Economics 452
Midterm examination

Dr. L. Welling — Time: 50 minutes
February 15, 2006 — Total value: 45 marks

DO ALL QUESTIONS. Please double-space all written answers, and use one-half page for each diagram you want marked. Be sure to label all diagrams, and show all your work.

Part A: (10 marks)

Each of the following statements is either *true* or *false*. State which, and justify your choice in no more than one page double-spaced. Be precise.

a) A risk averse individual would never prefer a gamble to a certain outcome.

False: the expected utility of a person who is risk averse depends on both expected income (the outcome) and the variability of the outcome. A risk averse person prefers a certain outcome to a gamble which has the same expected outcome, since the variance in outcome reduces utility. However, this individual might prefer a gamble with an expected outcome of ***x*** if the alternative were a certain outcome significantly less than ***x*** - they would be willing to trade-off risk for higher expected outcome.

b) Parveen is more risk averse than Ed is. If they have the same income, and face the same probability of a given loss, Parveen will purchase more insurance.

***False*:** if both individuals are risk averse, and can purchase fair insurance - so sellers make zero expected returns on policies, and the insurance premium is equal to the probability of loss/accident - then both individuals would purchase full insurance. If Parveen is risk averse and Ed risk loving, the statement is true. It is also true if both are RA and fair insurance is not available: if the insurance premium exceeds the loss probability, so sellers make strictly positive profits, both Parveen and Ed would purchase less than full insurance. Since Ed is less risk averse than Parveen, she is willing to pay more to decrease risk - so she will buy more insurance.

Part B: (35 marks)

Sharda operates an essay proofreading service, and hires graduate students from English. Her payoff from this service is given by the utility function $u(x) = 15\ln x - tx$, where *x* denotes pages and *t* is cost per page.

She knows there are two types of students who apply for work: high productivity students, who can proofread and correct 12 pages per hour, and low productivity

students, who can read and correct 8 pages per hour. Each student's next best alternative job pays $16/hour. All students have the same preferences, and utility is linear in income. One quarter of the students have high productivity.

a) (3) Assuming students can be employed in their alternative job for fractions of an hour, what is the opportunity cost of a page to a high productivity worker? For a low productivity worker?

Let *c* denote opportunity cost. Then for high productivity student, $12c_h = 16 \Rightarrow c_h = 4/3$, while a low productivity student faces $8c_l = 16 \Rightarrow c_l = 2$. So students have different constant marginal costs for producing proofread essays.

Suppose Sharda can costlessly observe a student's type.

b) (8) If Sharda has all the bargaining power, how many pages would she hire a high productivity worker to proofread? A low productivity worker?

First best: for high productivity, choose x_h to max $15x_h - 4x_h/3$, which yields $x_h = 45/4 = 11.25$

for low productivity worker, choose x_l to max $15x_h - 2x_l$, which yields $x_l = 15/2 = 7.5$

c) (6) Describe the contract she would offer each type: $\{x_l, t_l\}$ for the low type, and $\{x_h, t_h\}$ for the high type. Illustrate these contracts in a diagram.

In first best world, workers would receive opportunity cost, and produce first best outputs, so $\{x_l, t_l\} = \{7.5, 2\}$ and $\{x_h, t_h\} = \{11.25, 4/3\}$

Diagram? Simplest way to do this is to put *x* on the horizontal axis, and *T=tx* on the vertical - as in class, plot total payment against quantity produced. The A's IC's are linear, with slope given by opportunity cost, while P's IC's are positively sloped, with slope decreasing in *x*.

In *(x,t)* space, diagram is more complex. P's IC's come from utility function: $15\ln x - tx$, so have slope $\frac{dt}{dx} = \frac{(15 - tx)}{x^2}$ which is defined for all strictly positive values of *x*, and has positive slope for *tx<15* and negative slope for *tx*>15 . So the P's IC's change slope along the rectangular hyperbola *tx=15*, and have zero slope at the optimal contracts (since they

lie on this hyperbola). A's IC's come from $U^A = (t - c_i)x, i = h, l$ and have slope $\frac{dt}{dx} = -\frac{(t - c_i)}{x}$. Again, these IC's change slope, here when $t = c_i$. To be precise, an agent of type *i* has IC's which look like Cobb-Douglas IC's above the horizontal line $t = c_i$, have horizontal IC for $t = c_i$, and for $t < c_i$ IC's are flipped - mirror image of IC's above the horizontal line. At the first best contract for type *i*, theIC's for P and the A of type *i* are tangent, so $\frac{dt}{dx} = \frac{15 - t_i x_i}{x_i^2} = -\frac{(t_i - c_i)}{x_i}$, so $x_i = 15/c_i$, and P's utility is max'd when $t_i = c_i$.

Unfortunately, Sharda cannot observe anyone's type, even though she knows each student is fully informed about their own type.

d) (2) Suppose Sharda posted a job advertisement offering the contracts derived in (b). Which contract would applicants choose, and why?

For a high productivity student,

$$U_h^A(x_h, t_h) = (4/3 - 4/3)11.25 = 0 < U_h^A(x_l, t_l) = (2 - 4/3)7.5$$

while for a low productivity student,

$$U_l^A(x_h, t_h) = (4/3 - 2)11.25 < 0 = U_l^A(x_l, t_l) = (2 - 2)7.5$$

therefore both types of students prefer the contract designed for the low productivity student.

e) (6) If Sharda wishes different contracts to be chosen by high and low productivity workers, briefly explain what constraints these contracts must satisfy.

If she chooses to hire both types, contracts must satisfy the participation constraint for each type of student, so each student is willing to accept their contract: $U_i^A = x_i(t_i - c_i) \geq 0, i = h, l$

Also, for self-selection, the incentive compatibility constraints must be satisfied:
for high productivity: $U_h^A(x_h, t_h) = x_h(t_h - 4/3) \geq x_l(t_l - 4/3) = U_h^A(x_l, t_l)$
for low productivity: $U_l^A(x_l, t_l) = x_l(t_h - 2) \geq x_h(t_h - 2) = U_l^A(x_h, t_h)$
These constraints specify that an agent receives a (weakly) higher utility from choosing the contract designed for their type than from choosing the contract designed for the other type.

f) (6) Which type of student receives "information rents" under the equilibrium contracts with asymmetric information? Why?

The high productivity student receives "information rents". Under the first best contracts, this student has an incentive to pretend to be of low productivity, since their utility is higher with this misrepresentation - in this example, it takes the high productivity less time to proofread the number of pages demanded of the low productivity student, and even though the pay is lower at proofreading the overall utility is higher, since they can use the extra time at the job paying $16/hr. Since information regarding type has value for the principal when type is not costlessly observable, this agent must be reimbursed for revealing this information.

g) (4) How would the information rents be affected if Sharda were not willing to hire low productivity students?

If only high productivity workers were wanted, then the first best contract for high productivity workers could be offered even with hidden types. Why? - because a low productivity student would not be willing to forego the $16/hr at the other job to proofread under this contract, so only high productivity workers would apply.