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The human development index as a criterion for optimal planning*

Merwan Engineer[1], Ian King[2] and Nilanjana Roy[1]

Abstract:

Purpose – The Human Development Index (HDI) and Gender-related Development Index (GDI) have become accepted as leading measures for ranking the human well being in different countries. The purpose of this paper is to identify the planning policies that improve these indices and to also suggest modifications to the indices that yield more sensible policies.

Design/methodology/approach – This paper solves the first-best welfare problem in which the planner maximizes a development index subject to resource constraints.

Findings – Planning strategies that maximize the HDI tend towards minimizing consumption and maximizing expenditures on education and health. Interestingly, such strategies also tend towards equitable allocations, even though inequality aversion is not modelled in the HDI. We show that the GDI generates optimal plans with similar properties, and we determine when the GDI and HDI generate consistent optimal plans. A problematic feature of the optimal plans is that the income component in the HDI (or GDI) does not play its intended role of securing resources for a decent standard of living. Rather, it acts to distort the allocation between health and education expenditure. We argue that it is better to drop income from the index. Alternatively, we consider net income, income net of education and health expenditures, as indicating capabilities not already reflected in the index. Finally, we compare how the modified indices and the HDI rank countries.

Originality/value – The paper integrates development indices into national development planning. This is the first paper, of which we are aware, to do so.

Keywords Human Development Index; Gender-related Development Index; Income; Inequality; Planning.

Paper type Research paper

JEL classification: O21; O15

[1] Department of Economics, University of Victoria, Victoria, Canada.

[2] Department of Economics, University of Melbourne, Melbourne, Australia.

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1. Introduction

The Human Development Index (HDI) is a composite index published annually by the UN Human Development Report Office, since 1990, which is designed to measure "human well being" in different countries.[1] The index combines measures of life expectancy, school enrolment, literacy, and income to provide a broader-based measure of well-being and development than income alone. Since its publication, this index has become widely cited and is commonly used as a way of ranking the quality of life in different countries.

The impact of the HDI ranking on policy is reflected by the fact that some national governments have taken to announcing their HDI ranking and their aspirations for improving it. For example, in a recent speech, the President of India, Dr. Abdul Kalam, exhorted Indians to work together to improve India's existing HDI rank of 127 to achieve a rank of 20; (see Kalam (2005)). The HDI is discussed in recent Indian budgets (e.g. Union Budget and Economic Survey 2004-2005) and changes in India's ranking are covered by the media (e.g. Parsai (2006)). In announcing Canada's number one ranking in 1998, Prime Minister Jean Chrétien stated: "While the HDI tracks Canada's impressive achievements, it also tells us where we can improve." (Chrétien (1998)).

In this paper, we consider the implications of using the HDI as a criterion for economic development plans. In particular, we examine the consequences of pursuing plans that maximize the HDI score for a given country. To do this, we construct an economic model where a planner chooses expenditures to maximize a well-defined objective function that includes the HDI index as a special case.[2] We get two main results. First, the planner tends towards minimizing consumption and maximizing expenditures on education and health. We get this result despite the fact that the HDI includes an income index as one of its components. Second, the optimal plan tends to imply equitable allocations even though inequality aversion is not explicitly modelled in the HDI. This latter result is arguably a surprising beneficial consequence of using the HDI that addresses the concern for equity expressed in the Human Development Reports and literature (e.g. Anand and Sen (2000)). In contrast, the first result leads to what we consider to be a flaw with the HDI, but one that can be readily fixed.

The first result – that the planner tends towards minimizing consumption and maximizing expenditure on education and health – describes a lopsided allocation. The reason for the lopsided allocation is that consumption does not enter the index (the objective function) or the production technology, but costs the planner through the resource constraint, so the optimal plan will set consumption to meet minimum consumption requirements. From another perspective this lopsided allocation arises because the income component in the HDI only has one role in the optimal plan and that is to indirectly affect the allocations of funds between education and health.

The logic of the first result can be readily explained in our basic model where income can be decomposed into expenditures on consumption, education and health. As consumption is at its minimal level, the remaining expenditures are allocated to education and health. Education and health expenditures are directly valued in those components of the HDI and are also indirectly valued through the income component. The HDI is flawed because the income component "double counts" the value of allocating expenditures to education and health and because it does not effectively value expenditures on other items.

For these reasons we argue that it is better to drop income from the index in considering optimal plans rather using the HDI as it stands. However, dropping income from the index falls short of the original vision for the HDI as an overall composite index. The income component of the HDI was originally justified as an indirect proxy of "command over resources to enjoy a decent standard of living" (Human Development Report 1990, p. 1). Anand and Sen (2000, p. 86) state:

> The use of `command over resources' in the HDI is strictly as a residual catch-all to reflect something of other basic capabilities not already incorporated in the measures of longevity and education. … For example, going hungry is a deprivation that is serious not just for its tendency to reduce longevity, but also for the suffering it directly causes. Similarly, resources needed for shelter and for being able to travel may be quite important in generating the corresponding capabilities.

In the later part of the paper we argue for a modified HDI that replaces the income component with a net income component; i.e. income that is net of expenditures on education and health. Plans that maximize the modified index directly trade-off the allocations to consumption, education and health against each other. This leads to plans that balance expenditures across the three components.

Using data that includes OECD and a few other countries, we construct and calculate a modified human development index with net income component as well as a modified human development index without an income component. The rank orderings of nations given by these modified indices is compared with the HDI. Interestingly, the modified index without income yields the greater change in rankings.

The paper continues as follows. Section 2 develops the model. Section 3 solves the planner's problem, and Section 4 examines equity and extends the analysis to the Gender-related Development Index (GDI). Section 5 explores the role of income in the HDI index, and provides a critique of the income component. Section 6 compares the modified human development indices both theoretically and empirically with the HDI. Section 7 concludes.

2. The model

We consider a static closed economy model, where a planner acts to maximize the following objective function, which nests the HDI:

$$I(w,W) = wI^{y}(y) + (1-w)[WI^{e}(e) + (1-W)I^{l}(l)] \tag{1}$$

Here, $I^{y}(y)$, $I^{e}(e)$, and $I^{l}(l)$ represent indices of per capita income (y), educational attainment (e), and life expectancy (l) respectively. The indices are each bounded between 0 and 100, and are assumed to be differentiable, increasing and concave in their respective arguments.[3] The weight parameters w and W are bounded $0 \le w \le 1$ and $0 \le W \le 1$. We give income a separate weight w because we concentrate on that part of the index. Observe that the index can be rewritten as $I(w,W) = wI^y(y) + (1-w)I(W)$, where $I(W) = WI^e(e) + (1-W)I^l(l)$ is the part of the index not containing income. The HDI corresponds to a special case of this index, where w = 1/3 and W = 1/2, so that each of the three component indices are equally weighted.

Educational attainment is assumed to be a differentiable increasing function of per capita expenditures on both education (E) and health (H). Thus:

$$e = e(E,H), \quad e_E > 0, \; e_H \geq 0 \qquad (2)$$

Similarly, life expectancy is differentiable and increasing in both of these arguments[4]:

$$l = l(E,H), \quad l_E \geq 0, \quad l_H > 0, \quad l(0,0) > 0 \qquad (3)$$

To simplify the analysis, we are assuming that the economy in question has a level of per capita income high enough so that neither income nor consumption substantially affect life expectancy and educational attainment as measured in the HDI. This is formalized by the following assumption about individual consumption c:

$$c \geq c_{\min} \qquad (4)$$

where $c_{\min} > 0$ is a parameter which identifies the level of consumption beyond which no further increments in consumption will increase educational attainment or life expectancy.[5] Later we relax constraint (4) and show that the results become stronger.

