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TAXES, PUBLIC GOODS, AND THE RULING CLASS:
AN EXPLORATION OF THE TERRITORY BETWEEN
BRENNAN AND BUCHANAN’S LEVIATHAN
AND CONVENTIONAL PUBLIC FINANCE*

MERWAN ENGINEER**

I. INTRODUCTION

Brennan and Buchanan [1977, 1978, 1980, and 1985] have developed a body of theory of what could be called “Neo-Hobbesian” public finance. They see government as a revenue-maximizing Leviathan, but depart from the strict Hobbesian perspective by assuming that the Leviathan can be partially constrained by a constitution. This constitution is chosen by subjects rather than by rulers and places definite constraints upon the tax base. It is in the interest of the subjects to choose an inefficient tax base when they cannot choose tax rates along with the tax base.

Brennan and Buchanan’s Neo-Hobbesian public finance stands in sharp contrast to the traditional public finance in which tax rates and public goods are assumed to be chosen in the interest of all the governed. On that assumption, there would be no reason to restrict the tax base in a constitution because a tax that does not contribute to the maximization of social welfare would never be employed.

This paper develops Brennan and Buchanan’s Leviathan model in a general equilibrium optimal tax framework and compares it to the conventional utilitarian public finance. The distinctive feature of the Leviathan model is that the government acts solely on the behalf of a subset of the population called rulers. Selfish rulers choose the supply of public goods and set tax rates on all goods within the tax base, but cannot choose the tax base. The tax base is established by the subjects in a constitution that rulers are bound to respect. If all goods could be taxed, the rulers would set rates high enough to expropriate the entire wealth of the subjects. They cannot do so when some goods are excluded from the tax base, because high rates on goods within the permitted base provoke subjects to divert expenditures from taxed to untaxed goods.

Subjects are usually best off if they limit the number of goods that the rulers can tax so as to leave themselves with some disposable income and the rulers with some revenue with which to provide the public good. In all cases, subjects want to exclude goods with a positive minimal subsistence requirement from the tax base, because subjects cannot avoid a tax on such goods. Other things equal, goods that are complements with the public good should be the first to be included in the tax base,
for rulers have an incentive to provide more of the public good when it stimulates consumption of private goods in the tax base. Brennan and Buchanan's "Tax Constitution for Leviathan" describes these features of the subjects' optimal tax base.

The analysis is aided by a numerical example when utility is Cobb-Douglas. The greater is the number of rulers and the more restricted is the excise tax base, the smaller is the gap between the utilities of rulers and subjects. The subjects' optimal choice of tax base is calculated for different numbers of rulers. The number of goods in the optimal base initially declines as the number of rulers increase. Eventually, a point is reached where it is optimal for rulers to spend all tax revenues on the public good. Thereafter, as the number of rulers increase, the subjects optimal tax base increases in a way that maintains the equality of treatment of rulers and subjects. The example demonstrates that equality of treatment can result from limited technological or "political" restrictions on the taxing powers of rulers rather than from utilitarian policies.

II. THE RULERS' CHOICE OF TAXES AND PUBLIC GOODS

The population consists of \( H \) identical people, except that the first \( R \) are rulers and the remaining \( H - R \) are subjects. Each person is endowed with the same amount of labour time, \( x_i \), which is the only factor of production and is the numeraire. The output per unit of labour of good \( i \) is fixed at \( p_i \), which is also the producer price of the good. There are \( I \) private goods in the economy. The last good is leisure and has a price of unity, \( p_i = 1 \). There is one nonrivalrous public good, \( G \), that requires \( p_G \) units of labour to produce.

Rulers raise revenue through excise taxation on a given tax base and use the proceeds to buy public goods and make non-negative lump-sum payments \( m \geq 0 \) to themselves. Payments to subjects are never made because rulers are selfish. Of the \( I \) goods in the model, the first \( B \leq I \) goods are included in the tax base and the remaining \( I - B \) goods are exempt from taxation. The tax vector is \( t = (t_1, ..., t_i, ..., t_B) \), and the tax rate \( \tau_i = t_i / q_i \), where \( q_i = p_i + t_i \).

