Choice under Uncertainty

• Theoretical Concepts/Techniques
  – Expected Utility Theory
  – Representing choice under uncertainty in state-contingent space
  – Utility Functions and Attitudes towards Risk
    – Risk Neutrality
    – Risk Aversion
    – Risk Preference
Choice under Uncertainty

• Analyzing Decision Making under Uncertainty
  – The Desire to Buy Insurance
  – Insurance and asymmetric information: The Role of government in the insurance market
The Sadistic Philanthropist

• A patient learns he has two more days to live unless he gets a heart operation.
• The patient needs $20,000 to get the surgery.
• His friends and relatives cannot help him.
• He meets the sadistic philanthropist who makes the following proposal
The proposal

• Gamble A:
  – Get $10,000 with probability .5
  – Get $15,000 with probability .5

• Gamble B:
  – Get $0 with probability .99
  – Get $20,000 with probability .01
The patient’s choice

• Gamble A has a higher expected monetary value than B:
  • Gamble A: $0.5 \times 10,000 + 0.5 \times 15,000 = 12,500$
  • Gamble B: $0.99 \times 0 + 0.01 \times 20,000 = 200$.
• But he won’t be able to afford surgery if he chooses gamble A.
Expected monetary value versus expected utility

• In utility terms, the patient is better off with gamble B. With gamble B he has a small chance of surviving, while with gamble A, he will certainly die.

• His expected utility is therefore higher for gamble B than for gamble A.

• Illustrates that higher expected payoff of one gamble compared with another gamble does not necessarily imply that this gamble also yields a higher expected utility than the other gamble!
Maximizing Expected Utility

• When making decisions in situations involving uncertainty, agents do not simply choose the option that maximizes their expected monetary payoff; they evaluate the utility of each payoff.

• When people are faced with risk, they assess the possible payoffs in terms of utility and then choose the gamble that yields the highest expected utility.

• This is called the expected utility hypothesis.
Maximizing Expected Utility

• Given a risky situation we find the expected utility of this situation by summing up the utility if event $i$ – denoted by $v(y_i)$ – occurs multiplied by the probability of event $i$ – denoted by $\pi_i$ – occurring.

$$\bar{v} = \sum_{i=1}^{n} \pi_i v(y_i) = \pi_1 v(y_1) + \ldots + \pi_n v(y_n)$$
Maximizing Expected Utility: Cardinal Utility

- Recall that for choice under certainty, we only need ordinal utility.
- When there is uncertainty in the world, we need a stronger utility concept than ordinal utility.
- We need what economists call a cardinal utility function because it is necessary to place more restrictions on the types of utility numbers we use.
Example: Cardinal Utility

- Suppose an individual likes higher monetary payoffs more than lower monetary payoffs.
- Her utility should increase as payoffs get higher.
- Suppose her utility over money (denoted by $y$) is $v(y) = y^{1/4}$.
- Find the expected utility of investments A and B.
- Which investment will she choose?
Maximizing Expected Utility

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<th>Payoff in $</th>
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Find the expected utility of each investment if \( U = y^{1/4} \)

Investment A:

\[ .1 \cdot (10)^{25} + .3 \cdot (20)^{25} + .2 \cdot (30)^{25} + .2 \cdot (40)^{25} + .2 \cdot (50)^{25} = 2.3151 \]

Investment B:

\[ .3 \cdot (20)^{25} + .4 \cdot (30)^{25} + .3 \cdot (40)^{25} = 2.325 \]

Investment B maximizes expected utility!
Maximizing Expected Utility

<table>
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<tr>
<th>Investment A</th>
<th>Investment B</th>
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<td>Plant Wheat in the North</td>
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Find the expected utility of each investment if utility over money changes to \( v(y) = y^{1/2} \)

**Investment A:** maximizes expected utility!

\[
.1(10)^{0.5} + .3(20)^{0.5} + .2(30)^{0.5} + .2(40)^{0.5} + .2(50)^{0.5} = 5.4324
\]

**Investment B:**

\[
.3(20)^{0.5} + .4(30)^{0.5} + .3(40)^{0.5} = 5.4299
\]
Marginal utility of money

• With the first utility function, the individual prefers investment B to investment A.
• With the second utility function, the individual prefers investment A to investment B.
• Comparing both utility functions over money $y$, the utility of getting an additional $ is different at any given level of money for the two functions. This difference is not just a matter of scaling!
• The shape of the utility function over money tells us about the attitude of risk of an individual.
Attitudes towards risk

• We have seen that the marginal utility of money determines a person’s attitude towards risk.