In this simple static economy, we abstract away from capital and assume full utilization of labour. All individuals work and the total number of workers in the economy is normalized to one unit. Given this, output per capita is determined by the following differentiable production technology:

$$y = f(e,l,H), \quad f_e \geq 0, \; f_l \geq 0, \; f_H \geq 0 \qquad (5)$$

Here, education attainment affects output through human capital in the usual way. Increments in life expectancy increase the effective size of the labour force and thereby increase production. Life expectancy also increases production to the extent that it proxies healthier more productive workers.[6] However, insofar as life expectancy is an imperfect indicator of worker health, we include health expenditures as a variable that might also increase productivity.

Once produced, the single good in the economy can be allocated to three possible uses: aggregate consumption ($l \cdot c$), education expenditure (*E*), and health expenditure (*H*). Therefore, the economy must respect the aggregate constraint:

$$lc + E + H \leq y \tag{6}$$

Observe that consumption, *c*, is on items other than health and education and that we allow total consumption to be proportional to life expectancy. We only analyze situations where there is at least one feasible allocation (*c, E, H*) satisfying equations 2-6 and the non-negatively constraints $E \geq 0$ and $H \geq 0$. A sufficient condition for this is that $lc_{\min} \leq y$ when $E = H = 0$; i.e. minimum output can meet minimum consumption at minimum life expectancy.

3. Efficiency

Using equations 1-6, the planner's problem can be formulated as the programming problem P1:

$$\underset{\{c,E,H\}}{Max}\, I(w,W) = wI^{y}(f(l(E,H),e(E,H),H)) + (1-w)\left(WI^{e}(e(E,H)) + (1-W)I^{l}(l(E,H))\right)$$

subject to: i) $l(E,H)c + E + H - f\left(l(E,H),e(E,H),H\right) \leq 0$

ii) $-c \leq -c_{\min}$

Recall that the objective function (1) in the planner's problem is an index that nests the HDI as a special case. Thus, the following proposition also applies to the HDI.

Proposition 1. Consider the planner's problem P1.

(a) *Optimal plans, which maximize I(w, W), allocate the minimum to consumption,* $c^{*} = c_{\min}$*, and the remaining output to education and health expenditure,* $E^{*} + H^{*} = y^{*} - l^{*}c_{\min}$.

(b) *Incremental reductions in consumption c towards* $c_{\min}$ *increase I(w,W).*

The proof to this proposition is in the Appendix. The intuition behind these "efficiency" results is quite straightforward. Consumption does not enter the objective function or the production technology, but costs the planner through the resource constraint. Thus, reductions in consumption that are optimally allocated to education and health expenditures will increase the index. The optimal plan will set consumption to its minimal allowed value. Our formulation with a minimum consumption requirement clearly reveals that all remaining resources are allocated to those expenditures, education and/or health, which increase the objective function.[7]

The fact that the minimum consumption constraint is binding at the optimum implies that in the absence of the constraint that the planner would allocate even less to consumption and more to expenditures on education and health; i.e. if $c^* < c_{\min}$ then $E + H = y - lc^* > y - lc_{\min}$. Indeed, without the constraint, the planner's optimal choice would be $c^* = 0$ so that $E^* + H^* = y^*$. This unrealistic corner solution arises only because we have excluded consumption from the education, health and production functions on the grounds that $c \geq c_{\min}$ is sufficiently high not to affect these functions. If consumption $c < c_{\min}$ has a sufficiently positive effect (on any of education, health, or production) then the optimal choice would be $c^* \in (0, c_{\min})$ in the absence of the constraint. Minimal output is allocated to consumption when consumption only plays an instrumental role and is not valued directly.

Even under our simplifying assumption $c \geq c_{\min}$, the stark result $c^* = c_{\min}$ needs further qualification. Most obviously, we have assumed that there is sufficient income for this is to be a feasible allocation. Less obviously, our result depends on the efficacy of expenditures on education and health being valued by the planner; i.e. we have assumed that $e_E > 0$ and $l_H > 0$ as well as $I_e^e(e) > 0$ and $I_l^l(l) > 0$. These conditions ensure that the shadow prices of expenditures on education and health are always positive.[8] Then we get the result $c^* = c_{\min}$ because the shadow price of consumption is zero.[9]

4. Equity

Probably the most common concern raised with regard to the HDI is that it uses only per capita income and, therefore, is insensitive to large income disparities within countries. Thus, two countries with the same average income would be scored the same by the index, even though one country might have far more poor whose meagre 'command over resources' substantially inhibits their human development. For this reason Anand and Sen (2000), Foster *et al* (2005), and others, have argued that some sort of income inequality aversion should be built into the index explicitly. What seems to be missing in the literature is an analysis of how policies that promote human development as measured by the HDI affect inequality.

Our analysis has the surprising implication that policies that maximize the HDI score should dramatically reduce consumption inequality, *ceteris paribus.* The optimal policy according to Proposition 1 requires $c^* = c_{\min}$. This implies the optimal plan is egalitarian, at least with respect to consumption, even though no inequality aversion appears explicitly in the HDI itself. Though income inequality may still remain in a market economy, consumption inequality is what matters since a government following the optimal plan with access to non-distortionary taxation would tax away all disposable income leaving $c^* = c_{\min}$.[10]

To make the argument formally, we model individuals and derive how the planner would allocate expenditures to them. Suppose there are i =1, 2, … N individuals in the economy and each has an individual consumption constraint

$$c_i \geq c_{\min}(i) \tag{4'}$$

We also distinguish education and health expenditures and attainments by individual:

$$e_i = e_i(E_i, H_i) \tag{2'}$$

$$l_i = l_i(E_i, H_i) \tag{3'}$$

where these attainment functions are now assumed to be increasing and strictly concave in their arguments.[11] Except for differences in these attainments, we make the standard aggregate production assumption that individuals are equally productive and average education (e), longevity (l), and health expenditure (H) enter production as before. Since only averages enter the production and objective functions, the planner only cares about individual allocations insofar as they improve average output, education and longevity.

4.1 Homogenous Individuals

In our model, individuals are homogenous if they have the same minimum consumption needs, $c_{\min}(i) = c_{\min}$, and their education and longevity functions take the same form, $e_i = e(E_i, H_i)$ and $l_i = l(E_i, H_i)$. We have the following proposition.

Proposition 2. Consider an economy of homogenous individuals.

(a) *Optimal plans, which maximize I(w, W), feature*:

  (i) *Egalitarian allocations,* $c_i^* = c_{\min}$, $E_i^* = E^*$ *and* $H_i^* = H^*$ *for all i,*

  (ii) *Equal attainments,* $e_i^* = e^*$ *and* $l_i^* = l^*$ *for all i, and*

  (iii) *Expenditures* $E^* + H^* = y^* - l^* c_{\min}$.

(b) *Incremental reductions in average consumption from* c^1 *to* c^2*, where* $c^1 > c^2 > c_{\min}$*, increase the HDI score but need not be consumption inequality reducing.*

The proof to this proposition is in the Appendix. With respect to consumption, both 2(a) and 2(b) are straightforward generalizations of Proposition 1. However, there is an important proviso in 2(b): incremental reductions in average consumption may not reduce inequality. This follows simply from the fact that a reduction in average consumption can be achieved by reducing the consumption of a subset of individuals. Consider an example, where initially there is consumption equity with $c(i)=c^1=2c_{\min}$. The planner can increase the HDI score by reducing average consumption, say, to $c^2=1.5c_{\min}$. In achieving this incremental goal the planner is indifferent between any consumption distribution with $c(i) \geq c_{\min}$. Thus, the planner could reduce one half of the population to $c_{\min}$ and leave the other half of the population unchanged at $2c_{\min}$. In this example, an increase in the HDI score creates a substantial increase in inequality. Of course, the scope for dispersion in individual consumption narrows as average consumption approaches the minimum, $c^*(i) = c_{\min}$.

With respect to education and health, egalitarian expenditures are solely due to the concavity of the individual attainment functions $e(E_i,H_i)$ and $l(E_i,H_i)$ and does not spring from either imposing horizontal equity or from a Utilitarian objective function.[12] Overall, Proposition 2 indicates that, if governments use the existing HDI as an objective function to devise their plans, then this leads to equitable allocations and attainments – through the implied emphasis on maximizing funding to education and health.

