Multi tasking and incentives (L&M, ch. 5.2):

Set-up: single agent, multiple tasks for principal
Ex: retail: stocking shelves and selling
Academic: research and teaching
Government: long-term research vs day-to-day tasks

Problem: how does tech'l interaction between tasks affect incentives?
How do incentive considerations affect optimal mix of efforts?

Simplest case: two tasks, with efforts

Efforts: $\{e^1, e^2\}$, with $e^i \in \{0,1\}, i = 1,2$

Tasks: symmetric, same stochastic returns:
Prob ($s^i = \overline{s}) = \pi\left(e^i\right)$. So if
$e^i = 1, \pi(e^i) = \pi_1; e^i = 0, \pi(e^i) = \pi_0$
(notation: recall $\Delta\pi = \pi_1 - \pi_0$)

Outcomes? - three possible: $\{2\overline{s}, \overline{s} + \underline{s}, 2\underline{s}\}$

(outcome could be nonlinear for P, so
$u^P(s^1, s^2)$)
(notation: recall $\Delta s = \overline{s} - \underline{s}$)

Contract – three levels of payment, depending on aggregate effort exerted by A (notice importance of symmetry, here)

Payoffs:

- A's disutility of effort: Ψj, where j=0,1,2 denotes the number of high level efforts the agent provides. $\Psi_1 > \Psi_0$, since providing one level of high effort is more arduous than providing none. Are tasks complements or substitutes for the agent?

  Substitutes if $\Psi_2 > 2\Psi_1$, so providing high level of effort at one task is more costly if already providing high level at other task;

  Complements if $\Psi_2 < 2\Psi_1$, so providing high effort at one task is less costly if already performing at high level on the other task.

1: P,A both Risk Neutral

i) First best:
High on both preferred if:

$$\Delta\pi\Delta s \geq \max\{0.5\Psi_2, \Psi_2 - \Psi_1\}$$

LHS: marginal benefit of two rather than one level of high effort;

RHS: second term is marginal cost of high effort on two tasks rather than only one; first term is average cost of two high efforts;
if tasks are substitutes, second term is max;
if complements, incremental benefit must be at least as high as the average cost of high levels of effort on both tasks.

(Think about relationship of average and marginal costs.)

.ii) Moral hazard and limited liability for A:

If P wants both efforts high, should give A positive payoff only when both tasks are successful, and t=0 otherwise.

Constraints:

- both high better than only one high for A if $\pi_1^2\bar{t} - \Psi_2 \geq \pi_1\pi_2\bar{t} - \Psi_1$ (LM call this the "local incentive constraint"

- both high better than both low for A if $\pi_1^2\bar{t} - \Psi_2 \geq \pi_0^2\bar{t}_1$ (LM: global incentive constraint)

- together, these imply $\bar{t} \geq \frac{1}{\Delta\pi}\max\{\frac{\Psi_2 - \Psi_1}{\pi_1}, \frac{\Psi_2}{\pi_1 + \pi_0}\}$

- note that $\frac{\Psi_2 - \Psi_1}{\pi_1} \geq \frac{\Psi_2}{\pi_1 + \pi_0}$ iff $\frac{\pi_0\Psi_2}{\pi_1 + \pi_0} \geq \Psi_1$

Outcomes?

- P chooses low effort more often than under full info

  - if $\frac{\pi_0\Psi_2}{\pi_1 + \pi_0} \geq \Psi_1$, so binding constraint is both high better than one high, (so local incentive constraint is binding), *then* P induces both high less often than under full info; why?

2nd best cost of inducing high effort on additional task exceeds the incremental cost under 1st best.

- otherwise, both high better more often than under full info – "incentives create a complementarity between tasks which goes counter to the technological diseconomies of scope" (LM, p 207)

if tasks are complements: global constraint always binding, so always harder to induce both efforts under asymmetric info (moral hazard, here) than under full info – case much like that of a single activity – so under all circumstances P induces both constraints less often under asym. info.

5.2.3: risk averse agent:

Introduce notion of diseconomies of scope : both in participation, and incentives.

If tasks are substitutes, the incentives diseconomies of scope further reduce the set of parameters for which high effort on both tasks is optimal under asym info.

If tasks are complements, there would be incentives economies of scope (text does not work out this case, but says the analysis would be straightforward).

5.2.4: Asymmetric tasks

Differences:

- Prob ($s^i = \overline{s}) = \pi^i \left(e_k^i\right)$.

- Now need a fourth element in contract, to distinguish between outcomes $(\overline{s}^1, \underline{s}^2)$ and $(\underline{s}^1, \overline{s}^2)$

- Now two local incentive constraints (one to induce high effort on both rather than high on 1st, low on 2nd and one to induce high on both rather than low on 1st, high on 2nd)

LM introduce specific expression for inverse of A's utility function:

- $h(u) = u + \frac{ru^2}{2}$, with $r > 0, u \geq -1/r$
  - o recall that LM's notation has A's utility function as $U = u(t) - \Psi(e)$, with u(t) increasing and concave, normalized so that u(0) = 0. Frequently analysis is easier if change variables to work with

inverse of A's utility function:

$$h = u^{-1}, \text{ with } h', h'' > 0$$

When tasks are substitutes, local incentive constraints and participation constraint are all binding; global incentive constraint is always slack (Prop'n 5.7, LM p. 217; note: proof is in appendix 5.3, pp 238-9)