Notes on the Diamond and Dybvig (1983) Model

1. Asset Structure
   - There are two kinds of assets: a storage asset and a productive asset.
   - The following flow table describes gross returns:

<table>
<thead>
<tr>
<th>Asset</th>
<th>Period T</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Storage to period 1</td>
<td>-1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Storage from 1 to 2</td>
<td></td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Storage from 0 to 2</td>
<td></td>
<td>-1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Productive asset liquidated</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Productive asset matures</td>
<td>-1</td>
<td>0</td>
<td></td>
<td>R &gt; 1</td>
</tr>
</tbody>
</table>

Here the productive asset weakly dominates storage in rate of return over all time horizons. As such it is described as a “liquid” asset.

The productive asset is said to be “illiquid”, if it pays less than 1 in period 1 (liquidated prematurely).
Exercise: Show that the bank equilibria below are unaffected if the productive asset is illiquid.

2. Agents and Preferences
   
   - *Ex ante* (i.e. time 0) agents do not know their own type.
   
   - *Ex post* (i.e. time 1) each agent discovers its own consumption type (consumption shock is realized)

There are two types of consumers:

- Type 1 only want to consume in period 1
- Type 2 only want to consume in period 2
- The proportion $t$ are type 1 consumers, and the remaining proportion $(1-t)$ are type 2 consumers.

Mathematically. *Ex post* : fraction

\[
\begin{align*}
t & \text{ type 1 agents each have utility } U(c_1^1) \\
1-t & \text{ type 2 agents each have utility } \rho U(c_1^2 + c_2^2)
\end{align*}
\]

where:

- $c_1^1$ consumption by type 1 in period 1,
\[ c_1^2 + c_2^2 \] consumption by type 2 in period 2; goods received in period 1 are stored to period 2 and then consumed,

\[ \rho < 0 \] is the subjective “discount rate”.

More assumptions:

1. Agents randomly realize their types in period 1 (not period 0).
2. Individual type is not observable or verifiable by others.
3. \( t \) is a fixed known number.
4. Utility function is twice continuously differentiable, increasing, strictly concave which implies \( u'' < 0 \)
5. The relative risk–aversion coefficient \[ \frac{|cu''(c)|}{|u'(c)|} > 1 \]
6. \( R\rho > 1 \), the return is sufficiently great.
3. Competitive Markets; i.e. Without Banks
   Type 1 consume at rate 1 in period 1
   Type 2 agents consume at rate R in period 2
   This outcome arises because there is no trade in competitive equilibrium:
   - At time 0, agents are identical and will not trade.
   - They invest in the productive asset as it dominates in return.
   - At time 1, type 1 agents liquidate their asset and consume.
   - At time 1, type 2 agents do not liquidate their asset but let it mature for return R > 1 in period 2.
   - There is no advantage of trade in period 1 or 2.

4. Efficiency
   We can show the competitive outcome is generically inefficient.

   Consumption Possibilities Frontier for Society:
   - At time 0, the planner invest everything in the productive asset (as it dominates in return).
- At time 1, to provide for types 1, $tc_1^1$ must be liquidated, leaving $1-tc_1^1$ still invested;

- At time 2, remaining productive assets matures ands and yield $(1-tc_1^1)R$; as this is greater than the return to storage it is efficient when $c^*_1 = 0$, where * denotes efficiency.

- There are $(1-t)$ type 2 and each gets $c^2_2$ so that $(1-t)c^2_2$ is the total consumption by type 2’s.

Thus, the feasibility constraint facing society is:

$$(1-tc_1^1)R = (1-t)c^2_2.$$  

This can be rearranged as the equation of a line.

$$c^2_2 = \frac{R}{1-t} - \frac{t}{1-t}c^1_1R \quad (1)$$

This line is simply the CPF.

Exercise: Draw this frontier and identify the intercepts and slope. Show that the competitive outcome is simply one point on the frontier.