4.2 Heterogeneity and the Gender-related Development Index (GDI)

Heterogeneity exists in society and it is a concern in human development that disadvantaged individuals and groups should not be overlooked. To this end the Human Development Report publishes, in addition to the HDI, several other human development indices that focus on gender and poverty. Of these indexes, the Gender-related Development Index (GDI) is the only other index that includes all individuals in society and hence is comparable to the HDI. Here, we explore the implications of gender heterogeneity for optimal plans and examine if there is a conflict between the optimal plans implied by the GDI and the HDI.

The GDI differs from the HDI by separately evaluating females and males and by explicitly controlling for aversion to inequality by gender. First, dimension indices are calculated for females and males. Second, for each dimension an "equally distributed index" is created from the harmonic mean of the female and male dimension indices. The GDI is the sum of the equally distributed indices cross the three dimensions and can be expressed generally as follows.

$$GDI = \sum_{d=y,e,l} w^d \left[p_f (I_f^d)^{1-\varepsilon} + p_m (I_m^d)^{1-\varepsilon} \right]^{1/(1-\varepsilon)},$$

where p_f and p_m are the female and male population shares and w^d, $0 < w^d < 1$, is the weight on dimension $d=y,e,l$. The parameter $\varepsilon \geq 0$ measures the aversion to inequality and in the index $\varepsilon = 2$. The female and male dimension indices for income and education (I_f^y, I_m^y, I_f^e and I_m^e) take the same form as in the HDI. However, the life expectancy index has different goalposts for each gender. This dimension index takes the linear following form:

$$I_g^l = \frac{l_g - LB_g}{UB_g - LB_g} \quad ,$$

where LB_g and UB_g are the lower bound and upper bound goalposts by gender $g = f, m$. These goalposts are specified: $LB_f = 27.5$ versus $LB_m = 22.5$ and $UB_f = 87.5$ versus $UB_m = 82.5$.

Proposition 3. Consider an economy where individuals are homogeneous within gender groups but may have heterogeneous abilities and needs according to gender.

(a) Optimal plans, which maximize the GDI, feature:

(i) *Minimum consumption according to gender,* $c_f^* = c_{f,\min}$ *and* $c_m^* = c_{m,\min}$ *, and*

(ii) *Unequal life expectancies for females and males*; *i.e.* $l_f^* \neq l_m^*$.

(b) Incremental reductions in average consumption (*toward the minimum weighted average* $p_f c_{f,\min} + p_m c_{m,\min}$) *increase the GDI score but need not be consumption inequality reducing.*

With respect to minimum consumption, 3(a)(i) follows directly from the logic of Propositions 1 and 2(a). However, as minimum consumption is now distinguished by gender, equality with respect to consumption obtains only if $c_{f,\min} = c_{m,\min}$. With respect to incremental reductions in consumption, 3(b) indicates a greater scope for inequality than 2(b). Here consumption equality does not necessarily decrease with the final increment to the optimum. For example, suppose $c_{f,\min} < c_{m,\min}$ and initially there is consumption equality at $c(i) = c_{m,\min}$. Then lowering female consumption to $c_{f,\min}$ maximizes the GDI score but creates consumption inequality.

In 3(a)(ii) we generally get unequal life expectancies because the life expectancy sub-indexes are different by gender. To see how this must generally drive unequal attainments, suppose males and females are homogenous and are given equal allocations *i.e.* $y_g^* = y^*$, $E_g^* = E^*$ *and* $H_g^* = H^*$ *for g = f, m*. Then $l_f = l_m$, implies I_m^l > I_f^l. However, with inequality aversion, the planner can increase the GDI by reallocating expenditures from males to females to increase female life expectancy.[13]

Though the GDI generates unequal life expectancy attainments, we now show that it does so because it aims for equal treatment after considering inherent gender differences. The GDI and the different goalposts by gender were adopted initially in the Human Development Report 1995 partly in response to Anand and Sen's (1994) criticism that the life expectancy bounds for females should be higher as the evidence suggests that females on average live longer *ceteris paribus.* This inherent difference in gender is formalized in the choice of the life expectancy bounds. Key features of the bounds which describe the lifespan are: (1) the range of achievement from the lower to upper bound for females and males is the same, $UB_f - LB_f = UB_m - LB_m = 60$, and (2) the difference between the upper bound values is the same as the difference between the lower bound values, $UB_f - UB_m = LB_f - LB_m = 5$.

To get a sense of how the life expectancy sub-index works, suppose the male life expectancy function is $l(E_m , H_m)$, and the female life expectancy function is

$$l_f(E_f , H_f) = 5 + l(E_f , H_f) ,$$

where the component $l(E_g , H_g)$ for $g = f, m$ is strictly concave. Under this specification, females live 5 years longer than males when the genders receive equal allocations, i.e. $E = E_f = E_m$ and $H = H_f = H_m$. This is consistent with the features on the life expectancy bounds as well as females living longer *ceteris paribus.* Although equal allocations result in females living 5 years longer, I_g^l measures the life expectancy achievements by the genders equally as explained below.

In the spirit of the GDI, which only distinguishes females and males asymmetrically with respect to life expectancy, we assume the following:

Assumptions About Gender (Attainments and Consumption).

(i) $l_f(E_f, H_f) = 5 + l(E_f, H_f)$ and $l(E_m, H_m)$ describe female and male life expectancy functions (where $l(E_g, H_g)$ is strictly concave for $g = f, m$); and

(ii) $e(E_g, H_g)$ for $g = f, m$, i.e. females and males have the same education attainment functions (and this function is strictly concave).

(iii) $c_{f,\min} = c_{m,\min} = c_{\min}$, i.e. females and males have the same minimum consumption.

Proposition 4. Under the Assumptions About Gender, females and males are inherently similar in all dimensions except life expectancy. Then optimal plans, which maximize the GDI, have the following features:

(a) *Egalitarian allocations; i.e.* $c_g^* = c_{\min}$, $E_g^* = E^*$ *and* $H_g^* = H^*$ *for g = f, m.*

(b) *Equal educational attainment,* $e_g^* = e^*$ *for g = f, m.*

(c) *Unequal life expectancy; i.e. females live on average 5 years longer than males,* $l_f^* = 5 + l_m^*$.

(d) *Plans which maximize the GDI also maximize the HDI provided that the solutions to both planning problems satisfy the HDI goalposts,* $25 \le l_g^* \le 85$ *for g = f, m.*

The proof to 4(a) is in the Appendix. The result obtains because the planner effectively approaches the sexes symmetrically. First observe that Assumption (i) implies that the life expectancy index for females, $I_f^l = (5 + l(E_f, H_f) - 27.5)/60 = (l(E_f, H_f) - 22.5)/60$, reduces to the same function as for males, $I_m^l = (l(E_m, H_m) - 22.5)/60$. The planner also effectively approaches the sexes symmetrically when considering the production function and resource constraint: by Assumption (i) the marginal component of life expectancy is the same for both sexes. As the attainment functions are strictly concave, optimization involves equal expenditure allocations across genders. Total income, $p_f y_f + p_m y_m = y$, is also allocated equally $y_f = y_m = y$. This is because in the GDI the logarithm of income is used in I_g^y which makes it strict concave in income.

Interestingly, aversion to inequality plays no role because equal allocations to males and females are consistent with equal achievements in all dimensions. With equal allocations, both 4(b) and 4(c) immediately follow. Equal allocations to the genders generate equal achievements as measured by the GDI but unequal attainments in the one dimension of inherent difference, life expectancy.

