Conceptual stochastic climate models

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1 Introduction

The past 30 years have seen an increasing recognition that human activities may have substantial, and possibly even catastrophic, effects on the climate system. This recognition has highlighted the importance both of understanding the climate system and of being able to accurately model it. These two goals are made particularly difficult by the fact that the Earth's climate system displays substantial variability on spatial and temporal scales over many orders of magnitude, from turbulent eddies in the ocean surface driven by breaking waves, through mid-latitude cyclonic storms hundreds of kilometres in extent and lasting for days, to global-scale millennial rearrangements of ice cover and ocean circulation. Because of the complexity of the physics underlying the dynamics of the system, there are important couplings between aspects of climate variability at these different scales. This coupling of variability across scales poses a significant challenge for any attempt to quantitatively model the climate system, as in any computationally tractable climate model there will be a broad range of scales that cannot be explicitly resolved but whose aggregate effect on the resolved scales must be accounted for. The inability to explicitly represent all potentially relevant processes in climate models has many sources, including: finite spatial and temporal resolution of numerical approximations to the underlying differential equations, imposed by computing limitations; filtering of high-frequency processes out of the original equations of motion, also for computational reasons; poor understanding of the basic physical processes operating on smaller scales, following from a paucity of adequate observations (especially in the oceans); and (in the case of more conceptual models) the desire to avoid the problem that deep understanding of climate models tends to decrease as the number of explicitly resolved processes increases. In any event, for any climate model there will be a class of processes or scales of variability that may have an effect on the resolved variables but which are themselves not represented explicitly (or are simply ignored altogether).

In the construction of a climate model, then, decisions must be made as to which scales are represented explicitly, and which are to be (in the jargon of modelling) "parameterised". How this separation is made depends largely on where the model falls in the "hierarchy of climate models" [36]. This hierarchy ranges from heuristic conceptual models designed to capture the essence of important processes in the climate system, simple enough for their parameter space to be thoroughly investigated but too simple to be treated as quantitatively accurate, through Earth System Models of Intermediate Complexity (EMICs) which represent some climate subsystems (e.g. the ocean or the atmosphere) in detail while parameterising the dynamics of other subsystems [14], to full General Circulation Models (GCMs) which attempt to represent the climate system in as great detail as computational and conceptual limitations allow. Models at all levels of this hierarchy play important roles in the development of our understanding of the climate system. In particular, the utility of simple heuristic models is (at least) twofold. First, such models allow the investigation of basic climate processes in a reduced setting, facilitating the development of a conceptual vocabulary with which to understand the dynamics of the climate system, or of its representation by EMICs or GCMs (the output of which can be almost as complex as that of the climate system itself). Secondly, simple heuristic models can predict basic phenomena which may then be sought out in more complex models or in the climate system itself; classic examples of this are the prediction of multiple states of the thermohaline circulation by Stommel [78] and of the phenomenon of sensitivity to initial conditions by Lorenz [45]. One of the earliest simple climate models (in the modern sense) was that of the global atmospheric circulation discussed by Hadley [26]. While wrong in many details, this model is sufficiently instructive to retain a place of pride in introductory expositions of the general circulation of the atmosphere. In the development of our understanding of the climate system, *numerical investigations and the creation of an ever more trustworthy virtual reality* and *mathematical qualitative research of reduced models retaining only some aspects of reality* have to go hand in hand.

Central issues in climate modelling are thus which processes to represent explicitly and which to parameterise, and how to construct accurate parameterisations. The traditional approach to the solution of the parameterisation problem represents those terms in the dynamical equations associated with unresolved phenomena by deterministic functions of the resolved variables, introducing new parameters (e.g. eddy diffusivities, bulk transfer coefficients, etc.) whose numerical values are determined from observational data, from theory, or by being tuned until the model behaves “realistically”. A theoretical justification for this approach to the parameterisation problem follows from an analogy to the connection between statistical mechanics and thermodynamics. An ideal gas is characterised by variability both on the molecular microscale, and on the macroscale on which it is sensible to define quantities such as pressure and entropy. The aggregate effect of essentially random molecular motion on the microscale is to produce essentially deterministic dynamics on the macroscale. In the climate system, it is argued, if the (generally turbulent) unresolved scales are in statistical equilibrium with the resolved scales, then the parameterisation of their effects on the resolved variables by deterministic functions of the resolved variables is appropriate (e.g. [29], [39]). However, an essential difference between a typical statistical mechanical system and the climate system is that, in the former, there is an effectively infinite separation of scales between the micro and the macro (i.e., there is a “thermodynamic limit”), while no such clear separation of scales obtains in the climate system. Consequently, it is not *a priori* obvious that the aggregate effect of the turbulent, unresolved scales on the resolved scales is, and can be modelled by, a deterministic function of the resolved variables. Instead, the appropriate solution to the parameterisation problem may involve an element of randomness.

