Bond Practice Questions and Answers

1. What is the present value of the following payments?
   (a) $1000 two years from now when the effective annual interest rate is 10%.
   (b) $1000 two years from now when the bond equivalent yield is 10%.
   (c) $1000 one-half year from now when the yield on a discount basis is 10%.
   (d) Which of the above payments would you prefer?
   If the above were bonds then, under our assumption that the yield corresponds to the price, $P = PV$.
   (e) Given the prices found in (a)-(c), derive the corresponding rate.
   (f) What is the bond equivalent yield corresponding to the YTM in (a).
   (g) What is the YTM corresponding to the bond equivalent yield in (b)-(c).

2. Fill in the yields in the table for discount bonds with $F = $1000

<table>
<thead>
<tr>
<th>Price P</th>
<th>Maturity n</th>
<th>Yield on a discount basis</th>
<th>YTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>900</td>
<td>1 year</td>
<td></td>
<td></td>
</tr>
<tr>
<td>950</td>
<td>½ year</td>
<td></td>
<td></td>
</tr>
<tr>
<td>975</td>
<td>¼ year</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. The YTM on the bond is 5% and the coupon rate is 2% with annual coupons.
   (a) What can you say about the price of the bond?
   (b) What is its price if it is a 30-year bond and has a face value of 100,000?
   (c) What is the Holding Period Return ($HPR$) on the bond after 1 year if the yield to maturity drops to 2%?
   Actual bonds are quoted at the bond equivalent yield. The quoted yield is 5% and the coupon rate is 2% with semi-annual coupons. The bond has term to maturity of 30 years. Suppose you don't know the face value.
   (d) What is its price as a percentage of face value?
   (e) What is the holding period return on the bond after 6 months if the quoted yield drops to 2%? (This drop from 5% to 2% is what happened in 2009.)

4. Consider two bonds. A consol with yield 10%. A two-year coupon bond is selling at par (i.e. face value) has yield to maturity 10%. Both bonds pay coupons yearly. At the end of the first year, the yields on all bonds fall to 5%. Which bond earns a higher holding period rate of return?

5. As part of a promotion you are offered a car loan for $30,000 at a YTM of 2% with payments made annually over 5 years.
   (a) What is the annual fixed payment?
   (b) How much would you save per year relative to taking out the same loan at a bank at 7%? How much would you save in present value terms?
   Instead of a promotion with a lower interest rate you are offered the car at a lower price but at 7%.
   (c) At what price are you better off buying the car at the higher yield?
More realistically, the car loan for $30,000 is quoted 2% (i.e. bond equivalent yield) interest with payments monthly over 5 years.

(d) What is the monthly fixed payment?

(e) How much would you save in present value terms relative to taking out the same loan at the bank's quoted rate of 7%?

(f) If the promotion wasn't offered, how much should the person be willing to pay for the lower interest rate?

Answers

1. What is the present value of the following payments:

   (a) $1000 two years from now when the effective annual interest rate is 10%.
      Given: simple loan \( i = .1, n = 2, F = 1000 \).
      Find: \( PV = \frac{F}{(1+i)^n} = \frac{1000}{(1+.1)^2} = \$826.446 \)

   (b) $1000 two years from now when the bond equivalent yield is 10%.
      Given: \( ibey = .1, n = 2, F = 1000 \).
      Assuming semi-annual compounding, then \( i_{1/2} = ibey/2 = .1/2 = .05 \).
      Find: \( PV = \frac{F}{(1+i_{1/2})^{n(2)}} = \frac{1000}{(1+.05)^4} = 822.703 \)

   (c) $1000 one-half year from now when the yield on a discount basis is 10%.
      Find: \( PV = \frac{F}{(1+i_{1/2})^{n(2)}} = \frac{1000}{(1+.05)} = \$952.381 \)

   (d) Which of the above payments would you prefer? – (c) ; i.e. in ½ year.
      Ceteris paribus, receiving money sooner allows for reinvestment sooner.
      $1000 reinvested at positive interest after ½ year produces more than $1000 in two years.

      If the above were bonds then, under our assumption, \( P=PV \).