To establish 4(d) we have to show that maximizing the HDI also yields equal allocations among the genders. Note that an internal solution $25 \le l_g^* \le 85$ for $g = f, m$ implies $25 \le l_m^* \le 80$ and $30 \le l_f^* \le 85$. Suppose $25 \le l(E_g, L_g) \le 80$. Then we can decompose the HDI into female and male cohorts without loss of information: $HDI = p_f HDI_f + p_m HDI_m$, where HDI_g takes the same functional form as the HDI. Unlike with the GDI, here $I_g^l = (l_g - 25)/(85\text{-}25) = l_g/60 - 25/60$. Observe that I_g^l is linear in l_g which is in turn is linear in $l(E_g, L_g)$. Since I_g^l is weighted by 1/3 in HDI_g independent of gender, the planner faces the same marginal incentives to allocate expenditures to either gender. Thus an equal allocation to all individuals is optimal when there is an internal solution. It is reassuring that there exist reasonable assumptions for which optimal plans are the same under either HDI or GDI criteria.

In proving Proposition 4, assumption (iii) specifying $c_{f,\min} = c_{m,\min} = c_{\min}$ was only needed to establish that all allocations were egalitarian in 4(a). Allowing differences in minimum consumption by gender would weaken 4(a) to describing egalitarian expenditures on education and health. Otherwise, the proposition is unchanged. Finally, note that though individuals are allocated income in this planner's problem, they don't get to spend it as everything above their minimum consumption is taxed away. In this sense it is an irrelevant indicator, which leads to the critique below.

5. Critique of the Role of Income

The emphasis on education and health expenditures in optimal plans naturally leads us to consider what role income plays in the HDI. Recall that the income index $I^y(y)$ in the HDI was originally justified as an indirect proxy of "command over resources to enjoy a decent standard of living". Here we argue that income is a poor proxy for this purpose.

Proposition 5. Consider the weight w < 1 *on the income index* $I^y(y)$ *in the HDI. In determining optimal plans, the weight w affects only the trade-off between expenditures on education E and health H.*

This proposition immediately follows from the fact that in the optimal plan $c^* = c_{\min}$, so that remaining problem in P1 is how to allocate resources to *E* and *H*. This allocation is affected by $I^y(y)$ only because of the effects of *E* and *H* on production. In particular, the effects of *E* and *H* on production are entirely indirect, through life expectancy $l(E,H)$ and education $e(E,H)$, when $f_H = 0$. By way of contrast, both $l(E,H)$ and $e(E,H)$ have *direct* impacts on the indices $I^e(e)$ and $I^l(l)$ respectively.

The way that changing *w* affects the trade-off between *E* and *H* is generally complicated and ambiguous, since it depends on all partial derivatives of the functions. In the special case $f_l = f_e = f_H = 0$, where none of the inputs affect production, changing *w* has no affect on the optimal plan. Then $I(w, W)$ is simply an affine transformation of $I(W)$. Recall that $I(W)$ is the part of the index that excludes the income component. Hence in this special case, optimal plans are determined solely on the basis of parameter *W*, the relative weight on education versus health.[14]

Another way to see the problematic role of income is to examine how increases in output are apportioned. In the basic model $y = l \cdot c_{\min} + E + H$ and income growth does not affect the individual rate of consumption, *c*. Rather, $\Delta y = \Delta(E+H) + \Delta l \cdot c_{\min}$, so that output growth is correlated with expenditures $E+H$ and longevity *l*. Thus, the income component leads to the direct double counting of life expectancy. As the income component includes the inputs that increase $e(E, H)$ and $l(E, H)$, it indirectly double counts education and life expectancy. In the optimal plan, income does not fulfil its intended role of securing "command over resources to enjoy a decent standard of living".

5.1 Comparing the HDI with and without the Income Component

One response to our critique -- that the HDI effectively discounts the influence of income – might be that this is good news. Some who advocate the human development approach are critical of the role of income in the index on the basis that it isn't an achievement or functioning *per se* but an indirect proxy.[15] They are primarily concerned with how income is converted to education and health achievements. Proposition 5 reveals that with optimizing behavior, the weight on income w does not discourage education and health expenditures. However, the proposition also reveals that w distorts the choice of these expenditures. The problematic role of income in the HDI suggests that it should be excluded from the index.

Consider $I(W)$, the part of the index that excludes the income component. The discussion following Proposition 5 shows that maximizing $I(W)$ generally yields different allocations than maximizing $I(w, W)$. However, since optimization always implies $c^* = c_{\min}$ in both cases, the only difference between the allocations is in the division of E and H expenditures. These expenditures determine education and life expectancy.[16] This allocation should be done on the basis of the parameter W that weighs the achievement indexes for education and life expectancy. It should not depend through the constraints on another parameter w that is independent of these achievement indexes in the objective function. This suggests that income plays no useful role in the HDI and that a development index is better specified as $I(W)$.

5.2 HDI Rankings of Nations

The propositions have positive and normative implications for how the HDI ranks countries. First, consider two nations that are identical in every respect except policy. The country with the higher HDI will be the one that is doing a better job of maximizing the HDI. However, if we take the stance that $I(W)$ is the better development index, then we cannot say that the country with the higher HDI is doing a better job of improving human development. To make an assessment we must somehow control for the bias induced when $w > 0$.

Now consider countries that are only different according to multifactor productivity (e.g. $y = Zf(e,l)$, where Z is multifactor productivity). If all countries were optimizing their HDI, then the country that has a greater multifactor productivity would also have the greater HDI. Indeed, since the relationship is monotonic, the ranking of countries by HDI and ranking by y would be identical. Thus, under our assumptions, we get the prediction that the "development gap ranking", the rank of Gross Domestic Product (GDP) per capita less the rank of HDI, should be zero.[17] If this difference is not zero, some countries are not optimizing.

Again, however, if $I(W)$ is considered the better development index, we cannot be immediately assured that higher GDP per capita that leads to a higher HDI ranking is promoting development. In principle it is possible that more income can lead to a worse outcome when the incorrect objective function is being optimized. By similar reasoning we also cannot be generally assured that a reduction in the development gap ranking is promoting development. This is because the correct development gap ranking involves using the rank of $I(W)$ rather than the ranking of $I(w, W)$.

6. Modified Human Development Indices

This section proposes two indices that are minimal modifications of the HDI.[18] First, in general terms, consider the modified index $I(W)$, which is constructed by simply dropping the income component from the generalized HDI index $I(w,W)$. We argued in the previous section that this was a better index than the HDI since the income component played no useful role and was problematic. A further argument in favour of using $I(W)$ is that it otherwise has similar features to $I(w,W)$. This is because Propositions 1 and 2 include the special case $w=0$.

Whereas $I(W)$ may yield a better index, it neglects the dimension of "command over resources to enjoy a decent standard of living". A simple way to represent this dimension and at the same time avoid the income critique is to replace the income variable with a "net income" variable that removes the elements (education and health) that are double counted. Net income, defined as income less expenditures on education and health, is a variable that captures the net command over resources for all other purposes than those already represented in the index. For this reason, the second index that we consider is a modified HDI with net income

In our basic model, net income is the same as total consumption, $lc=y-(E+H)$, i.e. expenditures on all other items other than education and health. When either lc or c is used, instead of income, in the objective function, it is no longer true that optimization requires $c = c_{\min}$. By putting reasonable economic structure on the problem (i.e. strictly concave objective function and convex constraints), it is straightforward to establish an internal solution $c > c_{\min}$ as the optimal plan.[19] Here the planner trades off expenditures on education and health for more net income.

Whereas using net income in a modified index can yield internal solutions, it can also yield allocations that are less equitable. This is because when optimal average consumption is above the minimum, $c^* > c_{\min}$, there is room for dispersion of consumption. For example, it is possible for half of the individuals to be at the minimum and the other half to be at $c^* + (c^* - c_{\min})$. When inequality is a concern, the modified HDI with net income should perhaps be further modified to incorporate distribution information (e.g. Gini coefficient). In our calculations below we leave inequality adjustments for future work.[20]

6.1 Calculating and Comparing the Modified Indices

To calculate the $I(W)$ index we simply set W=.5 and use the education and health sub-indices that are contained in the HDI (as found in the Human Development Report 2007). Table 1 below lists the value of the $I(W=.5)$ index, denoted HDIW, for the year 2005. The table covers OECD nations as well as other several nations for which we have data with which to compare all the indices. There is a substantial difference in the rankings of the HDIW with the HDI.[21] The specific country differences are captured by "Rank of HDI less Rank of HDIW" (in the second to last column of Table 1). It is instructive to look at the top 25 HDI ranked nations in our table since 23 of them are in the top 25 nations in the overall ranking of all nations. From the top 25 countries, the notable losers are Ireland, and the US which fall by 8 and 11 places respectively, whereas, Italy, Spain and New Zealand gain 6, 7 and 14 places.