The parameterisation by stochastic processes of variability not resolved explicitly, first suggested in the seminal paper of Hasselmann [27], has recently been argued for on basic theoretical grounds (e.g. [59], [73]). It has demonstrated considerable success in the study of, for example, the dynamics of El Niño and the Southern Oscillation [62], decadal variability of the thermohaline circulation [71], upper tropospheric dynamics [15], the dynamics of baroclinic waves [16], interannual variability of open ocean deep convection [43], and glacial climate variability [21]. This list is merely a sampling of the work that has been carried out incorporating the representation of unresolved variability by stochastic processes, and is not by any means exhaustive. The field of “stochastic climate models” is diverse and growing.

From July 9 through July 11, 2001, 50 researchers from the mathematics and climate communities met in Chorin, Germany to discuss the idea of stochastic representation of unresolved variability in climate models. The foundation of stochastic climate models through stochastic reduction of multi-scale deterministic processes describing climate phenomena was one of the main focal points of the forerunner meeting in Chorin in June 1999. Its importance was underlined by the fact that a whole chapter in [35] was devoted to it under the name *The emergence of randomness: chaos, averaging, limit theorems*. This special issue of *Stochastics and Dynamics* collects some papers presented at this second meeting on Stochastic Climate Models, all dealing with reduced stochastic models of climate phenomena.

2 Simple stochastic climate models in theory

The paradigm of stochastically reduced climate models designed to increase the conceptual physical and mathematical understanding of highly complex phenomena is heuristically deduced in Hasselmann's visionary paper [27]. Its mathematical underpinning was postulated in [35] and named *Hasselmann's program* after Arnold's article [2]. The realisation of Hasselmann's program would allow the rigorous deduction of the noise terms used to close deterministic equations leading to stochastic conceptual models, such as the ones presented in Olbers' gallery [57]. These conceptual models have the advantage of easier analytical tractability, due to the arsenal of methods of the mathematical areas of stochastic analysis and random dynamical systems. An idealised formulation of Hasselmann's idea involves processes on (at least) two scales, typically a fast one (the weather scale) and a slow one (the climate scale). In reality, the fast scale may be on the order of several days, while the slow one may be on the order of years or more. Mathematically, one can introduce a scaling parameter ϵ, corresponding to the ratio of the (response) time scales of the slow and fast variables, and write an idealised GCM system composed of slow (x) and fast (y) component by the equations

$$\begin{aligned} \dot{x} &= f(x,y), \\ \dot{y} &= \frac{1}{\epsilon} g(x,y). \end{aligned}$$

The passage to statistical dynamical models (SDM) involves the mathematical technique of *averaging* out the fast variables while the slow ones are frozen. Defining $u = \langle x \rangle$, the average of x taken with respect to an invariant measure of the system, and $F(x) = \langle f(x,y) \rangle$, the corresponding SDM states

$$\dot{u} = F(u).$$

In this framework, Hasselmann's idea may be paraphrased by postulating that there is a realm between GCMs and SDMs populated by *stochastic climate models* which are stochastic differential equations (ordinary or partial) for the *error in averaging*, i.e. for the appropriately normalized deviation of the solution x^ϵ of the slow component from the average u. The Khasminskii deviation

$$v^\epsilon = \frac{1}{\sqrt{\epsilon}}(x^\epsilon - u)$$

converges to a linear stochastic differential equation with state vector, say v, in the small noise limit, as has been well known since 1966. In contrast to Khasminskii's deviation, Hasselmann's deviation leading to the non-linear stochastic differential equations uses, on the same time ranges, a diffusion approximation based on strong limit theorems rather than an approximation corresponding to the central limit theorem. Under highly simplified conditions, i.e. with an underlying ergodic

