Extension to basic model:

Agent's cost function is non-linear:

$U^A(t,q;\theta) = t - C(q,\theta)$ with $C_i > 0, i = q,\theta$,

$C_{qq} > 0$: increasing marginal cost of quantity

$C_{qq\theta} > 0$: MC increases more quickly for higher θ

Single crossing property / Spence Mirlees property:

$C_{q\theta} > 0$

(Do we really need the condition on the third derivative of the cost function??)

What does this change in the problem?

Nothing, really: the same constraints are binding, and the optimal contract looks the same:

i) first best quantity for efficient A
ii) lower than first best q for inefficient A
iii) efficient A gets positive info rents
iv) ineffcient A receives res'n utility

Examples:

1. Monopoly: nonlinear pricing (second degree price discrimination)

Setting:
consumers vary wrt willingness to pay for high quality: *vertical differentiation*;
demand one unit (think large consumer durables, membership in golf clubs,…)

Question: what qualities will firm put on the market?

1. Monopoly and 2nd degree price discrimination:

basic setting: single seller, heterogeneous consumers (ex: utilities, different sizes of laundry detergent…)

can set single price, *or* establish a nonlinear price schedule: unit price varies with quantity

Model: (LF give general; Campbell, ch 3.5, specific)

Principal = producer/seller
agent = consumer

Monopoly producer of electricity
cost: $1/unit (constant marginal cost

Consumers:

high benefit: $U(p,q;\overline{\theta}=4)=4\sqrt{q}-p$

low benefit: $U(p,q;\underline{\theta}=2)=2\sqrt{q}-p$

<u>First best</u> (perfect price discrimination):

monopoly charges each group max'm willingness to pay (takes all consumer surplus) - max's social surplus = profits + CS

high benefit consumer: $\max_q(4\sqrt{q}-q)\Rightarrow q=4$

low benefit consumer: $\max_q(2\sqrt{q}-q)\Rightarrow q=1$

prices? p_h=8, p_l =2

Contracts: (q,p)={(4,8),(1,2)}

surplus produced? High benefit: 2, low benefit = 0

<u>With asymmetric information</u>:

Who has incentive to misrepresent benefits?

Check to see if first best contracts satisfy SS constraints:

high benefit: U_h(4,8)=0; U_h(1,2)=2>0: SS violated low benefit: $U_l(1,2)=0$; $U_l(4,8)=-4<0$: SS satisied

If hold $U_l=0$ with asym contract (q_l, p_l), then SS for high benefit consumer is

$$4\sqrt{q_h} - p_h \geq 4\sqrt{q_l} - p_l$$

$$= 2\sqrt{q_l} + 2\sqrt{q} - p_l = 2\sqrt{q_l}$$

Spse v=0.25: 1/4 of all consumers are high benefit

Then P's problem under asym info: choose $\{q_l, q_h\}$ to max $0.25\times\{4\sqrt{q_h} - q_h\} + 0.75\times\{2\sqrt{q_l} - q_l\}$
$-0.25\times 2\sqrt{q_l}$

FOC's:

for q_h, same as first best - so $q_h=4$

for q_l, $\frac{\partial}{\partial q_l} = 0.75[\frac{1}{\sqrt{q_l}} - 1] - 0.25\frac{1}{\sqrt{q_l}}=0$

so $q_l = 4/9 < 1$ = first best quantity

Prices? $U_l = 0$, so $p_l = 2\sqrt{q_l}$ =4/3 < first best price (=2)

$$U_h = 2\sqrt{q_l} = 4/3, \text{ so}$$

$$p_h = 4\sqrt{q_h} - 4/3 = 20/3$$ < first best price (=8)