• Will now more systematically analyze this question.
Risk Neutrality

• A person is risk neutral if a certain amount for sure is as valuable to this person as a gamble with the same expected value.
• A risk neutral person is willing to take on a fair bet.
• A person who is risk neutral has a constant marginal utility of money.
Risk Neutrality

- A risk neutral person has a linear utility function over money, \( v(y) = ay + b \) with \( a, b \) constants, and \( a > 0 \).
- The expected utility of a gamble with two outcomes, say 20 with probability 1/2 and 60 with probability 1/2 is equal to 
\[ .5 \cdot v(20) + .5 \cdot v(60) = .5 \cdot (20a + b) + .5 \cdot (60a + b) = 40a + b. \]
- Note that this is the same as \( v(40) = 40a + b \).
- If a person is risk neutral only the expected monetary value matters, but not the risk.
- Note that setting \( a = 1, b = 0 \), will not change the ranking of any two gambles.
- **Only** for a risk neutral person, maximizing expected utility is equivalent to maximizing expected payoff!
Strict Risk Aversion

• A person who has a decreasing marginal utility of money is risk averse. This means $v(y)$ is a strictly concave function.

• Strict risk aversion means that this person is not willing to take a fair bet.

• A strictly risk averse person is only willing to gamble if the expected payoff of the gamble is higher than the money you offer her for sure. That is, the person needs to be compensated with a higher expected return compared to the safe return to take on some risk.
Risk Preference

• People with a risk preference (or risk loving people) have an increasing marginal utility of money. This means $v(y)$ is a strictly convex function.

• People with a risk preference are willing to take on an unfair(!) bet.

• People with a risk preference are only willing to accept a money value for sure if this value is higher(!) than the expected money value from a gamble.
Graphical representation

• Suppose there are only two states of the world.
• Can use indifference curves in state-contingent space to analyze choice under uncertainty.
State-Contingent income space

$y_1 = y_2$

45 degree line is the certainty line: same income in both states of the world.
Indifference curves in state-contingent space

- We find indifference curves the same way as always.
- Hold expected utility fixed, see what combinations of \((y_1, y_2)\) give us same expected utility.

\[
V(y_1, y_2) = \pi_1 v(y_1) + \pi_2 v(y_2) = V^0
\]
Slope of indifference Curves

\[
\pi_1 v(y_1) + \pi_2 v(y_2) = V^0
\]

\[
\pi_1 v'(y_1)dy_1 + \pi_2 v'(y_2)dy_2 = 0
\]

\[
\frac{dy_2}{dy_1} = -\frac{\pi_1 v'(y_1)}{\pi_2 v'(y_2)}
\]

Slope is equal to the negative ratio of the marginal expected utility in state 1 to the marginal expected utility in state 2.
Slope of indifference curves at 45 degree line

if \( y_1 = y_2, v(y_1) = v(y_2) \)
and therefore \( v'(y_1) = v'(y_2) \).

Then \( \frac{dy_2}{dy_1} = -\frac{\pi_1 v'(y_1)}{\pi_2 v'(y_2)} = -\frac{\pi_1}{\pi_2} \)

The slope of the indifference curves at the 45-degree line is equal to the negative of the risk ratio: \(-\frac{\pi_1}{\pi_2}\)
Indifference curves and attitudes towards risk

• For a strictly risk averse person, the indifference curves in state-contingent space have decreasing MRS, i.e. the indifference curves are strictly convex.
• For risk neutral person, indifference curves are straight lines with slope equal to the negative risk ratio.
• For a risk loving person, indifference curves have increasing MRS, i.e. the indifference curves are strictly concave.
• No matter what the attitude towards risk, the absolute slope of an indifference curve where it intersects with the 45 degree (certainty) line is equal to the risk ratio.
State-contingent income space and indifference curves of a strictly risk averse person

Slope = $-\frac{\pi_1}{\pi_2}$

$y_1 = y_2$

y in good state

y in bad state

$25$
The demand for insurance

• Let $L$ be the amount of loss in the bad state, and denote Brian’s wealth in the good state by $y$.

• Without insurance, Brian has $y$ in the good state and $y-L$ in the bad state.

• The bad state occurs with prob. $\pi_2$ and the good state with prob. $\pi_1$.

• Suppose Brian can insure himself at any amount of coverage $q$ paying $p$ for each $1$ of insurance coverage.