   (e) Given: \( P \) in (a)-(c). Derive: interest rate given above.
      \[
      \begin{align*}
      (a) & \quad i = \left( \frac{F}{P} \right)^{1/n} - 1 = \left( \frac{1000}{826.446} \right)^{1/2} - 1 = 0.1, \text{ similarly for (b)&(c)} \\
      (f) & \quad \text{Given: } i = .1. \text{ Find: } ibey_{1/2} = i_{1/2}(2) = 0.048809(2) = 0.097618 \text{ where } (1+i) = (1+i_{1/2})^2 \text{ implies } i_{1/2} = (1+i)^{1/2} - 1 = 1.1^{1/2} - 1 = 0.048809 \\
      (g) & \quad \text{Given: } ibey = .1. \text{ Find } i = (1+ibey/2)^2 - 1 = 0.1025 \\
      \end{align*}
      \]

2. Fill in the yields in the table for discount bonds with \( F=\$1000 \)

<table>
<thead>
<tr>
<th>Price ( P )</th>
<th>Maturity ( n )</th>
<th>Yield on discount basis</th>
<th>YTM</th>
</tr>
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<tbody>
<tr>
<td>900</td>
<td>1 year</td>
<td>(1000/900)-1 = .111</td>
<td>.111</td>
</tr>
<tr>
<td>950</td>
<td>½ year</td>
<td>( 2i_{1/2} = 2[(1000/950)-1] = .052632 )</td>
<td>.1080332</td>
</tr>
<tr>
<td>975</td>
<td>¼ year</td>
<td>( 4i_{1/4} = 4[(1000/975)-1] = .025641 )</td>
<td>.1065767</td>
</tr>
</tbody>
</table>
3. Given: \( i = .05 \), CouponRate = .02 = \( C/F \), \( n = ? \), \( F = ? \)
   (a) Find: \( P < F \iff i = .05 > .02 = \text{CouponRate}; \) i.e., price is less than face value.
   Given: \( n = 30 \) and \( F = 100000 \)
   (b) Find: \( P = \frac{C}{i} \left( 1 - \frac{1}{(1+i)^n} \right) + \frac{F}{(1+i)^n} \)
      \[ \begin{align*}
      \frac{.02(100000)}{.05} & \left( 1 - \frac{1}{(1+.05)^{30}} \right) + \frac{100000}{(1+.05)^{30}} = 53883.0
      \end{align*} \]
   Given: \( i \) drops to \( i = .02 \) at the end of the year.
   (c) Find: \( HPR = \frac{C + P - P_1}{P} = \frac{2000 - 100000 - 53883}{53883} = 0.89299 \)

   where \( P_1 = F \) as \( i = \text{CouponRate} \).
   Given: CouponRate = .02 = \( 2C/F \), \( i_{be}y = .05 \) (quote is bey), \( n = 30 \), \( F = ? \)
   (d) Find: \( P \) as percentage of \( F \)
      \[ \begin{align*}
      P = \frac{C}{i_{1/2}} \left( 1 - \frac{1}{(1+i_{1/2})^{n_{1/2}}} \right) + \frac{F}{(1+i_{1/2})^{n_{1/2}}} \n      \end{align*} \]
      \[ \begin{align*}
      &= \frac{.02F/2}{.05/2} \left( 1 - \frac{1}{(1+.05/2)^{60}} \right) + \frac{F}{(1+.05/2)^{60}} = 0.5363702F
      \end{align*} \]
      Quotes are often made as a percentage of face value, 53.64%.
      Given: \( i_{be}y \) drops to \( i_{be}y = .02 \) at the end of the year.
   (e) Find: \( HPR = \frac{P_{1/2} + C - P_0}{P_0} = \frac{F + (0.02F/2) - 0.5363702F}{.5363702F} = .883 \) or 88.3%

   where \( P_{1/2} = \frac{.02F/2}{.02/2} \left( 1 - \frac{1}{(1+.02/2)^{29.5/2}} \right) + \frac{F}{(1+.02/2)^{29.5/2}} = F \)

   (Advanced: It turns out that \( \text{CouponRate} = i_{be}y \iff P = F \) for semi-annual compounding.)

4. Consider two bonds. A consol yield to maturity 10%. A two-year coupon bond is selling at par (i.e. face value) has yield to maturity 10%. Both bonds pay coupons yearly. At the end of the first year, the yields on all bonds fall to 5%. Which bond earns a higher holding period rate of return (\( HPR \))?

