Comparative Static Analysis

1. What?
   - compare equilibria
   - model gives equilibrium conditions
     - first order conditions
     - frequently in implicit form
   - if exog. variables change, how does new eq'm compare to original?
     - check changes in endog. variables wrt exog. variables through use of eq'm conditions .
2. Why?
   - comparative dynamics are hard
   - belief that economic system is in equilibrium

Example: basic consumer theory:

- 2 goods, (x,y);
- budget set given by $\{p_x, p_y, M\}$
- choose x,y to max u(x,y) s.t. $p_x x + p_y y \leq M$
- properties of demand functions?

1. set up Lagrangean, obtain FOC's

$$u_x(x,y) - \lambda p_x = 0$$
$$u_y(x,y) - \lambda p_y = 0$$
$$p_x x + p_y y - M = 0$$

These 3 equations implicitly define Marshallian/Walrasian demand functions $x(p_x, p_y, M), y(p_x, p_y, M),$ and the marginal utility of income, $\lambda(p_x, p_y, M)$.

2. Totally differentiate equations wrt exog variables (p_x, p_y, M) and endog variables (x, y, λ); write in matrix form.

$$\begin{bmatrix} u_{xx} & u_{xy} & -p_x \\ u_{yx} & u_{yy} & -p_y \\ -p_x & -p_y & 0 \end{bmatrix} \begin{bmatrix} dx \\ dy \\ d\lambda \end{bmatrix} = \begin{bmatrix} \lambda dp_x \\ \lambda dp_y \\ xdp_{xx} + ydp_y - dM \end{bmatrix}$$

Determinant of LHS matrix can be signed:
SOC for constrained maximization > 0

3. Can find slopes of demand curves and engel curves using Cramer's rule.

Alternative (if only two choice variables):
- use constraint-as-equality (from theory) to eliminate one variable here: $y = (\mathrm{M} - p_x x)/p_y$

  so $u(x,y) = u(x,y(x)) = u(x, \frac{M - p_x x}{p_y})$
- treat as unconstrained max'm

problem: FOC is

$$\frac{du}{dx} = u_x(x,y) + u_y(x,y)\frac{dy}{dx}$$

$$= u_x(x,y) + u_y(x,y)(-\frac{p_x}{p_y}) = 0$$

This yields FOC for max: $\frac{u_x}{u_y} = \frac{p_x}{p_y}$

- once again, can derive properties of demand functions, recognizing that FOC defines $x(p_x, p_y, M)$, and

$$x(p_x, p_y, M) = (M - p_x x(p_x, p_y, M))/p_y$$

Sometimes, need to do more…:

Demand for insurance - with no asymmetric information:

Contingent commodities / state space

Consider the standard insurance model

- accident occurs with prob. $0 < p < 1$
- loss of L

- Individual's endowment point, E, (if buys no insurance):
    - if accident (state 1), ind'l has $y_1 = y_0 - L$;
    - if no accident(state 2), ind'l has $y_2 = y_0$

- If individual buys insurance paying out I in state 1, at cost δ per dollar:

    - state 1: has $y_1 = y_0 - L - \delta I + I = y_0 - L + (1-\delta)I$
    - state 2: has $y_2 = y_0 - \delta I$

EU without insurance?

$$EU = pU(y_0 - L) + (1-p)U(y_0)$$

EU with insurance?

$$EU = pU(y_0 - L + (1-\delta)I) + (1-p)U(y_0 - \delta I)$$

Optimal insurance? Choose I to max

$$\frac{\partial EU}{\partial I} = pU'(y_1)(1-\delta) - \delta(1-p)U'(y_2) = 0$$

I^* satisfies $$\frac{pU'(y_1)}{(1-p)U'(y_2)} = \frac{\delta}{1-\delta}$$

What is value of delta?

Cost of insurance:

Assume: 1. identical individuals
2. risk neutral firms
3. competitive insurance markets
4. no administrative costs

(1) + (2) implies firm max's expected profit on representative contract;

Eprofit = $p(\delta I - I) + (1-p)\delta I$
$= I(-p(1-\delta) + (1-p)\delta)$

(3) implies Eprofit = 0

Therefore: in competitive insurance market,

$$\frac{p}{(1-p)} = \frac{\delta}{(1-\delta)}$$

Sub'g this back into FOC for consumer, tells us

I^* satisfies $\frac{U'(y_1)}{U'(y_2)} = 1$: RA individual will buy full insurance if "fair insurance" is available.

Suppose insurance is unfair, so $\delta > p$. Then Eprofit > 0 on the average contract (assuming no fixed costs).

Will consumer still want full insurance?

<u>Comparative statics of insurance:</u>

From above, FOC for optimal insurance is

$$pU'(y_0 - L + (1-\delta)I^*)(1-\delta) - \delta(1-p)U'(y_0 - \delta I^*) = 0$$

Rearranging:

$$\frac{(1-\delta)U'(y_0 - L + (1-\delta)I^*)}{\delta U'(y_0 - \delta I^*)} = \frac{(1-p)}{p}$$

This defines demand for insurance as the implicit function $I^* = f(y_0, L, \delta, p)$.

Properties of this function?

1. How much insurance?
   i) if $\delta = p$, ind'l buys full insurance at all income levels.
   ii) if $\delta > p$, so insurance is unfair (and ins. firm makes strictly positive profit), then ind'l buys less than full insurance, so $y_1 < y_2$.
2. How does insurance vary with *y*?
   - totally differentiate FOC wrt I^* and endogenous & exogenous variables
   - look at partial wrt initial income:

$$\frac{dI^*}{dy_0} = \frac{\delta(1-p)U''(y_2) - p(1-\delta)U''(y_1)}{(1-\delta)^2 pU''(y_1) + \delta^2(1-p)U''(y_2)}$$

Sign? denominator <0, numerator….?

- sign depends on relative sizes of U", evaluated at different income levels.
- coeff. of ARA useful
- to use, need to bring in U'( ) - how?

i) rearrange FOC to solve for

$$\delta(1-p) = \frac{p(1-\delta)U'(y_1)}{U'(y_2)}$$

ii) sub. into (dI^*/dy_0) to obtain

$$p(1-\delta)U'(y_1)\left[\frac{U''(y_2)}{U'(y_2)} - \frac{U''(y_1)}{U'(y_1)}\right]$$

$$= p(1-\delta)U'(y_1)[A(y_1)-A(y_2)]$$

iii) sign depends on term in [ ], which is coeff. of ARA at two values;

- have $y_2 > y_1$, so…
- CARA - no change in insurance
- DARA - I* down as y up
- IARA - I* up as y up