Though rulers can set \( t, m, \) and \( G \) given the tax base \( B \), it is the subjects who choose the tax base. Thus, public finance becomes a "game" between rulers and subjects. The outcome of the game depends critically on the number of rulers. Brennan and Buchanan's archtypical case is that of a single ruler or king. The selfish king acts like a Leviathan by choosing tax rates to maximize tax revenues. He provides the public good only to stimulate tax revenues and for his own pleasure. Brennan and Buchanan also stress that many rulers can also act like a Leviathan. They briefly analyze the case where rulers form the majority. Subjects fare better under this regime because rulers lower tax rates to reduce the excess burden of
taxation on themselves. These cases form part of a spectrum with a king \((R = l)\) on one end and a society with universal enfranchisement \((R = H)\) at the other. The king's choice of taxes and public goods is analyzed before turning to the more general rulers' problem.

II.A. The King's Problem

The king knows that any taxes he pays accrue back to him. Therefore, his effective wealth is his endowment and the tax revenues from his subjects. The king's budget constraint is

\[
(1) \quad \sum_{i = 1}^{L} p_i x_i + p_G G = \bar{x}_i + (H - l) \sum_{i = l}^{B} t_i x_i(t, G)
\]

where \(x_i\) is the king's consumption of good \(i\) and \(x_i(t, G)\) are the subjects' demand functions for good \(i\). The king derives utility from the consumption of private goods and the public good:

\[
(2) \quad U(x_1, ..., x_t, G).
\]

The king's problem is to choose excise taxes, the public good and his consumption to maximize (2) subject to (1) and the nonnegativity conditions

\[
(3) \quad x_i \geq 0 \text{ for all } i, \quad t_i \geq 0 \text{ for all } i \leq B, \quad G \geq 0.
\]

If the tax base is unrestricted \((B = l)\), the selfish king expropriates the entire value of his subjects' labour endowment by imposing a uniform tax rate of one on all goods. With a uniform tax on all goods a subject cannot escape taxation by buying more untaxed goods. The king's wealth becomes \(\bar{x}_i + (H - l) \tau x_i\), where \(\tau\) is the common tax rate. A tax rate of one leaves the subject with no disposable income and the king with all of society's wealth. Out of his spoils, the king provides the public good solely for his own enjoyment, so that his marginal rate of substitution for the public good is equal to its price:

\[
(4) \quad \frac{\partial U}{\partial G} / \alpha = p_G
\]

where \(\alpha\) is the king's marginal utility of wealth.

Restrictions on the tax base \((B < l)\) afford subjects some protection against expropriation. If tax rates are too high, subjects can substitute toward untaxed goods. Assuming an internal solution, the king's revenue-maximizing tax conditions are

\[
(5) \quad x_i(t, G) + \sum_{i = l}^{B} t_i \frac{\partial x_i(t, G)}{\partial t_j} = 0 \quad \text{ for all } j \leq B.
\]
The marginal gain from an extra dollar of tax on any good \( j \) is just equal to the marginal revenue loss from the reduction in consumption of all taxed goods. If all taxed goods are zero gross substitutes, the optimum tax on the \( j \)th good corresponds to the demand for the \( j \)th good being unit elastic with respect to the tax. Of course, restrictions on the tax base only protect the subjects from the Leviathan if there are no goods with a positive subsistence requirement in the tax base. Subjects, if they had the choice, would choose to remove subsistence goods from the tax base.