• That is, if Brian pays $pq$ to the insurance he receives $q$ in the case his house burns down.
Opportunity cost of not buying insurance

• Suppose Brian can buy $1 of coverage for $p. What is the opportunity cost of income in the good state?
• The opportunity cost of $1 in the good state is equal to the amount of $ Brian foregoes in the bad state in order to keep $1 in the good state.
• Brian can convert $1 from the good state into $1/p dollars of coverage in the bad state. However, he still would have to pay $1 also in the bad state as insurance premium. Thus he is giving up $((1/p)-1) in the bad state for every $1 he keeps in the good state. Put differently, the opportunity cost of not paying $1 to the insurance company in the good state is $(1-p)/p$. 

Brian’s “budget constraint”

• As in the exchange economy, we can think of Brian’s initial wealth bundle \((y, y - L)\) as his initial endowment. If he chooses to consume at this point he buys no insurance. If he decides to buy coverage of \$q\) he must give up \(pq\) in the good state in order to receive \(q - pq\) in the bad state.

• His wealth in the good state is then \(y - pq\), and in the bad state it is \(y - L + q - pq\).

• We have identified two points on his budget constraint and thus can draw his budget line.
Insurance in Graph

Slope of budget line: 
\[-(1-p)/p\]

Income bundle with q coverage of insurance at premium p

Income bundle w/o insurance

\[y - L + q - pq\]

\[y - L\]

\[y - pq\]  \[y\]  \[y\]  \[y\]  \[y\]
Brian’s budget line

• Brian’s budget line identifies all the combinations of money holdings in states 1 and 2, that he can achieve by transferring money from state 1 to state 2 by buying insurance.
Insurance Demand in Graph

Income bundle w/o insurance

Optimal income bundle with premium $p$

$y$ in good state

$y$ in bad state

$y - L + q - pq$

$y - L$

$y - pq$
Actuarially Fair Premium Per $ of coverage

- The actuarially fair premium per $ of coverage is equal to the probability of the bad outcome happening.
- To see this, the insurance breaks even if for any amount of coverage $q$ the customer pays a premium $p$ such that $\pi_1 pq + \pi_2 (pq - q) = 0$

$$\pi_1 p + \pi_2 (p - 1) = 0$$

Note that $\pi_1 = (1 - \pi_2)$.

Then $p = \pi_2$
Insurance Demand in Graph

slope of budget line: $\frac{\pi_1}{\pi_2}$

Optimal bundle with actuarially fair premium: full coverage is optimal

Income bundle w/o insurance

y in bad state

y - pL

y - L

y - pL y y in good state

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Budget Line with Actuarially Fair Premium

• The expected payoff for any combination of state 1 - income and state 2 - income on the budget line with actuarially fair premium is always the same. This expected payoff is equal to

\[ \pi_1 (y - \pi_2 q) + \pi_2 (y - L - \pi_2 q + q) \]

\[ \pi_1 y - \pi_1 \pi_2 q + \pi_2 (y - L) + \pi_2 q (1 - \pi_2) \]

Note that \( \pi_1 = (1 - \pi_2) \)

Then expected payoff is equal to

\[ \pi_1 y + \pi_2 (y - L) \]
Insurance Demand in Graph

y in bad state

y - pL

y - L+q-pq

y - L

Optimal bundle with actuarially fair premium: full coverage is optimal

Optimal income bundle with actuarially unfair premium p > π₂: less than full coverage is optimal

Income bundle w/o insurance

y - pq

y - pL

y - L

y in good state
Demand for Insurance -
Summary

• If a person is strictly risk averse,

• with actuarially fair insurance, it is optimal
to be fully covered, \( q = L \).

• with actuarially unfair insurance, it is
optimal to be only partially covered, \( q < L \),
with \( q \geq 0 \).
Asymmetric Information

• Market failure due to Moral Hazard
• Market solutions to Moral Hazard: Deductibles and co-payments
• Government intervention due to MH
• Market failure due to Adverse Selection
• Market solutions to Adverse Selection: Deductibles and co-payments
• Government intervention due to AS
Moral Hazard-Hidden Action

• So far we have assumed that with a certain probability a bad event will happen for a person.
• In a lot of situations, however, the risk of a bad event happening depends on the actions taken by individuals.
• If these actions cannot be monitored by insurance companies, we have a problem of hidden action or Moral Hazard.
Behavior changes probabilities

• People can to some extent prevent bad things from happening to them by taking precautions or taking sufficient care.
• Examples: health, theft, fire etc.
• E.g., once insurance is bought, people might become careless and the probability of theft changes.
• An insurance company not taking into account the change in probability of thefts might end up losing money instead of making a profit.
Example: Insurance and Moral Hazard

