Contract with asymmetric information

Assume now that A has private info about own efficiency / P cannot costlessly observe θ

Is full-info optimal contract still optimal?

a) is it feasible?

Spse P offered two-part schedule
$\{(\overline{t}^*, \overline{q}^*), (\underline{t}^*, \underline{q}^*)\}$

Which contract would efficient A choose? inefficient A?

First-best contract is not implementable because it violates incentive compatibility

Viable contract menu under asymmetric info must satisfy
two sets of constraints:

i) participation constraints (to accept contract): (as above)

ii) self-selection constraints / incentive compatibility constraints:

$$U^A(\underline{t}, \underline{q}; \underline{\theta}) \geq U^A(\overline{t}, \overline{q}; \underline{\theta})$$

and $U^A(\overline{t}, \overline{q}; \overline{\theta}) \geq U^A(\underline{t}, \underline{q}; \overline{\theta})$

In this context, SS constraints are

$$\bar{\theta}: \quad \bar{t} - \bar{\theta}\,\bar{q} \geq \underline{t} - \bar{\theta}\,\underline{q}$$

$$\underline{\theta}: \quad \underline{t} - \underline{\theta}\,\underline{q} \geq \bar{t} - \underline{\theta}\,\bar{q}$$

If these both satisfied, neither type of agent/wkr has incentive to lie about level of own efficiency

Def'n: menu of contracts is *incentive feasible* if satisfies both participation and incentive constraints.

Properties of feasible menus?

1. Rearranging and combining SS constraints yields

$$\underline{\theta}(\bar{q} - \underline{q}) \geq \bar{t} - \underline{t} \geq \bar{\theta}(\bar{q} - \underline{q})$$

For this to hold, need $\underline{\theta}(\bar{q} - \underline{q}) \geq \bar{\theta}(\bar{q} - \underline{q})$

or $(\underline{\theta} - \bar{\theta})(\bar{q} - \underline{q}) \geq 0$

Since $\bar{\theta} > \underline{\theta}$, this requires $\underline{q} \geq \bar{q}$:

higher quantity demanded from more efficient agent

Since more efficient agent must work harder, must pay this type of worker more, to induce additional effort, so $\underline{t} \geq \overline{t}$ (so payment increasing in output).

Incentive feasible set in (q,t) space, given private info on θ? (picture?)

Def'n: *Information rent*: surplus transferred to agent under asymmetric info to ensure incentive compatibility, over and above payment required to satisfy participation constraint.

Principal's problem with asymmetric information?

max own payoff, given constraints:

choose $\{(\overline{t}, \overline{q}), (\underline{t}, \underline{q})\}$ to max

$$\nu(S(\underline{q}) - \underline{t}) + (1-\nu)(S(\overline{q}) - \overline{t})$$

st. 2 participation constraints:

and 2 self-selection constraints

Now: LM introduce a change of variables here:

Define: $\bar{U}$ is utility of agent with $\bar{\theta}$ in eq'm
If > zero (res'n utility), then inefficient agent benefits from the asym info - receives info rents.

Using this, in self-selection constraints, information rents accruing to efficient agent are:

efficient agent: $\underline{t} - \underline{\theta}\,\underline{q} = \bar{t} - \underline{\theta}\,\bar{q}$

$$= \bar{t} - \bar{\theta}\,\bar{q} + (\bar{\theta} - \underline{\theta})\bar{q}$$

$$= \bar{U} + \Delta\theta\,\bar{q}$$

Rewriting P's objective f'n in terms of $(\underline{U}, \bar{U})$:

$$\nu\,(S(\underline{q}) - \underline{U} - \underline{\theta}\,\underline{q}) + (1-\nu)(S(\bar{q}) - \bar{U} - \bar{\theta}\,\bar{q})$$

$$= \nu\,(S(\underline{q}) - \underline{\theta}\,\underline{q}) + (1-\nu)(S(\bar{q}) - \bar{\theta}\,\bar{q})$$
$$-(\nu\,\underline{U} + (1-\nu)\bar{U}) \qquad (1)$$

First line: surplus under full info
Second line: expected information rents

Why is this form useful?

Asymmetric info contracts different from those under full info *if* nec'y to pay agent for to induce revelation of payoff relevant information

- this reduces the surplus overall
- reduces payoff of P

In this form, P's chooses (quantity, info rent) pair for each type of agent.

Constraints? Participation and SS:

Participation: given res'n utility =0,

$$\bar{U} \geq 0 \text{ and } \underline{U} \geq 0 \qquad \text{(2a, 2b)}$$

SS: efficient agent: $\underline{U} \geq \bar{U} + \Delta\theta\, \bar{q}$ (3a)

inefficient agent: $\bar{U} \geq \underline{U} - \Delta\theta\, \underline{q}$ (3b)

(recall: $\Delta\theta \equiv \bar{\theta} - \underline{\theta} > 0$)

P's problem: choose $\{(\underline{U}, \underline{q}), (\bar{U}, \bar{q})\}$ to max (1)

subject to 2a-3b

Solution?

a) how to solve?
- set up lagrangian…use brute force

or
- think about problem, and try and reduce constraints.

What do we know?
- incentive for misrepresentation by efficient agent (at full info contracts)
- (inefficient agent has no incentive to misrepresent - worse off if claims to be efficient)

- therefore efficient agent must be compensated to induce self-selection, inefficient agent need not be

- since P's payoff is decreasing in expected info rents, P wants to minimize this

- therefore: in contract,
  i) participation constraint binding for inefficient agent - so $\overline{U} = 0$, and

ii) self-selection constraint binding for efficient agent - so $\underline{U} = \Delta\theta\,\overline{q}$.

Return to optimization problem, using these, P's objective function is:

$$\nu\left(S(\underline{q})-\underline{U}-\underline{\theta}\,\underline{q}\right)+(1-\nu)\left(S(\overline{q})-\overline{U}-\overline{\theta}\,\overline{q}\right)$$

$$=\nu\left(S(\underline{q})-\underline{\theta}\,\underline{q}\right)+(1-\nu)\left(S(\overline{q})-\overline{\theta}\,\overline{q}\right)-v\Delta\theta\,\overline{q}$$

Notes:

i) a matter of choosing only quantities produced by each type of agent - payment (t) determined by constraints once quantity det'd

ii) info rent (last term in obj've function) depends only on quantity produced by *inefficient* agent. (WHY?)

Solution to P's problem/ optimal solution with asym info?

1. wrt $\underline{q}$: same as first best

(notice: $\underline{t}$ here > first best, since $\underline{U}>0$)

2. wrt $\overline{q}$: lower than in first best

(note: $\overline{t}$ correspondingly lower, since $\overline{U}=0$)

So: consequences of asym info - gains and losses?

1. lower quantity produced by inefficient agent - loss of social surplus
2. inefficient agent receives same payoff - unaffected by asym info
3. efficient agent benefits - receives info rents in form of higher pay for same output
4. P worse off: social surplus lost, plus must pay more for efficient agent.