When taxation is distortionary, the king has an incentive to provide more of the public good if it is a complement with taxed goods,

\[
\frac{\partial U}{\partial G} \propto p_G - (H - 1) \sum_{i=1}^{B} t_i \frac{\partial x_i(t, G)}{\partial G}.
\]

Even if the king resides abroad and therefore does not consume the public good, he provides it as long as additional increments generate a profit for him. Brennan and Buchanan argue that goods with a strong complementarity to the public good should be "earmarked" for the tax base. A gasoline tax is a good example. The Leviathan builds roads to stimulate higher gasoline consumption which he taxes.

\[\text{II.B. The Rulers' Problem}\]

With an unrestricted tax base \((B = I)\), rulers can also use a uniform tax rate of one on all goods to extract each subject's full endowment. The nondistortionary excise taxation of the rulers at the same rate as subjects does not change the tax rule, because taxes paid by rulers are fully compensated by payments to rulers. With all of society's wealth, each ruler's share is \( H x_i / R \). Rulers provide the public good for their consumption pleasure according to

\[
R \frac{\partial V}{\partial G} \propto p_G
\]

where \( \alpha \) is the representative ruler's marginal utility of wealth.

When the tax base is restricted, Brennan and Buchanan argue that subjects fare better with a larger ruling class, because rulers lower tax rates to reduce the excess burden of taxation on themselves. Individual rulers "free-ride" by substituting away from taxed goods. With a relatively large number of identical rulers, it is reasonable to assume that individual rulers treat the policy variables as independent of their individual consumption decisions. Then the rulers' government budget constraint is

\[
R \sum_{i=1}^{B} t_i x_i(t, m, G) + (H - R) \sum_{i=1}^{B} t_i x_i^*(t, G) = Rm + p_G G
\]
where \( x_i(t, m, G) \) is the representative ruler’s demand function for the \( ith \) good. Expenditures on the public good and payments to rulers exhaust tax revenues. The representative ruler’s utility can be expressed as a function of the policy variables:

\[
V(t, m, G).
\]

The rulers’ problem is to choose excise taxes, payments, and the public good to maximize (9) subject to (8) and the nonnegativity conditions

\[
t_i \geq 0 \text{ for all } i \leq B, \quad m \geq 0, \quad G \geq 0.
\]

Concentrating on the internal solution, the first-order conditions respectively for excise taxes, payments, and the public good can be expressed as follows:

\[
\frac{\partial V}{\partial m} = 1 + \frac{1}{\sum_{i=1}^{B} \frac{\partial x_i(t, m, G)}{\partial m}} + \frac{(H - R)}{R} \frac{\partial x_j(t, G)}{\partial m} 
\]

\[
\frac{\partial V}{\partial \lambda} = 1 - \sum_{i=1}^{B} \frac{\partial x_i(t, m, G)}{\partial \lambda}
\]

\[
\frac{\partial V}{\partial G} = \frac{\lambda}{\frac{\partial V}{\partial \lambda}} \left[ p_g - \sum_{i=1}^{B} \frac{\partial x_i(t, m, G)}{\partial G} - (H - R) \sum_{i=1}^{B} \frac{\partial x_i(t, G)}{\partial G} \right]
\]

where \( \lambda \) is the value of a marginal increase in revenues in terms of the utility of the representative ruler.

Condition (13) describes a modified Samuelson rule for the provision of the public good. The rulers provide the public good up to the point where the rulers’ marginal valuation of the public good is equal to its marginal cost. The direct cost to the rulers of the last unit of the public good consists of the price of the public good less the extra tax revenues generated by the public good. More tax revenues will be generated the greater is the complementarity between the public good and taxed goods for subjects as well as rulers. The greater is the complementarity, the less are the direct costs encouraging the provision of the public good.

The direct cost of the public good is weighted by the rulers’ opportunity cost of funds which is the ratio \( \lambda \)/\( \frac{\partial V}{\partial \lambda} \). This opportunity cost is determined by (11) which is a modified Ramsey tax rule. Rulers are inhibited by setting taxes too high because of the deadweight loss imposed on the rulers. The higher the deadweight loss becomes, the greater is the opportunity cost of funds discouraging the provision of the public good.
Condition (12) describes the rulers' choice of payments, assuming that the non-negativity condition on payments, $m \geq 0$, is not binding. The allocation of tax revenues for payments and the public good are determined by the opportunity cost of funds. If the opportunity cost is too high, no funds may be allocated for payments and the constraint $m \geq 0$ binds. As is demonstrated in the example in Section III, this occurs when the number of rulers is relatively large and the public good is highly valued. In this case, the excess burden on rulers of raising taxes beyond that desired for the public good exceeds the benefit of the payments.