   Consol: \( HPR = \frac{P_1 + C - P_0}{P_0} = \frac{C/.05 + C}{C/.1} - 1 = \frac{1/.05 + 1}{1/.1} - 1 = 1.1 \),
   where \( P_1 = C/i \) at time 1 \( i = .05 \).
   Coupon Bond: Given \( P_0 = F, i = 0.1 \).
   As \( P_0 = F \iff i = \text{CouponRate} \) implies \( i_c = 0.1 = C/F \)
   \[ \begin{align*}
   HPR = \frac{P_1 + C - P_0}{P_0} = \frac{[(C + F)/1.05] + C}{P_0} - 1 = \frac{(C/F + 1)/1.05 + C/F}{P_0} - 1 \\
   = \frac{(1+1)}{1.05} + .1 - 1 = .147619
   \end{align*} \]

   where \( P_1 = (C+F)/1.05 \) as there is one year to go for the bond, and \( (C/F)=.1 \) is the coupon rate when coupons are paid annually. The consol has a much higher \( HPR \). This is because the term to maturity is much greater.

   Advanced: Is there a general solution when we don’t know the coupon rate? -Yes.
\[
HPR = \frac{P_t + C - P_0}{P_0} = \frac{[C + (C + F) / 1.05] + C}{(1 + .1)^2} - 1
\]

As \( \text{CouponRate} = C/F \), then \( C = \text{CouponRate} \cdot F \).

Then substituting \( C = \text{CouponRate} \cdot F \), the \( F \)'s cancel leaving
\[
HPR = \frac{[\text{CouponRate} + 1] / 1.05 + \text{CouponRate}}{(1 + .1)^2} - 1 = \frac{(1.1)^2 1 + (2.05)\text{CouponRate}}{1.05 + (2.1)\text{CouponRate}} - 1,
\]
for any \( i \geq 0 \) the return lies in the interval: \( 0.1294 < HPR < 0.152 \).

The consol always earns a higher rate of return.

5. Given: Fixed payment loan \( LV = 30,000 \), \( i = .02 \), \( n = 5 \), \( FP \) is yearly.

(a) Find:
\[
FP = \frac{LV(i)}{1 - (1+i)^{-n}} = \frac{30000(.02)}{1 - (1+.02)^{-5}} = 6364.8
\]
Note: \( LV = \frac{FP}{i} \left( 1 - \frac{1}{(1+i)^n} \right) \)

(b) Find:
\[
FP_{i=.07} - FP_{i=.02} = 7316.7 - 6364.8 = 951.9 \text{ per year},
\]
where \( FP_{i=.07} = \frac{30000(.07)}{1 - (1+.07)^{-5}} = 7316.7 \)

Find: PV of savings \( PV = \frac{951.9}{.07} \left( 1 - \frac{1}{(1+.07)^5} \right) = 3899.3 \)

using payment loan formula and the bank rate as the opportunity cost of funds. Because the savings are discounted they are less than the accounting total saving \( 951.9(5) = 4755.45 \).

(c) Find: \( LV = 26,100.7 \) at \( i=7 \) is when indifferent between the loans.
This \( LV \) at \( i=7 \) implies the same fixed a payment as \( FP_{i=.02} \)
\[
FP_{i=.07} = \frac{LV(07)}{1 - (1+.07)^{-5}} = \frac{26100.7(.07)}{1 - (1+.07)^{-5}} = 6365.7
\]
Note: 30000 - 3899.3 = 26,100.7 .
Repeat given: \( i_{bei} = .02 \), \( n = 5 \), \( FP \) is monthly.

(d) Find:
\[
FP = \frac{LV(i_{1/12})}{1 - (1+i_{1/12})^{12(n)}} = \frac{30000(00166)}{1 - (1+.00166)^{60}} = 525.724 ,
\]
where \( i_{1/12} = (1 + i_{bei}/2)^{1/6} - 1 = (1 + .01)^{1/6} - 1 = 1.65976 \times 10^{-3} \)
Note: \( LV = \frac{FP_{i_{1/12}}}{i_{1/12}} \left( 1 - \frac{1}{(1+i_{1/12})^{12(n)}} \right) \)

(e) Find:
\[
FP_{i=.07} - FP_{i=.02} = 592.622 - 525.724 = 66.898 ,
\]
where \( FP_{i=.07} = \frac{LV(i_{1/12})}{1 - (1+i_{1/12})^{12(n)}} = \frac{30000(00575)}{1 - (1+.00575)^{60}} = 592.622 \)
where \( i_{1/12} = (1 + i_{bei}/2)^{1/6} - 1 = (1 + .035)^{1/6} - 1 = 5.750039 \times 10^{-3} \)

Find: The present value savings is:
\[
66.898 \left( 1 - \frac{1}{(1+.00575)^{60}} \right) = 3386.542;
\]

(f) An individual should be willing to pay up to $3386.54 for the lower interest rate. Note monthly payment lead to less savings than in part (b).