- Suppose Brian’s wealth is $w$.
- There is a chance $\pi^0$ that Brian loses $L$ of his wealth so that he’s left with $w - L$ if he has an expenditure on care equal to 0.
- However, if Brian spends a positive amount $a$ on care, there is a smaller chance $\pi^0 > \pi^a$ that Brian loses $L$ of his wealth so that he’s left with $w - L$. 
Income bundles w/o insurance

- Without expenditure on care
  \[ y_G = w \]
  \[ y_B = w - L \]

- With expenditure \( a \) on care
  \[ y_G = w - a \]
  \[ y_B = w - L - a \]
Competitive insurance market

- If the expenditure on care is observable, competition in the insurance market will drive the premium per $ of coverage down to the actuarially fair premium.
Income bundles with insurance if expenditure level on care can be observed

- **Without expenditure on care** \( p = \pi^0 \)
  
  \[
  y_G = w - \pi^0 q
  \]
  
  \[
  y_B = w - L + q - \pi^0 q
  \]

- **With expenditure** \( a \) **on care** \( p = \pi^a \)
  
  \[
  y_G = w - a - \pi^a q
  \]
  
  \[
  y_B = w - L - a + q - \pi^a q
  \]
Optimal coverage when expenditure on care is observable

• Since the premium is actuarially fair, Brian would choose full coverage at each expenditure level of care.

• Assume that Brian would receive a higher utility level under full coverage and the higher level of care, so he would choose the higher level of care.

\[
V^a > V^0
\]

\[
V^a = v(y - \pi^aL - a) > v(y - \pi^0L) = V^0
\]
Why spend money on care?

• While it costs Brian \( a \) to take care, it lowers his insurance premium. The incremental benefit of care exceeds its incremental cost.

\[
V^a > V^0 \\
\left(\pi^0 - \pi^a\right)L > a
\]
Full Information Equilibrium

\[ y_G = y_B \]

\[ V^a > V^0 \]

\[ x^a = \pi^a L \]

\[ x^0 = \pi^0 L \]

\[ x^a < x^0 \]
Expenditure on care unobservable

• Next assume the insurance companies cannot observe the level of expenditure on care.
• What happens if the insurance companies offer an insurance premium of $\pi^a$?
• Brian will get full coverage and not take care because

$$v(y - \pi^a L) > v(y - \pi^a L - a)$$
Offering the actuarially fair premium for a is not an equilibrium

- Since Brian has no incentive to take care, the insurance company will no longer break even by charging the actuarially fair premium assuming Brian will take care.
- Since the insurance companies are fully aware that Brian’s utility, once he has bought full coverage, is higher if he doesn’t take care, they will not make this deal.
Optimal Insurance contract if care expenditure is unobservable

- What could they offer instead?
- Competition will drive insurance profits down to zero. Insurance contract must be set such that insurance companies know which level of care Brian will take.
- Brian will be better off buying insurance and taking care than buying insurance and not taking care
- Brian will be better off buying insurance and taking care than not buying insurance
- Whenever faced with a choice between two contracts, Brian will take the contract that maximizes his expected utility.
What is a deductible?

- With a deductible, Brian needs to pay damages up to a certain amount by himself and damages beyond that amount are covered by the insurance.
- Another way of putting this is that the insurance companies do not offer full coverage.
Incentive Compatibility

• If the insurance company wants to break even, it needs to ensure that Brian will behave in the way it wants.

• That is, premium and coverage should make sure that Brian is getting a higher utility from behaving in the way it wants him to behave.

• At the actuarially fair premium if Brian takes care, what is the highest amount of coverage that the insurance company can offer without Brian switching to zero care costs?
Imperfect Information

\[ y_G = y_B \]

\[ V^0(\pi^a) > V^a \]

\[ x^a = \pi^a L \]

\[ x^0 = \pi^0 L \]

\[ x^a < x^0 \]

Offering full coverage at \( \pi^a \) is not an equilibrium; it will not result in point A.
Incentive compatibility

\[ y_G = y_B \]

- Since IC’s cross at 45 degree line, the two indifference curves yield the same utility level.
- If coverage leading to point A is offered, Brian has no incentive to reduce care costs to zero.
Participation Constraint

• It has to be in Brian’s interest to buy insurance at the premium and deductible offered. His utility from buying insurance and taking care must be at least as high as his utility with the next best alternative.