(Table 1 here)

Figure 1 provides a histogram comparison over all jurisdictions (177 countries and territories) covered in the Human Development Report 2007. Here the variation is quite marked. The outliners are Saudi Arabia (-37), Oman (-36), Georgia (+33), and the Occupied Palestinian Territories (+51). The largest changes in the ranking of populous countries are Turkey (-19), Russian Federation (-16), Philippines (+16), and Vietnam (+23). Both India and China gain one rank. From the perspective of the model, the fact that the HDIW index ranking is quite different suggests that income matters. As income matters only when it is problematic in the theory, the case for using HDIW over HDI, we feel, is quite compelling.

(Figure 1 here)

Now consider the modified index with net income which we denote HDIN. Constructing HDIN involves specifying the net income variable and then defining a net income sub-index. In keeping with our minimalist approach, we start with the income variable used in the HDI; i.e. GDP per capita measured in purchasing power parity equivalent dollars. Net income simply adjusts GDP per capita to net out the percentage of GDP spent on private and public expenditures for education and health. The data for health is available in Table 6 of the Human Development Report 2007, and the data on education is available in Education at a Glance 2006 and 2007, OECD publications.[22]

To construct a sub-index for net income, we follow the methodology for the income sub-index in the HDI, which is constructed as follows:

$$I^{y}(\mathrm{y}) = \frac{\log(\mathrm{y}) - \log(100)}{\log(40000) - \log(100)} .$$

This "achievement" sub-index uses "goalposts" \$100 as a lower bound and \$40,000 as an upper bound on income. The form of the index and the choice of goalposts have varied over the years (as described by Anand and Sen (2000)). The logarithm of income ensues that there is considerable concavity for this sub-index. In contrast, the other indices for education and health are linear in their variables. The reason for this specification was to de-emphasize income to alleviate the concerns discussed in the previous section.

For net income, denoted y^N, we use the following sub-index:

$$I^y\left(\mathrm{y}^N\right) = \frac{\log\left(\mathrm{y}^N\right) - \log(86)}{\log(34317) - \log(86)} .$$

Here we have deflated the lower and upper bound valued by the factor .86. This factor corresponds to the average of net income over the income across countries in the sample. We choose this adjustment as it is in the spirit of the income index and works from the previous bounds in a way that roughly maintains the same overall weight as the income index. It is important to control for the weight otherwise that factor would be driving the differences with the HDI. With this net income index, all the countries are well away from the lower bound, and Norway and Luxembourg are at the upper bound. In comparison, with the income index these two countries and also the United States are at the upper bound.

Though there is substantial variation in our net income series, it does not show up strongly in changing the rankings of net income relative to income. The largest shift is the United States, which loses 3 places.[23] The lack of change in the sub-index shows up in the values and ranking of HDIN in Table 1. The difference in the ranking of this series with HDI is listed in the last column. There are relatively few differences and the country that registers the largest difference is, again, the United States, which looses 2 places.

Of our two modified indices, clearly the change in ranking with the HDIW is much larger than with the HDIN. This is perhaps not surprising as HDIN has a GDP per capita component. (However, there is a literature that argues that the GDP per capita is redundant in the HDI and that omitting the income component doesn't make a large difference.)[24] Conversely, the variation in net income is not insubstantial. It would appear that a key factor that is driving the results is that the logarithmic concavity of the (net) income index and the linearity of the education and life expectancy indices. Here the lower bound on (net) income appears to be a particularly strong determinant as it much smaller than the minimum in the series.[25]

We believe that using net income in a modified index like HDIN is important because it includes the additional dimension of command over resources. However, using a very similar methodology yields only a minor change in the ranking of nations. This methodology (of using log income and a very small lower bound) is intended to de-emphasize income. This methodology should perhaps be reconsidered, with an explicit effort to control for inequality, when designing a human development index that incorporates net income rather than income.

7. Conclusion

The Human Development Index (HDI) is a widely cited statistic that is commonly used as a measure of well being in different countries. Here, we have examined some of the implications that follow if government planners decide to use maximization of the HDI as a criterion for optimal plans. We have found that, if they do so, planners will tend to heavily emphasize expenditures on education and health by lowering consumption. This eventually leads the economy towards a more egalitarian allocation – even though inequality aversion does not appear explicitly in the HDI itself.

The other human development index published in the Human Development Report is the Gender-related Development Index (GDI). In contrast to the HDI, the GDI explicitly differentiates individuals and also includes aversion to inequality. The strategies that optimize the GDI are examined and are shown to be similar to the HDI. Reassuringly, we find "reasonable" assumptions under which both criteria lead to identical optimal development plans. Under these assumptions, the optimal plan prescribes egalitarian consumption, education and health allocations. Females and males education attainment is the same. However, females life expectancy is five years greater as they are assumed to enjoy longer average lifetimes *ceteris paribus*.

A problematic feature of optimal plans is that the income component in the HDI (and GDI) only plays a role indirectly in determining the trade-off between expenditures on education and health. The income component effectively double counts education and health achievements, components that are already in the HDI. Because the income component does not play its intended role of securing resources for a decent standard of living, we argue that it is better to drop income from the index in considering optimal plans rather than using the HDI in its current form.

While dropping income from the HDI is a better basis for human development planning, this approach falls short of the original intent that the HDI cover dimensions beyond education and health. We also considered net income (income net of education and health expenditures) as an indicator of decent standard of living. When net income is used in a modified HDI index, the optimal plan captures the direct trade-offs between allocations for consumption, education and health. This yields a balance of expenditures on the three components.

Using data from primarily OECD countries we constructed and calculated two modified human development indices, one with a net income component and another without any kind of income component. The rank ordering of nations given by these modified indices is compared with the HDI. As one might expect, the modified index without any income component yields a far greater change in rankings. We attribute the lack of change in the ranking when using the net income modified index to the methodology of constructing the income sub-index.

In this paper we have taken the somewhat unusual methodological approach of evaluating a well-known achievement index, the HDI, in terms of the optimal plans it implies. We believe this approach has been quite revealing in uncovering unintended consequences of using the index as a guide for development planning. The critique of the optimal plans implied by the index lead us to modify the index in several ways, and we selected amongst the alternatives according to the modified index that best fulfilled the original intent motivating the HDI. Adopting the modified index with net income would provide better guidance to development planners.

The analysis of planning with multi-dimensional objectives is inherently complex. This is particularly so when there are unusual arguments like life expectancy in the objective function which feedback to production and other elements of the economy. We have explored the implications of optimal plan with the HDI in the simplest possible environment to get a feel for the issues. While we believe that this simple approach has shed significant light on the questions we raise, our analysis falls short in a number of important ways. In particular, our static normative analysis ignores capital accumulation and growth issues. From a dynamic perspective, the analysis might be best thought to correspond to some sort of steady state. Further we do not directly examine the issues around individual incentive and participation constraints. We intend to pursue all these issues in richer models in future research.

References

Anand, S. and Sen, A.K. (1994) "Human Development Index: methodology and measurement", Human Development Report Office Occasional Paper 12, UNDP, New York; reprinted in Fukuda-Parr, S. and A. K. S. Kumar (Eds.), *Readings in Human Development*, 138-151, Oxford University Press, New Delhi.

Anand, S. and Sen, A.K. (2000), "The income component of the Human Development Index", *Journal of Human Development*, Vol. 1, No. 1, pp. 83-106.

