Output, Inflation, and the Quantity Theory of Money

Ref: Chap 20 p454-57, 466-73
& Chap 21 p481-89
Aggregate Output (Income)

Aggregate Output is measured by GDP, Gross Domestic Product: The market value of final goods and services produced in a country during a year.

Aggregate Income is measured by GNI, Gross National Income: Total income of factors of production (land, capital, labour) during a year.
GDP and GNI are often very about the same magnitude.

Questions: Why do they differ in magnitude, and when are they the same? When is it better to use GDP?
• Real versus Nominal GDP:
  - Distinguish changes in prices from changes in quantities.
  - Real GDP uses base-year prices and isolates change in quantities.

\[
\text{GDP Deflator} = 100 \times \frac{\text{Nominal GDP}}{\text{Real GDP}}
\]
$Y_t$ denotes *Aggregate Real Output*

Note: $t$ indicates output over a time period. e.g. year $t = 2012$ so $Y_{2012}$ is aggregate real income from the beginning to the end of the year 2012.

Note: Output is a “flow variable” (in units per period of time)
The Price Level

$P_t$ denotes the *Aggregate Price Level*

\[
P_t = \frac{\text{Aggregate Nominal Output}}{\text{Aggregate Real Output} (Y_t)}
\]

- A price is a level variable, so subscript $t$ corresponds to a specific time in period $t$, usually the end of period $t$.
- $P_t$, like the $CPI_t$, is in dollars per unit quantity of goods and services.
\[ \pi_t \equiv \Delta P \equiv \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1 \]

where "\( \equiv \)" denotes “defined as”.

- The inflation rate for period \( t \) is the change between end of period \( t \) and end of period \( t-1 \).
- This inflation rate is expressed as a fraction. To express as a percentage multiply by 100.
- The text denotes the percentage inflation:
  \[ \% \Delta P \equiv \frac{\Delta P}{P_{t-1}} \times 100 \]
• Proportional change in the price level from the previous period.
• An increase (decrease) in the price level is referred to as “inflation” (“deflation”).
• “Hyperinflation” describes a very high rate of inflation, usually 100% or more a year i.e. $\pi_t \geq 1$.

  e.g. p322, Bolivia in 1985 $\pi_t = 110$
  Ukraine in 1983 $\pi_t = 50$

(FYI: my paper, Currency Transactions Costs and Competing Fiat Currencies explains why domestic money continues to be used in hyperinflations.)
e.g. In German in 1923, the rate of inflation hit $3.25 \times 10^6$ percent per month (prices doubled every two days).

Question: At this rate, what is the annual (net) inflation rate? (In your calculations, first work out the gross inflation rate.)

- Deflation corresponds to a negative inflation rate: $\pi_t < 0$.
  - In a deflation, money increases in value!

See [Inflation Propaganda Film](up to 9:25)
The Value of Money

\( v_t \) denotes the Value of Money: in units of goods and services per dollar.

\[ v_t = \frac{1}{P_t} \]

e.g. A doubling of the price level results in the value of money going down by half. Thus, money buys 50% less goods and services.

Ex. What is the relationship between the rate of inflation, \( \pi_t \), and the rate of return on money, \( \nu_t \), where \( \nu_t \equiv \frac{v_t - v_{t-1}}{v_{t-1}} \)?
The Bank of Canada (BOC) follows an inflation-targeting policy, which aims at 2% inflation and to keep (core) inflation within bounds of 1% to 3% (see Ch. 20).

Ex. What are the implications of this policy for the path of prices and the value of money. Start at $t = 0$ with $P_0 = 1$.

(FYI: For my evaluation of inflation policies see [When is Price-Level Targeting a Good Idea?](#))
Equation of Exchange

\[ M_t V_t = P_t Y_t \]

Quantity of Money \((M_t)\)

\( \times \) Velocity of Money \((V_t)\)

\( = \) Nominal \(GDP_t\)

where Nominal \(GDP_t\)

\( = \) Price level \((P_t)\) \(\times\) Real Output \((Y_t)\)
Equation of Exchange (Cont’d)

\[ M_t V_t = P_t Y_t \]

implicitly defines the Velocity of Money:

\[ V_t = \frac{P_t Y_t}{M_t} \]

\( V_t \) is the average number of times a year that a dollar is spent.

Caution: Be careful to distinguish \( V_t \), \( v_t \) and \( \nu_t \).
Equation of Exchange (Cont’d)

\[ M_t V_t = P_t Y_t \]

implies the approximation \((\approx)\) used in Ch. 21:

\[
\%\Delta M_t + \%\Delta V_t \approx \%\Delta P_t + \%\Delta Y_t
\]  

\((15)\)

\%Money Growth + \%Velocity Growth

\approx \%Inflation + \%Real Growth

It gives close to the exact value for small changes.