• Alternative contract involves paying the actuarially fair premium when care costs are zero.
Participation constraint

\[ y_G = y_B \]

• Alternative contract involves paying the actuarially fair premium when care costs are zero.
• This gives Brian a lower utility than paying for partial coverage in A.
Equilibrium is not efficient

- A is the equilibrium outcome; nobody has an incentive to deviate.
- Still, Brian would be even better off if he could get full coverage at actuarially fair premium when care costs are $a$ and he indeed pays care costs of $a$ without making insurance companies worse off.
Moral Hazard leads to Market Failure

- We do not have a Pareto efficient situation here: Brian could be made better off without making the insurance company worse off, if he could commit to not changing his behavior after he gets insurance.

- We encountered here a case of market failure due to Moral Hazard. The competitive equilibrium is not Pareto efficient.
Government Intervention

• Is there anything the government could do to restore efficiency?
• Yes, there is.
• Government can tax and subsidize.
• Subsidize care and then lump-sum tax individual to balance the budget.
Does the government need to know more than insurance companies?

• Government can change behaviour of individuals without knowing more than insurance companies.

• By subsidizing people to take care and hence lowering the costs of care, individuals will take more care than before.

• People are taxed no matter if they take advantage of subsidy or not.
Policy to subsidize care and LS-tax.

- Let the price of care be 1, so that a per unit subsidy lowers price of care from 1 to $1-s$.
- Each individual is being taxed $sa$ no matter what actions he/she takes.
- Let $s = 1$, then care is costless.
Subsidy and LS-tax yield efficiency

The diagram illustrates the concept of Pareto efficiency. B is a Pareto improvement over A.

The equation $x^a = \pi^a L$ represents the relationship between the tax paid ($x^a$) and the yield ($\pi^a L$).

No longer the cost of care, but the LS tax a person has to pay.
How to deal with HS

- Tax bad behavior (e.g. increase price of cigarettes to improve health, soda tax)
- Subsidize taking care
- Then levy lump-sum tax/subsidy to balance budget.
- Note that Brian will then find it in his best interest to buy insurance and exert the optimal level of care.
- Note that many government programs provide insurance and thus govt has to consider the presence of MH itself.
Adverse Selection

• While the moral hazard problem occurs because one party is unable to monitor the behaviour of the other party after the trade, the adverse selection problem occurs because one party is unable to recognize certain characteristics of the other party before the trade.
Health Insurance and Deductibles

We’ll analyze how the competitive insurance market copes with adverse selection.

We’ll assume two different risk groups and allow insurance companies to offer two different policies: one targeted at the safe types, the other targeted at unsafe types.

If premium and deductibles are set properly, each type will choose the policy that is intended for its type.

We’ll see that such an outcome is not Pareto efficient.
Pool vs. Separating Equilibrium

- If we have an equilibrium where sellers offer only one policy that both types buy, this is known as a **pooling equilibrium**.
  - Risk across the two types is pooled.

- If we have an equilibrium where sellers offer two policies, one which is only purchased by safe types and another which is purchased only by unsafe types, this is known as a **separating equilibrium**.
  - The policies separate the types.

- We will see that in insurance markets with adverse selection:
  - There will *never* exist a pooling equilibrium.
  - There *might* exist a separating equilibrium.
    - Depends on number of safe types relative to unsafe types.

- To start thinking about equilibrium in insurance markets with asymmetric info, recall the full info equilibrium.
Notation

- $U =$ Unsafe type
- $S =$ Safe type
- FIBL: (actuarially) Fair Insurance Budget Line. That is, insurance company charges actuarially fair premium for this type.
- $W_G$: wealth in good state
- $W_B$: wealth in bad state
- $E\Pi$: Expected Profit of Insurance companies
Assumptions about Wealth

• Assume that both types have the same wealth in good and bad state without insurance.

• To make things even more simple assume that wealth in bad state is zero without insurance. That is, Loss in bad state is equal to wealth in good state w/o insurance.
Assumptions about insurance market

• Perfect competition drives expected profits to zero.

• If current policies are such that by offering a new policy an insurance company can steal customers away and make expected profits greater than zero, the current policies cannot be a competitive equilibrium.
Actuarially Fair Premium for each Type

With full info, safe types are offered full insurance at premium \( x_s \) and unsafe types are offered full insurance at

Question: Could the policies A and B be equilibrium policies with asymmetric info?

No: If offered choice of policy A or B, both S and U types would want A (obviously: A offers same coverage, but is cheaper!).