Anand, S. and Ravillion, M. (1993), "Human development in poor countries: on the role of private incomes and public services", *Journal of Economic Perspectives*, Vol. 7, pp. 133-50.

Bourguignon, F. and Fields, G. (1990), "Poverty measures and anti-poverty policy", *Recherches Economiques de Louvain*, Vol. 56 No.3-4, pp. 409-27.

Cahill, M.B. (2005), "Is the Human Development Index redundant?" *Eastern Economic Journal*, Vol. 31, No. 1, pp. 1-5.

Chrétien, J., 1998, in Former Prime Minister's Newsroom Archive (1995-2003)), September 9, 1998.

Deaton, A. (2003), "Health, inequality, and economic development", *Journal of Economic Literature*, Vol. XLI, pp. 113-57.

Education at a Glance 2006, OECD Publishing, Paris. Available at: http://www.oecd.org/edu/eag2006

Education at a Glance 2007, OECD Publishing, Paris. Available at: www.oecd.org/edu/eag2007

Engineer, M., Fink, S., and Roy, N. (2008), "Adjusting the Human Development Index for Morbidity", mimeo, University of Victoria. Available at:

http://web.uvic.ca/~menginee/mypapers/HDIwithMortality.pdf

Foster, J., Lopez-Calva, L., and Szekely, M. (2005), "Measuring the distribution of human development: methodology and an application to Mexico", *Journal of Human Development*, Vol. 6, No. 1, pp. 5-25.

Human Development Report 2007: Fighting climate change: Human solidarity in a divided world, Palgrave Macmillan, New York, NY. Available at: http://hdr.undp.org/en/media/hdr_20072008_en_complete.pdf

Kalam, A., 2005, address at the: "International seminar on income and employment security in India", April 6, 2005, New Delhi Index.

Kanbur, R. and Mukherjee, D. (2007), "Poverty, relative to the ability to eradicate it: An index of poverty reduction failure", *Economics Letters*, Vol. 97, pp. 52-57.

McGillivary M. and White, H. (1993), "Measuring development? Yet another redundant composite development indicator?", *Journal of International Development*, Vol. 5, No. 3, pp. 183-92.

Osberg, L. and Sharpe, A. (2005), "How should we measure the "economic" aspects of well-being?", *Review of Income and Wealth*, Vol. 51, No. 2, pp. 311-36.

Ogwang, T. (1994), "The choice of principal variables for computing the Human Development index", *World Development*, Vol. 22, pp. 2011-14.

Parsai, G. (2006), "India one notch up in its Human Development Index", *The Hindu*, 11 October, available at: http://www.thehindu.com/2006/11/10/stories/2006111018392400.htm

Rao, C.R. (1973), *Linear Statistical Inference and Its Applications*, John Wiley & Sons, New York, NY.

Sen, A.K. (1981), "Public action and the quality of life in developing countries", *Oxford Bulletin of Economics and Statistics*, Vol. 43, pp. 287-319.

Sen, A.K. (1985), *Commodities and Capabilities,* North-Holland, Amsterdam.

Union Budget and Economic Survey 2004-2005, Ministry of Finance, Government of India, New Delhi. Available at: http://indiabudget.nic.in/es2004-05/chapt2005/chap14.htm.

United Nations Development Programme (1990), *Human Development Report 1990: Concept and Measurement of Human Development*, New York: United Nations.

APPENDIX

Proof of Proposition 1. The Lagrangian for problem P1 is:

$$L_{HDI} = wI^{y}(f(l(E,H),e(E,H),H)) + (1-w)\Big(WI^{e}(e(E,H)) + (1-W)I^{l}(l(E,H))\Big)$$
$$+\lambda_1\left(f(l(E,H),e(E,H),H) - E - H - l(E,H)c\right) + \lambda_2\left(c - c_{\min}\right) \quad (7)$$

Among the Kuhn-Tucker conditions are the following:

$$c(\lambda_2 - \lambda_1 l(E,H)) = 0 \quad (8)$$

$$\lambda_2(c - c_{\min}) = 0 \quad (9)$$

$$\lambda_1 \geq 0, \quad \lambda_2 \geq 0 \quad (10)$$

We first show that, at an optimum, $\lambda_2 > 0$. Suppose not. Then, by (10), $\lambda_2 = 0$. By (8), since $l(E,H) > 0$ and $c > 0$, this implies that $\lambda_1 = 0$. Since the objective function is strictly increasing in E and H, the resource constraint (6) binds, and so $\lambda_1 > 0$. This is a contradiction. Thus, $\lambda_2 > 0$. By (9), this then implies that $c = c_{\min}$. It remains to be shown that the constraint qualification when constraint (ii) in problem (P1) is not binding. If (i) is also not binding the constraint qualification is trivially satisfied. If (i) is binding then the gradient vector to (i) is $\nabla g_1(c, E, H) = (l, l_E c + 1 - f_l\, l_E - f_e\, e_E, l_H c + 1 - f_l\, l_H - f_e\, e_H - f_H)$ for $c > c_{\min}$. When there is only one vector, it is linear dependent only if $\nabla g_1(c, E, H) = (0, 0, 0)$. The assumption in (3), $l(0,0) > 0$, rules this out. Even without this assumption, the gradient must be linearly independent. A necessary condition for linear dependence is $l_E c + 1 - f_l\, l_E - f_e\, e_E = 0$. But this condition cannot correspond to local maximum because decreasing c to increase E and/or H along the constraint increases the objective function. Hence, the constraint qualification is satisfied and the Kuhn-Tucker conditions are necessary for a maximum. Thus, at an optimum $c^* = c_{\min}$.

Suppose policy limits the level to which consumption can be reduced to some level c_p, where $c_p > c_{\min}$. Then the planner's problem yields a Lagrangian that is the same as

(7) except for c_p replacing $c_{\min}$. As above $\lambda_2 > 0$. Thus, the marginal value of restricting the policy level c_p further is $\frac{dL_{HDI}}{dc_p} = -\lambda_2 < 0$. It follows that reductions in c_p increase the Lagrangian, which is just the constrained optimal HDI score. ■

Proof of Proposition 2. The Lagrangian for this problem is:

$$L_{HDI} = wI^y(f(l,e,H)) + (1-w)\left(WI^e(e) + (1-W)I^l(l)\right)$$
$$+\lambda_1\left(f(l,e,H) - E - H - \sum_{i=1}^{N} l_i c_i\right) + \sum_{i=1}^{N} \lambda_{2i}\left(c_i - c_{\min}\right) \qquad (7')$$

Among the Kuhn-Tucker conditions are the following:

$$c_i\left(\lambda_{2i} - \lambda_1 l_i / N\right) = 0 \qquad (8')$$

$$\lambda_{2i}\left(c_i - c_{\min}\right) = 0 \qquad (9')$$

$$\lambda_1 \geq 0, \quad \lambda_{2i} \geq 0 \qquad (10)$$

(a) With respect to consumption this is a generalization of the proof to Proposition 1 and is identical except for having to show that $\lambda_{2i} > 0$ for all i at an optimum. Given the strict concavity of the attainment functions $e(E_i, H_i)$ and $l(E_i, H_i)$, it follows that the Lagrange multipliers corresponding to E_i and H_i must be the same across individuals which requires equal expenditures and attainments for all individuals.
(b) The incremental result with respect to average consumption is also a generalization Proposition 1 but with individual values of $c_{pi} > c_{\min}$. The possibility of increasing inequality is demonstrated in the text. ■