Markovian motion in its invariant law playing the role of the fast variable, Kifer [40] has recently derived a Hasselmann type equation expressed in a state vector, say w. The comparison of v and w shows that the respective approximations v^ϵ and w^ϵ are on different scales: for some $\gamma > 0$ one has

$$v^\epsilon - w^\epsilon \sim \epsilon^\gamma,$$

with $\gamma \neq \frac{1}{2}$. Of course, different deviations on different time scales, in particular those including large deviations for the response times, could also be of great relevance. To reduce a supposedly chaotic or well mixing fast component to a tame ergodic Markov process as in the outset of Kifer's work in concrete situations remains, despite some recent progress made by Pène [61], the greatest obstacle to the verification of the decay of correlations of the fast variables.

In addition to these primary problems, the lack of sufficient numerical evidence makes substantial theoretical progress in the mathematical foundation of Hasselmann's program in the short run unlikely. This makes the consideration of simple case studies by means of numerical simulations a high priority. Hasselmann's program has to be verified in a number of numerical case studies, before work on a more conceptual level may be started. The program comprises the following steps:

1. study of the system after freezing the fast variables; investigation of *attractors and invariant measures*

2. *averaging* of the slow equation using the results of the previous step

3. investigation of the averaged equation, in particular of its local attractors

4. proving decay of correlations; computation of the diffusion matrix of the (linear) central limit approximation

5. study of the (non-linear) diffusion approximation; investigating whether it provides a better approximation than the linear one

6. study of the equation by the large deviations theory by Y. Kifer [40]; calculation of the action functional; comparison with results of the diffusion approximation.

To gain numerical evidence case studies were carried out with models based on work by Monahan [53]. In particular, the Maas-Lorenz model studied by Wu [88] is a simple coupled ocean-atmosphere model describing the angular momentum of an ocean in a rectangular basin. It is subject to thermal and mechanical forcing from a Lorenz model atmosphere. Based on deterministic bifurcation studies by van Veen [84], Monahan's simulations exhibit a complex spectrum of invariant structures and attractors including limit cycle regimes and chaotic behaviour. It is this interplay of multiple regimes with essentially different (fast) invariant structures triggered by variations of the slow variables which complicates matters considerably. Wu's

[88] numerical simulations clearly exhibit parameter ranges for the Maas-Lorenz model for which Hasselmann's nonlinear reduction with white noise forcing is in very good accordance with the full deterministic model. On the other hand, there seem to be regimes for which even a numerical decay of correlations for the Lorenz component does not hold. In any case, it is evident that more case studies need to be investigated before the full scale program of stochastic reduction following scale separation according to Hasselmann may be tackled mathematically.
A different approach of stochastic climate models and their effective reduction was made by Majda, Timofeyev and Vanden-Eijnden ([47], [48], [49]). Their approach consists of systematic strategies to reduce stochastic equations to a small set of resolved variables by eliminating a (typically) large number of unresolved ones from a complex nonlinear system with many degrees of freedom. Again, the correlation times between the resolved and unresolved quantities play a crucial role: the approach is mathematically rigorous if the correlation decays, but agreement can be good for even rather large ratios of correlation times.

3 Some simple stochastic climate models in practice

Though it is a vital issue for stochastic climate modelling, strategies of model reduction play a relatively minor role in this volume. The focus instead is on the treatment with methods from stochastic analysis and stochastic dynamics of reduced models with heuristically justified noise forcing. Given the range of reduced climate models parameterising unresolved scales by stochastic variability, it is impossible to thoroughly review this entire field; we proceed to discuss a few problems that have been found to be mathematically interesting. Extended discussions of simple models of the climate system can be found in Ghil and Childress [22] and in the recent monograph by Saltzman [72].