If everyone buys policy A, then \( E\Pi < 0 \) for seller.
Reason: Break even on safe types (A is on FIBL_s), lose money on unsafe types (A is above FIBL_U).
So if both types are going to be offered the same policy, it must lie between FIBL_U and FIBL_S in order for \( E\Pi = 0 \)
Average risk and average actuarially fair premium

• How do we calculate exactly where in between the FIBLs?
  – It will depend on the number of S types relative to U types.
• Suppose 75% of the population are safe and 25% are unsafe.
• The average level of risk in the population is the probability-weighted average of the different values of $\pi_B$.
  – i.e., $\pi_B^p = (3/4) \pi_B^S + (1/4) \pi_B^U$
• If we are going to sell one policy to both types, then if this policy has an implicit per dollar premium equal to $\pi_B^p$, then $E\Pi = 0$.
• Graphically, this policy will lie on a new FIBL, one that corresponds to average risk. That is, $\text{FIBL}_p$, which will have slope of $-(1 - \pi_B^p)/\pi_B^p$. 
FIBL for average risk

- Note that because $\pi_B^U > \pi_B^P > \pi_B^S$, FIBL_P lies somewhere between FIBL_U and FIBL_S.
- The greater the number of safe types relative to unsafe types, the closer FIBL_P lies to FIBL_S.
  - And vice versa.

If we sell one policy to all types and make $\Pi = 0$ then we are offering a policy that lies on a BL with slope $= \frac{- (1 - \pi_B^P)}{\pi_B^P}$.
Why we can’t have a pooling eq’m

- If there is a pooling equilibrium:
  - It must involve both types buying the same policy
  - It will consist of a policy that lies on FIBL_P, suppose on point A. That is, both types pay “X pooled” for full coverage.

Question: What does each consumer’s IC look like through point A?

Risk aversion implies strictly convex ICs

We also want to know the relative slopes.

MRS for each type = \(\left(\frac{\pi_G \text{MU}(w_G)}{\pi_B \text{MU}(w_B)}\right)\)

\(\pi_B^U > \pi_B^S\) implies that \(\text{IC}^U\) through any point is flatter than \(\text{IC}^S\) through that same point.
Why we can’t have a pooling eq’m

• Suppose policy A is offered to both types by firm 1.
• Can this be an equilibrium? Does anyone have an incentive to do anything differently?

On $FIBL_P$ so for firm 1 $E\Pi = 0$

Both consumers prefer A to their endowment, so they are happy to buy A
If this is the only policy offered.

Can another firm offer an alternative policy, steal some of firm 1’s customers, and make a profit?

If so, A can’t be an equilibrium.
Why we can’t have a pooling eq’m

- Can firm 2 offer an alternative policy, steal some of firms 1’s customers, and make a profit?
  - Suppose firm 2 offers a policy at point B
    
    All safe types prefer B to A, so they *all* switch policies, leaving firm 1 just selling to unsafe types, because unsafe types prefer A to B.

We have seen that another firm can offer an alternative policy, steal some of firms 1’s customers, and make a profit.
Thus, A can’t be an equilibrium.
No pooling eq’m exists

• So we know there cannot be a pooling equilibrium at point A.
• In fact, we have actually proven that there cannot be any pooling equilibrium.

The argument we have made is a very general one, and relies only on the fact that IC\[^{U}\] is flatter than IC\[^{S}\] through any point.

No matter where we are in this diagram, one policy cannot be sold to both types in equilibrium.

*Any* pooled risk policy offered by one firm allows the other firm to offer an alternative policy that steals away just the low risk consumers.
Finding the Separating Eq’m

• So if there is asymmetric information, then we will never have an equilibrium in which all types are offered the same policy.
  – That is, there is never a pooling equilibrium.

• What about a separating equilibrium?

• We will be looking at situations where every consumer is offered a choice of two policies, and where all the safe types choose one policy, and all the unsafe types choose the other policy.

• Insurance companies try to design these policies so that each type buys the “right” policy for their type. (Incentive compatibility)
Determining whether we have a separating eq’m

- In any separating equilibrium we need the following:
  1. Any policy sold to safe types must lie on $\text{FIBL}^S$ Need to have $\Pi > 0$
  2. Any policy sold to unsafe types must lie on $\text{FIBL}^U$
  3. Each consumer must be as happy with the policy they buy as they would be with the alternative policy (or else they would switch policies).
  4. There can be no possibility of a competitor insurance company offering an alternative policy that steals some customers and yields $\Pi > 0$.

- In order to ascertain whether any proposed pair of policies is actually an equilibrium pair of policies, we need to make sure that 1 through 4 above are satisfied.
  - If all are satisfied, then the proposed pair is an equilibrium.
  - If any statement is not, then the proposed pair cannot be an equilibrium.
**Incentive compatibility**

- Let’s consider an arbitrary pair of policies (A and B) and ask if they are equilibrium policies.