Proof of Proposition 4(a). From Proposition 3 and Assumption (iii), we have $c_f = c_f = c_{min}$ at the optimum. Then maximizing GDI subject to the resource constraints implies the Lagrangian:

$$L_{GDI} = \sum_{d=y,e,l} w^d \left[\sum_{g=f,m} p_g (I_g^d)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} + \delta_1 \left(f(e,l,H) - E - H - lc_{\min} \right)$$

$$+\delta_2 \left(f(e,l,H) - \sum_{g=f,m} p_g y_g \right)$$

Since the objective function is strictly concave and the constraints are convex, the first order-conditions are necessary and sufficient for optimization. The first-order conditions for y_g, E_g and H_g respectively imply (after cancelling constants) the following conditions:

$$w^y \left[\sum_{g=f,m} p_g (I_g^y)^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}} (I_g^y)^{-\varepsilon} I_g^{y'}(y_g) = \delta_2 \tag{11}$$

$$w^e \left[\sum_{g=f,m} p_g (I_g^e)^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}} (I_g^e)^{-\varepsilon} I_g^{e'} e_{E_g} + w^l \left[\sum_{g=f,m} p_g (I_g^l)^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}} (I_g^l)^{-\varepsilon} \frac{l_{E_g}}{60} = \delta_1 \{1 + l_{E_g} c_{\min}\} - (\delta_1 + \delta_2)\{ f_e e_{E_g} + f_l l_{E_g} \} \tag{12}$$

$$w^e \left[\sum_{g=f,m} p_g (I_g^e)^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}} (I_g^e)^{-\varepsilon} I_g^{e'} e_{H_g} + w^l \left[\sum_{g=f,m} p_g (I_g^l)^{1-\varepsilon} \right]^{\frac{\varepsilon}{1-\varepsilon}} (I_g^l)^{-\varepsilon} \frac{l_{H_g}}{60} = \delta_1 \{1 + l_{H_g} c_{\min}\} - (\delta_1 + \delta_2)\{ f_e e_{H_g} + f_l l_{H_g} + f_H \} \tag{13}$$

where $e = \sum_{g=f,m} p_g e(E_g H_g),\ \ l = p_f 5 + \sum_{g=f,m} p_g l(E_g H_g),\ \ E = \sum_{g=f,m} p_g E_g,\ \ H = \sum_{g=f,m} p_g H_g$

$$I_g^y = I^y(y_g), \quad I_g^e = I^e(e_g) = I^e(e(E_g, H_g)), \quad I_g{}^l(l) = \frac{l(E_g, H_g) - 22.5}{60}$$

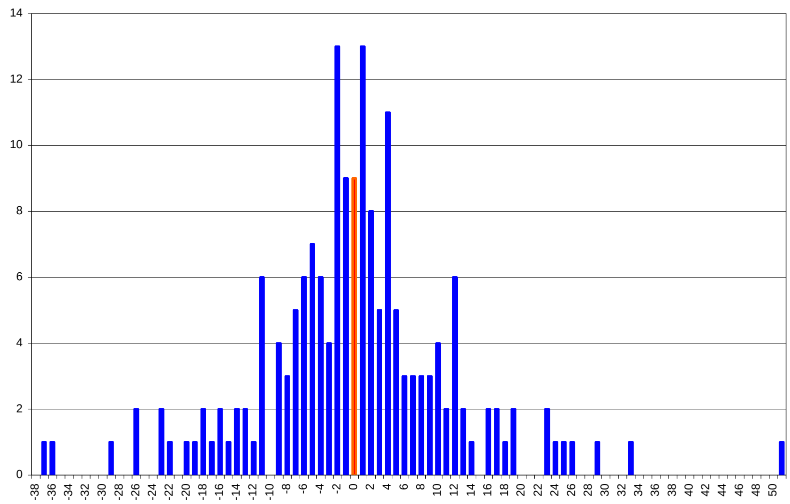
We now confirm that equal allocations ($y_g=y$, $E_g=E$, $H_g=H$ for $g=f,m$), satisfy (11)-(13). Clearly, (11) is consistent with $y_f = y_m = y$. Under equal allocations, $e(E, H)$ and $l(E, H)$ and the derivatives e_{E_g}, e_{H_g}, l_{E_g}, and l_{H_g} are the same across gender $g=f,m$. Then all terms across gender in (12)-(13) have the same magnitude. Hence, all the first-order conditions are satisfied under equal allocations. ■

Table 1:

Ranking of Human Development Indices

Country (OECD; non-OECD*)	Human Development Indices			Rank of HDI Indices			Rank Differences	
	HDI (Original 2005)	HDIW (Without Income)	HDIN (With Net Income)	Rank of HDI	Rank of HDIW	Rank of HDIN	Rank of HDI less Rank of HDIW	Rank of HDI less Rank HDIN
Iceland	0.968	0.960	0.966	1	2	2	-1	-1
Norway	0.968	0.952	0.968	2	6	1	-4	1
Australia	0.962	0.963	0.961	3	1	3	2	0
Canada	0.961	0.956	0.960	4	3	5	1	-1
Ireland	0.959	0.942	0.960	5	13	4	-8	1
Sweden	0.956	0.951	0.955	6	7	6	-1	0
Switzerland	0.955	0.942	0.953	7	12	8	-5	-1
Japan	0.95f	0.951	0.954	8	9	7	-1	1
Netherlands	0.953	0.946	0.952	9	11	9	-2	0
France	0.952	0.951	0.950	10	8	11	2	-1
Finland	0.952	0.946	0.952	11	10	10	1	1
United States	0.951	0.926	0.948	12	23	14	-11	-2
Spain	0.949	0.956	0.950	13	4	12	9	1
Denmark	0.949	0.938	0.948	14	15	13	-1	1
Austria	0.948	0.937	0.947	15	17	15	-2	0
United Kingdom	0.946	0.935	0.946	16	18	16	-2	0
Belgium	0.946	0.937	0.945	17	16	17	1	0
Luxembourg	0.944	0.916	0.944	18	25	18	-7	0
New Zealand	0.943	0.953	0.942	19	5	19	14	0
Italy	0.941	0.940	0.942	20	14	20	6	0
Germany	0.935	0.928	0.934	21	22	21	-1	0
Israel*	0.932	0.934	0.930	22	20	22	2	0
Greece	0.926	0.934	0.928	23	19	23	4	0
Korea (Republic of)	0.921	0.931	0.922	24	21	24	3	0
Slovenia*	0.918	0.926	0.918	25	24	25	1	0
Portugal	0.897	0.902	0.896	26	26	26	0	0
Czech Republic	0.891	0.892	0.892	27	29	27	-2	0
Hungary	0.875	0.879	0.875	28	30	28	-2	0
Poland	0.872	0.896	0.873	29	28	29	1	0
Chile*	0.867	0.901	0.868	30	27	30	3	0
Slovakia	0.862	0.871	0.864	31	32	32	-1	-1
Estonia*	0.862	0.872	0.864	32	31	31	1	1
Mexico	0.829	0.853	0.830	33	33	33	0	0
Russian Federation*	0.803	0.813	0.806	34	35	34	-1	0
Brazil*	0.800	0.830	0.801	35	34	35	1	0
Turkey	0.775	0.793	0.777	36	36	36	0	0

Figure 1:
Histogram of Number of Jurisdictions With Rank HDI less Rank of HDIN



Endnotes:

[1] For a detailed description see http://hdr.undp.org/statistics/indices/.

[2] Bourguignon and Fields (1990) minimize various poverty indices subject to redistribution constraints and show that the implied policies can differ radically depending on the index.

[3] In the HDI the income of PPP $100 US corresponds to the index being 0 and the "goalpost" of PPP $40,000 US corresponds to the index being 100. As of 2005 three countries, Luxembourg, Norway and the US, had achieved the $40,000 US goalpost. The income index is logarithmic in income between the bounds. The education and life expectancy indices are linear in their variables. The education variable e is a linear combination of literacy rates and school enrolment rates, and as of 2004, there were 5 countries that scored enrolment rates that yielded 100 on this index. No country has achieved the goalpost for life expectancy, which is currently 85. No country is at the lower bound for any of the indices. Our analysis goes through for any country as long as there is at least one index that falls short of 100. Indeed, we do not even need concavity for our results but assume it for simplicity.

