefly.

Ans: If dealing with a known-to-be ineffcient producer, Jason would choose *n* to max $10\ln(n) - 2n$, yielding *n*=5, and would offer p=2. Since the price exceeds the marginal cost of the efficient producer, this producer could make strictly positive profits by claiming to be inefficient, vs. zero profits if they claimed to be efficient. The inefficient producer, on the other hand, has no incentive to misrepresent their effciency.

c) (2) Using subscripts "e" and "i" for efficient and inefficient producers respectively, write down the constraints on the quantity-price contracts that will ensure that a producer who accepts a contract will be the type they claim to be.

Ans: SS for efficient producer: $t_e - n_e \geq t_i - n_i$

SS for inefficient producer; $t_i - 2n_i \geq t_e - 2n_e$

d) (5) Derive the contract menu that Jason would create in this situation.

Ans: Technique: i) assume binding constraints are participation constraint for inefficient agent and SS constraint for efficient agent, and solve for optimal contract;

ii) check to ensure that, at this solution, SS for inefficient agent and participation constraint for efficient agent are satisfied.

Problem: choose n_i, n_e to max $v[10\ln(n_e) - t_e] + (1-v)]10\ln(n_i) - t_i]$ s.t.

$t_e - n_e = t_i - n_i$ and $t_i - 2n_i = 0$; solution is

$$n_e = 10, n_i = \frac{10(1-v)}{2-v}, t_e = n_i + n_e = \frac{10(3-2v)}{2-v}, t_i = 2n_i = \frac{20(1-v)}{2-v}$$

e) (2) Explain what is meant by the term "information rents" in this context.

Ans: Information rents are the payment to the agent with the incentive to misrepresent their type, over and above their reservation utiltiy. Here, under asymmetric information the efficient agent receives a payoff greater than 10, because if they were to "choose" the contract designed for an inefficient agent, they would receive a payoff of 10.

f) (2) Suppose now that the marginal cost of efficient producers decreases, while that for inefficient producers is unchanged. Would this increase or decrease the problem caused for Jason by the asymmetric information? Explain.

Ans: increase, since it widens the gap between the two types of producers, and thus increases the incentive for an efficient producer to misrepresent. Thus a larger part of the surplus must be transferred to the efficient agent, in equilibrium.

3. (15) (from Macho-Stadler and Perez-Castrillo) For several months not a drop of rain has fallen in Bilbao, and daily water usage restrictions have had to be enforced. All classic methods of solving the problem have failed, and the mayor is searching for drastic and imaginative solutions. One of the mayor's aides tells him about a sorcerer from India who is able to make it rain. Naturally, the mayor is unconvinced, and is afraid the sorcerer will turn out to be a fake. Therefore, the mayor is trying to find a contract to offer the sorcerer such that he will accept only if he is authentic.

Everyone knows that a phony sorcerer has no power over the rain. If the mayor contracted with a fake, the probability of rain would remain unchanged. It is currently estimated that the probability of rain in the next week (which is the time required to adequately test the powers of a sorcerer) is 2/100. An authentic sorcerer, in spite of having powers, is not infallible, and will increase the probability of rain, within a week, to 20/100.

All sorcerers are risk averse, with utility functions given by $u(y) = y^{0.5}$, where *y* denotes income. No authentic sorcerer will work unless the contract gives him at least $\underline{U} = 10$; a fake, on the other hand, needs only $\underline{U} = 1$. The mayor of Bilbao, who is risk neutral, is concerned with designing a contract which would be accepted only by an authentic sorcerer, since the political cost of being publicly ridiculed, should anyone discover that a fake had been hired for the job, is prohibitively high.

Suppose the mayor offered a contract promising to pay a sorcerer y_r if it rains, and y_0 if there is no rain.

a) (2) Illustrate the sorcerers' indifference curves in a state-space diagram.

b) (4) What constraints must the payments in the contract satisfy? Why?

Ans: participation constraint for the authentic sorcerer, and a self selection constraint for the fake which ensures that a fake sorcerer cannot attain EU higher than one by accepting the contract. No need to worry about the other two constraints, because the mayor does not want to offer a contract that a fake would accept.

c) (5) Derive the optimal contract. (given the mayor has all the bargaining power).

Ans: Mayor will want to minimize the expected cost of hiring an authentic sorcerer, subject to the constraints above. So his problem is to choose (y_r, y_o) to min $0.2y_r + 0.8y_o$ subject to

i) participation constraint for authentic sorcerer: $0.2\sqrt{y_r} + 0.8\sqrt{y_0} \geq 10$

ii) SS for fake: $0.02\sqrt{y_r} + 0.98\sqrt{y_0} \leq 1$

To solve: 1. first find solution when (i) binds, ignoring (ii). If (ii) is satisfied at this solution, problem solved. Given A is risk averse, and P risk neutral, ignoring SS yields a lump-sum contract, with equal payments in both states: A has no uncertainty about income. Any fixed-fee payment which satisfies (i) violates (ii), so this will not work.

iii) so (ii) will bind, as well as (i). Since the sorcerer must receive a higher expected payment to compensate for differences between his income in the two states, minimizing the mayor's expected payment requires minimizing the gap between incomes in the two states, subject to satisfying the constraints. Thus the contract

will be determined by the intersection of the two constraints:
$y_0 = 0, y_r = 2500.$

d) (4) Calculate the costs of this contract compared to that offered under symmetric information.

Ans: under AI, the expected cost of the contract = $0.2 \times 2500 = 500.$ Under symmetric information, the mayor would offer the authentic sorcerer a lump sum payment, *y*, such that $\sqrt{y} = 10,$ so y=100. Thus the cost of the AI is 400.