If all consumers are offered the choice of A or B, they will all buy A (both on higher IC at A).

Problem here is that the policy we want only the safe types to buy (A) looks too good to unsafe types.

To ensure that unsafe types buy the policy designed for them (B), we need to make the policy designed for safe types look less attractive to the unsafe types, otherwise statement 3 on the previous slide is violated.
**Incentive Compatibility**

- How can we do this?
  - Increase the premium, decrease the coverage, or both

Note: Policy designed for safe types still needs to be on $\text{FIBL}^S$, for $\Pi = 0$

Means that, IF we want the unsafe types to buy the policy $B$, then the best policy we can offer for the safe types is at $A'$

Note: $A'$ is worse for the safe types than $A$.

So $A$ and $B$ cannot be equilibrium policies, but might $A'$ and $B$ be an equilibrium pair of policies?

We need to check to see if statements 1 through 4 hold.
Firm 2 has incentive to steal away customers

- Might A’ and B an equilibrium pair of policies?
- Each is on the relevant FIBL: statements 1 and 2 are satisfied.

Safe types would rather have A’ than B, and unsafe types are indifferent: statement 3 is satisfied.

\[ y_B = y_G \]

What about statement 4? Can a competitor steal customers and make a profit?

YES! A policy like B’ will steal all unsafe types (but not safe types) and is below \( \text{FIBL}_U \), so \( E\Pi > 0 \).

So A’ and B can’t be equilibrium policies either.
No firm has incentive to steal away unsafe customers

- But A’ and B’ cannot be equilibrium policies either, as B’ is not on FIBLU (i.e., $\Pi > 0$ for policies sold to unsafe types).

In order to have both:

- $\Pi = 0$ for policies sold to unsafe types (statement 2); and
- No possibility of a competitor stealing unsafe types (statement 4), we need to offer full insurance to unsafe types (B’’).

So maybe A’ and B’’ are equilibrium policies?

No……
Stealing away safe costumers

- Maybe A’ and B” are equilibrium policies?
- Both are on the relevant FIBL (statements 1 and 2 hold)

Safe types prefer A’ to B” and unsafe types prefer B” to A’ (statement 3 holds)

And we know there is no profitable way to steal unsafe consumers.

But can another seller steal away the safe types?

Yes, a policy such as A” is preferred by all safe types (but not unsafe ones) and is below FIBLS so $\Pi > 0$.

But A” can’t be part of an equilibrium either, because $\Pi > 0$. ...
No incentive to steal away customers

- So where are we heading?
- To the (newly labeled) points A and B

Each is on relevant FIBL (statements 1 and 2)

Safe types prefer A to B and unsafe types *weakly* prefer B to A (statement 3)

Any alternative policy that attracts *only* safe types yields $E\Pi < 0$.

Any alternative policy that attracts *only* unsafe types yields $E\Pi < 0$.

(statement 4)

So IF there is a separating equilibrium, it will be the policies A and B.

Unsafe types fully insure, safe types are offered less than full insurance.
The Separating Equilibrium

• Note the careful language:
  – IF there is a separating equilibrium…

    We haven’t actually proven that statement 4 holds

Any alternative policy that attracts only safe types yields $\Pi < 0$.

Any alternative policy that attracts only unsafe types yields $\Pi < 0$.

What about an alternative policy that attracts all types?

Look at the position of FIBL$_p$: Any policy that both types would prefer to their current policy would yield $\Pi < 0$ (say point C).

Now we have proven that statement 4 holds, and we know A and B are the equilibrium policies.
Separating eq’m may not always exist

- Why did we need to make sure there wasn’t a policy that other firms could offer to attract both types?

  Because under different circumstances there might be.

  Suppose we had more safe types, so FIBL_P pivots as shown.

  Now if policy C is offered, all types will buy it, and it yields $\Pi > 0$.

  So in *this* case, A and B won’t be equilibrium policies.

But we also know that C can’t be an equilibrium either, as we have already proven that there can never be a pooling equilibrium.

There is no equilibrium at all in this case……
Is the separating eq’m efficient?

- Back to the case where there is a separating equilibrium.
  - Is this equilibrium efficient?
    We fully satisfied demand for insurance by *unsafe* types, as we have a policy where \( MB = MC \) at the policy \( B \).

\[
\begin{align*}
  y_G &= y_B \\
  \text{FIBL}_P &\quad \text{FIBL}_U \\
  \text{ICU} &\quad \text{ICS}
\end{align*}
\]

Less than full insurance implies \( MB > MC \) at policy \( A \).