[4] We have taken the short cut of specifying life expectancy (education) as a function of education (health) expenditure rather than education (health) attainment. Functions (3) and (4) can be shown to be consistent with the more general specification under minor restrictions. Assuming that minimum life expectancy is positive simplifies the analysis.

[5] This assumption is consistent with Anand and Ravallion's (1993) "capability expansion through social services". According to this explanation (also see Sen, 1981), the public provision of essential goods and services leads to improved social outcomes and income matters if it is used to finance suitable public services and alleviate poverty. For example, Anand and Ravallion find in a sample of 22 developing countries that after controlling for health expenditures and poverty (as measured by the proportion of population consuming less than one dollar per day in 1985 at PPP US$), life expectancy is not affected by consumption. Even the unconditional plot of income against life expectancy displays an income threshold (roughly at PPP $10,000 US) beyond which there is no discernable relationship (e.g. Deaton, 2003)). Anand and Ravallion contrast schools of thought on the importance of social services versus private consumption for human development.

[6] In the HDI, life expectancy is a proxy for the dimension 'a long and healthy life' (see Human Indicators in the Human Development Report 2007). Engineer et. al. (2008) argue that life expectancy is an imperfect indicator of health as it doesn't capture morbidity.

7 While the optimal plan heavily emphasizes expenditures on education and health, it does not involve maximizing the combined expenditure $E + H$. Recall from the resource constraint that $E + H = y - lc_{\min}$ so that maximizing $E + H$ is the same as choosing E and H to maximize $y - lc_{\min}$ subject to the resource constraint. The planner's problem yields the same outcome as this special case in two instances: (1) when $l_E = l_H = 0$ and $w \to 1$ so that the planner maximizes output, and (2) when l is at its upper bound so that all remaining expenditures go to education which is the only way to increase output.

8 It is sufficient for our result if only one of education or health have a positive shadow price. In the HDI no country has reached the upper bound of 85 year so that the life expectancy index is below its upper bound, $I^l(l) < 100$. Further, even in the richest countries, incremental health expenditures (and education expenditures on health) appear to be efficacious in increasing life expectancy.

9 More elaborate models yield the same result. For example, our static formulation with full employment does not consider the possibility that persons do not work for a portion of their life, say beyond a retirement age, R, so that the amount of lifetime work is $\max[l,R]$. When life expectancy exceeds the retirement age, $l > R$, this might lead to $f_l = 0$ at the margin. Still the objective function is increasing in E and H so that the resource constraint binds and the proposition obtains.

If we use a rate of output y/l instead of y in the objective function, the function would be increasing in y/l as long as the elasticity $f_l l/f > 1$, which is arguably empirically plausible. If $f_l l/f < 1$, then we need at least one of the shadow prices on E or H to be positive to prove the proposition. For example, sufficient conditions for education to have a positive shadow price are $e_E > 0$ and $l_E = 0$.

10 In practice, the government may only have access to distortionary tax instruments, in which case it faces a second-best problem. In order to maintain a high level of income, the government would need to set taxes in a way that leaves those with higher incomes greater disposable income. However, the reason for leaving higher disposable income is simply to maintain output higher than otherwise, not to secure access to resources for a decent standard of living. We have chosen to not flesh out the second-best problem as this would involve specifying a detailed microstructure to the problem and the particular results would depend on the particular microstructure used. Second-best problems in taxation are well known to limit the ability of government to implement allocations.

11 Our results generalize to the modelling of external effects by including average education and health expenditure, $E = \frac{1}{N}\sum_{i=1}^{N} E_i$ and $H = \frac{1}{N}\sum_{i=1}^{N} H_i$, in the individual attainment functions.

12 The analysis assumes that education and health are rivalrous, to some extent. If they are considered pure public goods, $e_i = e(E,H)$ and $l_i = l(E,H)$, egalitarian outcomes obtain without assuming diminishing returns. If education and health facilities are equally accessible to everyone in the economy, perhaps because of their public good nature, then maximizing the HDI implies equality of treatment though not necessarily outcomes among heterogeneous individuals.

13 Interestingly, equal outcomes are possible in the homogenous case if there is no inequality aversion, $\varepsilon = 0$, and the life expectancy is within the bounds. With $\varepsilon = 0$, the GDI is additive in the dimension indexes. Since I_g^l is linear with denominator of 60, $l_f = l_m$ is consistent with the marginal conditions.

14 In the HDI, the education and life expectancy indices have the same weight, $W = (1 - W) = ½$. However, this does not imply that expenditures are equal on education and health. First, the resource constraint $E+H = f - lc_{\min}$ reveals that increasing life expectancy has the cost of increasing total consumption. This feature discourages expenditures that enhance life expectancy compared to education. Second, the education and health indices are not necessarily symmetric nor are the achievement functions $e(E, H)$ and $l(E, H)$.

15 The human development approach, or equivalently the "capabilities approach", de-emphasizes valuing income *per se* [e.g. Sen (1985), Anand and Ravallion (1993)]. Anand and Ravallion (1993, p.136-37) note that while the philosophy of the Human Development Report has been heavily influenced by the capabilities approach, the inclusion of income in the HDI is problematic because "…it is not a direct indicator of any achievement or functioning, …".

16 Expenditures E and H only effect the planner's problem through the attainments $e(E,H)$ and $l(E,H)$ when $f_H = 0$. Otherwise H can directly affect output y. This role for H is still problematic since y does not have the status as a direct measure of living standards.

17 This statistic is listed in the HDI statistics pages of the Human Development Reports.

18 Osberg and Sharpe (2005) devise a Index of Economic Well-Being (IEWB) which they use instead of the income component in the HDI to represent the command over resources. The IEWB consists of measures for dimensions: consumption flows, net societal accumulation of stocks of productive resources, income distribution and economic security. In contrast to the indexes we examine, the IEWB

is not minimalist in the sense that is a substantial departure from the income component and requires a number of sources of data. The motivation of the IEWB is unrelated to our critique.

[19] For example, the current HDI uses *ln*(GDP/capita) as a variable in the index. Replacing this with $ln(c)$ would yield sufficient concavity to generate an internal solution as long as c_{min} were not too large. Indeed, if one interpreted the lower bound of $100 of y in the index as the lower bound for c, almost all countries would be at an internal optimum (when the education and health achievement functions were responsive to expenditures).

[20] Anand and Sen (2000) and Foster *et al* (2005) provide explanations and references to ways to include inequality aversion for income directly in the HDI . In principle the very same methods could be used with our net income variable. However, in practice, there are many nations for which we lack of data with which to construct a net income variable at the individual level.

[21] The Spearman rank correlation coefficient between these two series is .9102, which indicates high but imperfect correlation. The conclusion of imperfect correlation comes following the testing procedure given in Rao (1973) and recently used by Kanbur and Mukherjee (2007). Given that 2/3 of the HDI is perfectly correlated with HDIW and that the nations out of the top 25 are substantially different from each other (rank well down in the complete list of countries), the value of .9102 suggests that there is a substantial difference between the rankings.

[22] Both health and education data are a percentage out of GDP for 2004. We assume this percentage has not changed for 2005. Data for Luxembourg includes only public non-tertiary education, and data for Estonia and the Russian Federation includes only public education expenditures.
We have limited our analysis to countries covered in their publication Education at a Glance, since we were unable to find another source that had comparable private education expenditure data. We choose not to expand the analysis to by excluding private education expenditures as this is likely relatively large in many poorer countries where public provision of education is minimal.

[23] These calculations are available in a spreadsheet upon request.

[24] Ogwang (1994) using Principal Component Analysis identifies life expectancy as the best principal variable when analysing the HDI. McGillivray and White (1993) find that the GDP per capita is very highly correlated with other components of the HDI. Cahill (2005) finds Spearman rank correlation coefficients of above .9 when comparing the HDI rank with variants that exclude the income index.

[25] In the income sub-index the lower bound is $100 and the net income lower bound is $86. Contrast this with Brazil, which has the lowest income at $8,407 and net income at $7,338. The country with rank 25 in Table 1 has income $22,275 and net income $18,933.