Finally, note that with full enfranchisement, $R = H$, the rulers' problem with a restricted base becomes the standard second-best tax problem (for example see Atkinson and Stern [1974]), and with an unrestricted base, Samuelson's rule for the provision of the public good applies.

III. AN EXAMPLE SHOWING HOW RULERS' CHOICES VARY WITH THE NUMBER OF RULERS AND THE EXTENT OF THE TAX BASE

Consider a simple economy with a population, $H$, of 1000, and 100 private goods of which the last is labour ($I = 100$). The per capita labour endowment, $x_i$, is 100 hours. Units are chosen so that it requires one hour of labour to produce one unit of any good, $p_i = 1$ for all $i$. There is a single public good, $G$, with a unit price of one thousand hours of labour, $p_G = 1000$. Finally, all utility functions, for both rulers and for subjects, are Cobb-Douglas and identical:

\[
U^h = \prod_{i=1}^{100} (x_i^h)^{0.1} G^{-5}
\]

where $x_i^h$ is the consumption of private good $i$ by individual $h$. With Cobb-Douglas utility all goods are zero gross substitutes implying that optimal tax rates are the same for all taxed goods (Sadka [1977]). Thus, it makes no difference to the rulers which goods are included in the tax base, but the welfare of the rulers increases with the size of the allowable tax base.

Utility levels of rulers and subjects and the optimal policy values are presented in Table 1 for an economy with one hundred rulers and nine hundred subjects as the tax base varies from 5 to 100 goods. When there are only five goods in the tax base ($B = 5$), rulers choose to use all tax revenues on the public good. Since the public good is nonrivalrous and both groups face the same tax rate, the utility of a ruler equals the utility of a subject. As the tax base increases, the rulers' optimal tax rate declines slightly and revenues increase almost proportionately with the increase in the base. Rulers transfer some of the revenues to themselves, and therefore are better off than their subjects. The rest of the additional revenue is used to finance
the public good. Subjects initially value the additional public good above the additional taxation costs. Subjects' utility continues to increase until \( B = 20 \), where it reaches its maximum and then declines as the base increases. Rulers' utility increases with the base. This is consistent with the Le Chatelier Principle which implies that rulers cannot lose from an expanded tax base.

When the base includes all goods including leisure, \( B = 100 \), a tax rate of one on all goods is used by the rulers to expropriate the entire value of each subject's endowment. With no disposable income, subjects' utility is zero. Though rulers face the same tax rate, they receive an infinite gross transfer so that in the limit, each gets

TABLE 1

<table>
<thead>
<tr>
<th>( B )</th>
<th>( V )</th>
<th>( V^2 )</th>
<th>( r )</th>
<th>( G )</th>
<th>( m )</th>
</tr>
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<tbody>
<tr>
<td>5</td>
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<td>1.89</td>
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</table>

Where \( B \) is the number of goods in the excise tax base, \( V \) is the utility of the representative ruler, \( V^2 \) is the utility of the representative subject, \( r \) is the rate on all taxed goods, \( G \) is the quantity of the public good and \( m \) is the gross payment to each ruler.

A net transfer of 566. Rulers spend one-third of the wealth on the public good and two-thirds on private goods. This solution is efficient since the tax base is nondistortionary.