3.1 Energy Balance Models

One of the simplest and earliest applications of ideas from the field of stochastic analysis to models of the climate system goes back to the energy-balance models considered in Nicolis [56] and Benzi et al. [3]. These analyses attempted to provide a qualitative explanation of Quaternary (the last ~ 2 million years) glacial cycles based on a deterministic differential equation for the global mean temperature describing a balance between the absorbed shortwave and emitted longwave radiative energies. It is heuristically reasonable that colder globally-averaged temperatures will be associated with greater ice extent on the earth's surface, and consequently with a reduced absorbed solar radiation due to the high albedo of ice (the so-called "ice-albedo feedback"). The absorbed solar radiation is therefore modelled as a (nonlinear) function of globally-averaged temperature. A periodic external forcing arises from slow periodic variations in solar insolation (Milankovitch cycles) caused by gravitational interactions with other planets in the solar system (e.g. [28]). Only the addition of a stochastic term as a second forcing makes spontaneous transitions between the otherwise isolated *metastable states* of temperature possible. The resulting stochastically and periodically perturbed differential equation was capable of describing at least one characteristic aspect of experience: the typically short and abrupt *transitions*, observed before by Kramers [42] in reaction-diffusion phenomena. While these models are not presently considered to provide a comprehensive descriptions of Quaternary glacial cycles (although they have seen something of a renaissance in recent debates over the "snowball earth" hypothesis [75]), the underlying concepts brought to light the phenomenon of *stochastic resonance*. Roughly, a system subject to a periodic modulation, and to random perturbations of intensity ϵ, is in stochastic resonance if ϵ is tuned in such a way that the randomly periodic output is optimal in the sense that the periodicity of the random output closely resembles that of the input signal. A very lively research field has developed around this concept, drawing examples from a wide spectrum of areas such as the bistable ring laser from optical systems, the analog electronic simulator from electronic systems, and the crayfish experiments by Moss et al. from biological and neuronal systems. See Gammaitoni et al. [20] for a review.

3.2 Thermohaline Circulation

Another physically important problem that has been a rich source of reduced climate models involves the thermohaline circulation (THC) of the world ocean. This collection of slow currents, driven by gradients in the density of seawater, dominates the deep circulation of the World Ocean and is believed to play a significant role in climate variability on timescales of decades and longer (e.g. [8], [13], [68], [76]). In its present configuration, the THC is associated with a net transport of warm surface water to the North Atlantic ocean and a net transport of cold deep water away from this region, thereby (it is believed) warming North Europe relative to other regions at this latitude [69]. A significant result from an early simple conceptual model due to Stommel [78], in which sea surface salinity and temperature differences between high- and low-latitudes are nonlinearly coupled, suggested that the present configuration of the THC may not be the only one it can take, but that for a broad range of surface buoyancy forcings a weaker circulation associated with a dramatic reduction in poleward heat transport is also dynamically permitted. Subsequent investigations with EMICs and GCMs (e.g. [30], [50], [66]) confirmed the existence of these multiple circulation states; these results have also been found to be broadly consistent with paleoclimatic data (e.g. [8], [13], [21], [77]). Parameterisation of high-frequency "weather" variability by stochastic processes in simple models of the THC leads to models characterised by *spontaneous rapid transitions* between *metastable states* of ocean circulation (e.g. [11], [24], [44], [54], [79], [82]). Because ocean temperature is actively coupled to the temperature of the air above, it adjusts on much faster timescales than salinity; heuristic scaling arguments then boil these simple models of the THC down to one-dimensional multistable stochastic differential equations ([54], [82]). In [52], Monahan considers versions of the simple Stommel box model of the THC perturbed by random noise, in particular for parameter combinations for which the stationary probability density function is bimodal. A discussion of the phenomenon of stabilisation by noise (well known from mechanical problems) leads to the problem of stochastic reduction effects of hysteresis loops and stochastic perturbation of bifurcation scenarios, analysed in detail in, for example, Berglund and Gentz [5].

3.3 Quaternary Climate Variability

Over the last few decades, our understanding of global climate variability throughout the Quaternary has increased dramatically, due largely to the analysis of paleoclimate proxy data from ice cores and ocean sediments (e.g.[7], [36], [64], [70]). In particular, we now have very good evidence that the last 400,000 years have seen four alternations between cold glacial (ice-age) conditions and warm interglacial (resembling the present-day) conditions. The 100,000 year period of these glacial/interglacial oscillations matches very well the timescale of the Milankovitch cycle associated with variations in the Earth's eccentricity. Furthermore, it has become clear that while the climate system is relatively stable during interglacials, as in the present Holocene (the last $\sim$ 10,000 years) the glacial climate is characterised by abrupt millennial-scale fluctuations between cold *stadial* and warm *interstadial* states; these have come to be known as Dansgaard-Oeschger (D/O) events. The realisation that the THC may be subject to abrupt and radical changes, coupled with the recognition of its importance in the global transport of energy, have led to its implication as a key player in these dramatic and rapid shifts in the climate system (e.g. [8], [76]). The existence of multiple stable or metastable states in the climate system plays a central role in various mechanisms proposed to explain glacial/interglacial cycles or D/O oscillations (e.g. [21], [58], [74]), although this is not the case in all proposed mechanisms (e.g. [23]). Ganopolski and Rahmstorf have recently brought the phenomenon of stochastic resonance home to climate research in this context, proposing that the D/O oscillations arise as a stochastically resonant response to an as-yet-unspecified 1500-year cyclic forcing [21]. Simple reduced climate models have played a central role in interpreting the paleoclimate record (e.g. [22], [23], [58], [72]) and will undoubtedly continue to do so; parameterisation of unresolved variability by stochastic fluctuations will presumably figure prominently in these studies.