Adam Smith Question: Is the competitive equilibrium efficient?
To determine this we suppose there is a benevolent planner who maximizes ex ante (expected) utility with respect to $c_1^1$, $c_2^2$ subject to the CPF:

\[
\max \quad \text{EU} = tu(c_1^1) + (1-t)\rho u(c_2^2) \\
\text{s.t.} \quad (1-tc_1^1)R = (1-t)c_2^2
\]

This can be expressed an unconstrained optimization problem in $c_1^1$:

\[
\max \quad \text{EU} = tu(c_1^1) + (1-t)\rho u\left(\frac{(1-tc_1^1)R}{(1-t)}\right)
\]

The f.o.c. is \( u'(c_1^{1*}) = \rho Ru'(c_2^{2*}) \); To solve we use the CPF: \( (1-tc_1^{1*})R = (1-t)c_2^{2*} \).

Diagrammatically, the efficient outcome is where the indifference curve is tangent to the budget set. In general it will not be tangent at point (1, R). Thus, the competitive equilibrium is inefficient.

DD do not use explicit functions but show that under assumptions 2&3: \( 1 < c_1^{1*} < c_2^{2*} < R \).
Here agents want to smooth consumption relative to returns.

Example: $u = -1/c$, satisfies all assumptions.

At the efficient outcome, the planner is pooling risks optimally.

The competitive equilibrium is inefficient because the market cannot pool risk.

DD then examine how good is a bank in pooling risk.

5. A Bank Model

*Demand deposit contract*: the bank promises to pay $c_i$ to withdrawers in period 1 on a first come first serve basis as long as funds last. In period 2, the bank pays the remaining depositors their per capita share of the remaining funds.

- The contract respects what is known as the *sequential service constraint*, where withdrawers are paid on a first-come-first serve basis.
Can the bank implement the efficient outcome: i.e. $c^*_1$ to types 1 and $c^*_2$ to types 2?

- Yes, if it can get type 1 agents to withdraw in period 1 and type 2 agents to withdraw in period 2.

6. Bank Equilibria

Depositors’ Strategies:

- Type 1 depositors have a dominant strategy to withdraw in period 1.

- Type 2 depositors strategies depend on what they believe other type 2 agents will do.

Observe that if all type 2 agents choose to withdraw in period 2, this yields the efficient outcome.

An Efficient Bank Equilibrium

Consider the best response of a type 2 agent (You) when they believe that all other type 2 agents will “stay” in period 1 and withdraw in period 2.
- if you withdraw in period 1, You get payoff $c_1^2 = c_1^{!*}$
  (assuming any one agent is *small* relative mass of depositors).
- if you withdraw in period 2, You get payoff $c_2^{!*} > c_1^{!*}$
- your best response is to withdraw in period 2

Since all type 2 agents are symmetrically situated under these beliefs, they all will choose to withdraw in period 2.

Actions and beliefs are consistent.

This is a Nash Equilibrium in pure strategies, and this Nash Equilibrium is efficient.

An *(Inefficient) Bank-run Equilibrium*

Consider the best response of a type 2 agent (You) when they believe that all other type 2 agents will “withdraw” in period 1.
- If you withdraw in period 1, You get payoff:
with probability $\frac{1}{c_i^*}$ (Run)

0 with probability $1 - \frac{1}{c_i^*} > 0$

- If you do not withdraw, you get payoff 0.
- Your best response is to withdraw in period 1.
- As all type 2 agents are situated symmetrically they all will choose to withdraw in period 1.
- This is a Nash Equilibrium in pure strategies that is inefficient.

The analysis can be represented in a payoff matrix where the actions in period 1 are either to withdrawal (w) or stay (s):

<table>
<thead>
<tr>
<th></th>
<th>w</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>You</td>
<td>Run,Run</td>
<td>$c_1^{\text{<em>}}, c_2^{\text{</em>}}$</td>
</tr>
<tr>
<td></td>
<td>0, Run</td>
<td>$c_2^{\text{<em>}}, c_2^{\text{</em>}}$</td>
</tr>
</tbody>
</table>

Other type 2 agents
Two Equilibria:
- Actions (w, w) with payoff (Run, Run)
- Actions (s, s) with payoff \((c_2^*, c_2^*)\).