Table 2 is a comparison of the utilities of rulers and subjects when the number of rulers is allowed to vary.\(^6\) As before, an expansion of the base makes the rulers no worse off and generally better off. Subjects' utility follows the same pattern as before; first rising then falling as the base is expanded. For any given base, the utility of rulers does not increase and generally declines as the number of rulers increase. In contrast, the utility of subjects generally increases and never declines as the number of rulers increase. As the number of rulers goes up the rulers' deadweight loss from taxation increases if tax rates are held constant. Thus, larger ruling classes choose lower tax rates.
The example shows what factors contribute to equality between ruler and subject. Generally, the greater is the number of rulers and the more restricted is the tax base, the greater is the equality between the groups. In fact, there exists a substantial region of base-ruler combinations where the non-negativity condition $m \geq 0$ binds implying complete equality between groups. This region lies to the right of the equality boundary in Fig. 1. Starting from any point in the equality region, an increase in the number of rulers has no effect upon utility of either party.

<table>
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<th>$R$</th>
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<td>76.98</td>
<td>38.49</td>
<td>25.66</td>
<td>19.24</td>
<td>12.83</td>
<td>9.62</td>
<td>7.70</td>
<td>6.41</td>
<td>4.81</td>
<td>3.85</td>
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</table>

Where $B$ is the number of goods in the excise tax base and $R$ is the number of rulers. The top number in each pair of entries is the utility of the representative ruler, and the bottom number is the utility of the representative subject.
An important implication of the example is that equality of treatment may emerge from restrictions on the taxing powers of rulers rather than as the consequence of utilitarian policies. This is particularly true of large ruling classes. Consider the case where rulers comprise one-half the population, \( R = 500 \). When \( B \leq 33 \), ruler and subject are treated equally. A majority voting rule yields the same result as a unanimity rule. Fiscal policy cannot be profitable used to discriminate between subjects and rulers.

Either "political" or technological restrictions on the tax base may generate the above results. However, political restrictions, unlike technological restrictions, can be loosened to increase efficiency. The size of the ruling class now becomes an important variable in the determination of how far the subjects would want to expand the tax base.

The subjects' optimal constitutional choice of the tax base as a function of the number of rulers (as determined from Table 2) is plotted in Fig. 1A. The
corresponding utilities of subjects and rulers are plotted in Fig. 1B. When the ruling class is small, rulers are able to make large payments to themselves, even from a small tax base. Rulers enjoy a very high level of utility, while subjects experience a very low level of utility. As the number of rulers increases, the utilities of rulers and subjects rapidly converge. When \( R \geq 170 \), rulers spend all their revenues on the public good implying equality of utility between subjects and rulers.

The number of goods in the subjects' optimal base contracts from 33 to 9 as the number of rulers increases from 10 to 170. Subjects yield a larger tax base to a smaller ruling class because they value the additional public goods that would be provided very highly. Beyond \( R = 170 \), the optimal tax base increases with \( R \) because subjects know that a larger ruling class has a greater incentive to provide the public good and a reduced incentive to raise the tax rate. Since a more efficient base is being used, the utility of both subjects and rulers increases. Note, however, that the tax base remains inefficient despite the fact that the utilities of rulers and subjects are the same. The tax base is not broadened because the subjects know that rulers would use the broadened base to exploit them. This is an example of what Faith and Thompson [1981] call a "paradox in the theory of the second-best". The paradox arises because the original reason for the second-best constraint is to defend the subjects against the rulers and the abandonment of the constraint would permit rulers to engage in behavior that would make the subjects worse off than if the constraint were in force.

NOTES

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1 The rulers face no agency problem. If bureaucrats are thought to control the government, they should in this framework be specified as the rulers. The role of the rulers is kept undefined for generality. This begs deeper questions of how that class rose to completely dominate government and the role that evolution might have in the subsequent weighting. Romer [1986], and Usher and Engineer [1987] develop models of revolutions where successful revolutionaries are rewarded with a positive weighting in the new regime's objective function.

2 Brennan and Buchanan [1978, 1980, Ch. 7] model a king who consumes the public good directly as a "nonsurplus-maximizer". Their analysis of the "king's problem" is a generalization of their earlier analysis of "government as a revenue-maximizer" (Brennan and Buchanan [1977, 1980, Ch. 3]) when provision of the public good is fixed and government as a "surplus-maximizer" (Brennan and Buchanan [1978, 1980, Chs. 4 and 7]) when the public good is variably provided.