\textit{e.g.} Inflation = 2\%; Real Growth = 2.25\%; Velocity Growth=−0.75\%. Then (15) implies

\[
\%\Delta M_t \approx 2\% + 2.25\% + 0.75\% = 5\%.
\]

Question: Find the exact rate of money growth?
Figure 2.4  Money Growth and Inflation, 1969-2009

[Graph showing the relationship between money growth rate and inflation two years later, with data points for 1969-1975 (triangle), 1976-1981 (diamond), and 1991-2007 (square).]
Money Growth and Inflation (cont’d)

A. All Countries

- Nicaragua
- Angola
- Bolivia
- Ukraine
- Armenia
- Brazil
- Belarus
- Argentina
- Azerbaijan

B. Moderate-Inflation Countries

- 45° line

Average Annual Inflation Rate (%) vs. Average Annual Money Growth (%)
Figure 20.8  The Velocity of Money

A. Long-Run Velocity

SOURCE: Bank of Canada M1+ v37258; M2++ v41552790, both Table 176-0020; Statistics Canada, nominal GDP, v498074, from Table 380-0001, accessed 17 June 2009.
The Quantity Theory of Money combines the Equation of Exchange

\[ M_t V_t = P_t Y_t \]

with “Classical Dichotomy” assumptions.

The basic Quantity Theory of Money assumes velocity and output are constant and implies:

\[ P_t = \frac{V_t}{Y_t} M_t = \frac{V}{Y} M_t \]

The price level is determined solely by the quantity of money! (Equation describes AD.)
In turn, this implies

\[ \Delta P_t = \frac{V}{Y} \times \Delta M_t \]

The price level changes solely from changes in the quantity of money. This implies

\[ \pi_t = m_t \equiv \frac{M_t - M_{t-1}}{M_{t-1}} \]

The inflation is given by the rate of money growth, \( m_t \)!

*Ex. Show the steps in deriving \( \pi_t = m_t \).*
The basic Quantity of Theory of Money predicts that inflation increases at the same rate as money growth rate.

- On a plot the data would lie on the 45% line.
- Helps to explain why high inflation and money growth go together.
- It does not explain why low and moderate inflation countries fall below the 45% line and hyperinflation countries lie above the line.
Quantity Theory of Money (cont’d)
Inflation and Money Growth

A. All Countries

B. Moderate-Inflation Countries

Average Annual Inflation Rate (%) vs. Average Annual Money Growth (%) for various countries.
“Classical Dichotomy”: Only real forces determine real variables, and only the money supply determines the price level.

- Behavioral assumptions. In the long run:
  a) Aggregate output is at the full-employment (equilibrium) level;
  b) Velocity is only determined by the rate of technological progress.

- Thus $V_t$ and $Y_t$ are independent of money supply $M_t$. Money is said to be “neutral” i.e. *money neutrality*.

- In the basic quantity theory, which is without economic growth or technological progress, velocity and output are assumed constant

$$V_t = V \text{ and } Y_t = Y.$$
One of the oldest and most successful economic theories.

- First described by David Hume (1752), the famous philosopher, in his paper “On Money”
- Formulated by Irving Fisher, Milton Friedman and Robert Lucas, America’s greatest economists.

 Appropriately modified, it can explain why:

- Low inflation countries have inflation rates below the 45% line – because of positive output growth.
- High inflation countries lie above the 45% line – velocity increases as people turn over their money faster to avoid losing purchasing power.
The Quantity Theory yields a transaction demand for money, that does not depend on the interest rate. For example, with velocity constant, money demand, $M_t^D$, increases proportionately with nominal income:

$$M_t^D = \frac{1}{V} (P_t Y_t)$$

- With this theory, Monetarists (e.g. Milton Friedman) recommend targeting money growth, setting targets for $M_t^S$ in order to control inflation (see Ch. 21).

Ex. How would you set $M_t^S$ to implement an inflation-target path of 2% inflation?
Quantity Theory of Money Demand (cont’d)

• Canada was the first country to adopt Milton Friedman’s policy of targeting money growth. The policy started in 1975 but was considered a failure and abandoned in 1981. Velocity became unsteady when the BOC tried to target the money supply.

• Currently, the BOC targets inflation by using an interest rate, the *overnight rate*, rather than money supply to control the money market and inflation.

• It is widely believed that interest rates affect money demand so that the Classical Dichotomy doesn’t hold, at least in the short run.

Question: In the *Inflation Propaganda Film* is money neutral?
Quantity Theory of Money Demand (Cont’d)
Is the Velocity Constant?

Figure 20.8 The Velocity of Money

B. Percentage Change in M2++ Velocity

SOURCE: Bank of Canada M1+ v37258; M2++ v41552790, both Table 176-0020; Statistics Canada, nominal GDP, v498074, from Table 380-0001, accessed 17 June 2009.