Equilibrium selection:
- We would like to believe that agents would coordinate on the efficient equilibrium.
- Theory and practice suggest the inferior equilibria can occur.
- We have to specify beliefs to get a determinate outcome.

7. Formally (as in DD)

Let \(r_1\) be the promised payment for period 1.

The period payoff functions for an agent \(j\) are:

\[
V_1(f_j, r_1) = \begin{cases} 
    r_1 & \text{if } f_j < r_1^{-1} \\
    0 & \text{if } f_j \geq r_1^{-1}
\end{cases}
\]
\[ V_2(f, r) = \max \{ R(1 - r_j f) / (1 - f), 0 \} \]

where \( f_j \) is the fraction of withdrawers' deposits serviced before agent \( j \);

\( f \) is the total number of demand deposits withdrawn in period 1.

The analysis in Section 6 uses \( r_i = c_i^* \).

8. Suspension of Deposit Convertibility

- Historically this was a common practice used to avert runs, as noted in Bernanke (1983).

Under suspension of convertibility:

- A bank has the right to suspend withdrawals after \( t \) proportion of people withdraw. If you are unable to withdraw in period 1, you can (attempt to) withdraw in period 2.
- Suspension allows productive assets to mature.

- Type 2 agents know that they will get $c_2^{2*}$ if they withdraw in period 2.

- Type 2 agents have a dominant strategy to wait to withdraw in period 2.

- The bank-run equilibrium is eliminated; only the efficient equilibrium exists.

More formally, with suspension of convertibility, the payoffs are:

$$ V_1(f_j, r_i) = \begin{cases} r_i & \text{if } f_j \leq \hat{f} \\ 0 & \text{if } f_j > \hat{f} \end{cases} \quad \text{where } \hat{f} = \frac{1}{r_i} $$

$$ V_2(f, r_i) = \max\{R(1-r_i\hat{f})/(1-f), R(1-r_i\hat{f})/(1-\hat{f})\} $$

We get the above analysis when $\hat{f} = t$ and $r_i = c_i^{*}$.

Note: the $V_2$ payoff is always positive unlike the case without suspension.
- type 2 agent that does not succeed in withdrawing in period 1 can always withdraw something in period 2.

- When \( \hat{f} = t \) and \( r_i = c_i^1 \), then type 2 agents can always withdraw \( c_2^* \), in which case they have a dominant strategy to withdraw in period 2.

9. Government Deposit Insurance

- Guarantees that the promised return will be paid to all who withdraw;

- Depositors have a dominant strategy to withdraw in the period in which they want to consume.

- The efficient equilibrium is the only equilibrium.

Why is deposit insurance provided by government instead of private company?

- DD show that gov’t can levying a tax/subsidy on the accounts of first period withdrawers \( ex \ post \) to achieve the optimal allocations

- This seems a strained argument as gov’t do not punish withdrawers.
• Private insurance could protect individual banks from non-systematic risk of a run.

• However, gov’t has deep pockets (backed up by taxation) and can support a whole banking system from a run; from systematic risk.

10. Random $t$ ("Aggregate Uncertainty")

No efficient equilibrium exists because the bank does not know $t$ in period 0 and hence cannot pick $c_1$ optimally.

- Under the sequential service constraint, the bank cannot first count and determine $t$ to then pay those wanting to withdraw in period 1 at the efficient rate.

- An near efficient equilibrium exists in which agents withdraw when they want to consume.

- A bank-run equilibrium exists.

- Suspension of convertibility prevents bank runs but leaves some type1 agents with nothing in period 1

- Government insurance works if it can tax first period withdrawals.