Equilibrium is INefficient

If only there was a way to solve the info problem, there could be Pareto improvements

If we could get *just* safe types to buy a policy at \( C \), sellers and buyers would both be better off.

Problem: unsafe types would also want \( C \), and we cannot stop them buying it.
Summary: Understanding why there are different insurance plans offered

• In insurance markets with asymmetric information:
  – There is never a pooling equilibrium.
  – There may be a separating equilibrium, as long as the proportion of safe types is not too high.

• If there is a separating equilibrium, it will be characterized by
  – Full insurance for unsafe types.
  – Less than full insurance for safe types (e.g., through deductibles)
  – Inefficiency
Dealing with the Adverse Selection Problem

• Policies to correct the market failure?
• Government provision of insurance is one possibility.
• Essentially the government can mandate that it be the only insurance provider, then provide one policy to everybody.
• Example: Basic health care in Canada.
  – We all receive the same basic level of coverage, and it is (more or less) impossible to go outside the system for this coverage.
  – That is, no competitor can steal away just the healthy consumers from the public system.
  – It would be profitable to do this, but insurers are (more or less) prevented from it by law.
Mandatory Health Insurance is efficient

- If all consumers are paying the actuarially fair premium for pooled risk, this is efficient, because

\[ \text{Average MB} = \text{average MC at policy C}. \]

Given C, we can only make one type better off by making the other type worse off.
Mandatory Health Insurance is efficient

- C is efficient and possibly a Pareto improvement (recall that we start from a situation in which the eqm does not exist)
Why is full coverage at pooled actuarially fair premium efficient?

- We need a balanced budget for any coverage we may want to offer, so the govt faces $\text{FIBL}_p$ as its constraint.
- On this constraint we could choose the optimal coverage for one type. If that’s the case we put all the weight on this type.
- Next we investigate what happens if we put equal weight on each type, so that we choose coverage that maximizes the sum of expected utilities.
Solving for optimal coverage

• Let the fraction of safe types be equal to $S$ and the fraction of unsafe types be equal to $(1-S)$.

• Maximizing the sum of expected utilities is efficient. (That’s the Utilitarian approach to finding the socially optimal policy)

• Given that both types need to receive the same policy, we can solve for optimal coverage.
Solving for optimal coverage

\[
\max_q S \left[ (1 - \pi_B^S) \nu \left( w - \frac{S \pi_B^S + (1 - S) \pi_B^U}{2} q \right) + \pi_B^S \nu \left( w + (1 - \frac{S \pi_B^S + (1 - S) \pi_B^U}{2}) q - L \right) \right] \\
+ (1 - S) \left[ (1 - \pi_B^U) \nu \left( w - \frac{S \pi_B^S + (1 - S) \pi_B^U}{2} q \right) + \pi_B^U \nu \left( w + (1 - \frac{S \pi_B^S + (1 - S) \pi_B^U}{2}) q - L \right) \right]
\]

\[
FOC : -S \left( \frac{S \pi_B^S + (1 - S) \pi_B^U}{2} (1 - \pi_B^S) \nu'(y_G) + (1 - \frac{S \pi_B^S + (1 - S) \pi_B^U}{2}) \nu'(y_B) \right) \\
+ (1 - S) \left( -\frac{S \pi_B^S + (1 - S) \pi_B^U}{2} ((1 - \pi_B^U) \nu'(y_G) + (1 - \frac{S \pi_B^S + (1 - S) \pi_B^U}{2}) \nu'(y_B) \right) = 0
\]

\[
-\frac{S \pi_B^S + (1 - S) \pi_B^U}{2} \nu'(y_G) (S(1 - \pi_B^S) + (1 - S)(1 - \pi_B^U)) + (1 - \frac{S \pi_B^S + (1 - S) \pi_B^U}{2}) \nu'(y_B) (S \pi_B^S + (1 - S) \pi_B^U) = 0
\]

\[
-v \left( w - \frac{\pi_B^S + \pi_B^U}{2} q \right) + \nu \left( w + (1 - \frac{\pi_B^S + \pi_B^U}{2}) q - L \right) = 0
\]

\[q^* = L.\]

Full coverage is optimal.
Compulsory Health Insurance

- With compulsory health insurance, people with different health risks pay the same premium.
- This makes people with higher health risks better off than the market solution and makes the people with lower health risk worse off than with the market solution (if there is a separating equilibrium).
- While efficient, it is not necessarily a Pareto-improvement over the market!