3 Brennan and Buchanan [1980, Ch. 8].

4 With a uniform tax rate on all goods, each ruler's disposable income is \((1 - r)(\tilde{x}_i + m)\). Since subjects do not receive a transfer, each has disposable income \((1 - r)\tilde{x}_i\). Excise tax revenues from subjects and
rulers will equal the transfers to the rulers (ignoring the public good): $(H - R) \rho \bar{x}_i + R \tau (\bar{x}_1 + m) = Rm. The budget constraint for a ruler can therefore be expressed as
\[
\sum_{i = 1} \rho_i \bar{x}_i = (1 - \tau) \bar{x}_1 + H \bar{x}_1 R.
\]
Now in the limit as \( r \to 1 \), each ruler’s disposable income goes to \( H \bar{x}_1 / R \), implying a net transfer of \( (H - R) \bar{x}_1 / R \). The gross transfer in this case goes to infinity as the tax rate goes to one.

This is often referred to as a “small agent assumption”. The intermediate case of a few rulers with partial free-riding is not treated because of the game-theoretic complications involved. As far as I know, an examination of the small agent assumption has not been attempted in the optimal tax literature.

\(^5\) If \( R = 1 \), the small agent assumption is assumed not to apply. The lone ruler will set the tax rate to maximize tax revenues given the excise tax base. Since all goods have unit price elasticity with a Cobb-Douglas utility function, the ruler sets the tax rate equal to one. Thus each subject’s utility in this case is zero if any part of the tax base is taxable. For \( R > 1 \) the small agent assumption is used.

\(^6\) Brennan and Buchanan develop their “Tax Constitution for Leviathan” from behind a Rawlsian veil of ignorance using a maximin rule. Since subjects are never better off than rulers, the maximin rule maximizes the utility of subjects. West and Corke (1980), in their discussion of Brennan and Buchanan’s work, analyze constitutional choice using an expected utility approach. Unfortunately, pursuing this approach in this paper is complicated by the fact that there exist many private goods and a public good in the model. A simple alternative that preserves some of the spirit of the expected utility analysis is to include \( V \) and \( V' \) as wealth arguments in a function
\[
EZ(V, V', \rho) = \rho Z(V) + (1 - \rho) Z(V')
\]
where \( Z \) is a concave function and \( \pi \) is the probability of being a ruler. The maximin rule corresponds to the “absolute risk averse” case, \( Z'' = -\infty \). Without absolute risk aversion the Rawlsian individual chooses policy considering the probability that he may become a ruler. More complex lotteries may also be considered. For example, the size of the post constitutional ruling class may be a random variable. A coup d'etat by an elite might be modelled as \( R = 100 \) with 10 per cent probability and \( R = 1000 \) with a 90 per cent probability. Now if \( Z(v) = v^2 \) and \( \rho = .1 \), the optimal base has about 85 goods. Even a modest degree of risk aversion can result in a cautious choice of base.

REFERENCES


**Summary: Taxes, Public Goods, and the Ruling Class: An Exploration of the Territory Between Brennan and Buchanan’s Leviathan and Conventional Public Finance.** — Alternative theories of government behavior yield conflicting prescriptions for public finance, ranging from Samuelson’s (1954) well-known rule for the provision of the public good to Brennan and Buchanan’s (1977) “Tax Constitution for Leviathan”. This paper presents a framework that subsumes these rules as polar cases. A subset of the population, called the “ruling class”, is modelled as maximizing its own welfare exclusively in the choice of public goods, non-negative lump-sum payments and excise tax rates, subject to a given excise tax base and to private sector behavior. At one pole, where everybody is a member of the ruling class, it is in the public interest for the rulers to use a comprehensive and efficient excise tax base. At the opposite pole, where there is only one ruler, the subjects’ interests are best defended by means of a restricted and inefficient tax base which enables subjects to substitute away from goods that are heavily taxed — Brennan and Buchanan’s “Tax Constitution for Leviathan”.