3.4 ENSO

The time-mean state of the tropical Pacific ocean is characterised by an east-west (*zonal*) gradient in sea surface temperature (SST), with warm waters in the west and cool waters in the east. Surface winds generally blow from east to the west (the *trade winds*), with surface convergence and rising air in the western Pacific, and surface divergence and sinking air in the eastern Pacific [65]. The coupled atmosphere-ocean system in the equatorial Pacific is highly variable around this time-mean state, characterised by dramatic warmings and coolings of the central and eastern equatorial Pacific, referred to respectively as El Niño and La Niña events. During an El Niño (La Niña), trade winds slacken (intensify), and convection and precipitation in the western Pacific decrease (increase). El Niños (La Niñas) tend to be phase-locked to the annual cycle, achieving their peak intensities around December or January, and occur irregularly with a typical recurrence time of 3-5 years, with successive events usually (but not always) separated by a La Niña (El Niño) event [12]. The atmospheric component of this variability is known as the Southern Oscillation, and the variability of the whole system is generally referred to by the acronym ENSO (El Niño and the Southern Oscillation). Changes in the intensity and location of atmospheric convection are associated with droughts and floods, and can have serious social and economic impacts, as can impacts on fisheries associated with changes eastern Pacific SST and thermocline depth.

Simple conceptual models have played a central role in the development of our understanding of ENSO. A large number of low-dimensional deterministic and stochastic models have been designed to describe its dynamics and predict its occurrence. These include multivariate Ornstein-Uhlenbeck processes (e.g. [38], [55], [62]), deterministic nonlinear differential equations (e.g. [37], [83]), deterministic delay-differential equations (e.g. [80], [86]), and stochastic differential equations with nonlinear drifts (e.g. [85]). These various studies have been able to address, with varying degrees of success, such aspects of ENSO as its characteristic recurrence timescale, its phase-locking to the annual cycle, and its irregularity. The relative merits of these different models have been a subject of debate; in particular, there was a controversy in the 1990's as to whether ENSO variability was best described as chaotic, as a perturbed limit cycle, or as a stable fixed point driven by stochastic fluctuations. Recent studies tend to support the hypothesis that to first order ENSO is best described as a multivariate Ornstein-Uhlenbeck process (e.g. [63], [81]), although variability in the tropical Pacific does not appear to be strictly linear (e.g. [10], [51]). Simple conceptual models have recently been employed to describe observed decadal-scale modulations of ENSO (e.g. [25], [87]); these studies have so far been deterministic, but unresolved high-frequency variability may conceivably play a role in this interdecadal variability.

3.5 Multistable Systems

A common mathematical structure behind many of these reduced climate models is stochastic (partial) differential equations with non-linear drift and relatively simple stochastic forcing. Often either additional external periodic forcing, or autonomous periodic forcing through limit cycles or delay is present. The effect of the non-linear drift on the system may be pictured in terms of white noise driven particles moving on a complex potential landscape, the minima of which provide finitely many *metastable states (attractor basins)*, to be interpreted as *climate states* in our setting. The effective dynamics is roughly governed by *Markovian transitions (hopping)* between climates happening at time scales characterized by the geometry of the potential. Tools for the mathematical treatment of these models are found in the arsenal of random dynamical systems and ergodic theory (see Arnold [1]), stochastic (partial) differential equations (see Zabczyk [89]), large deviations (Freidlin and Wentzell [19]), and the spectral theory of linear operators.

These mathematical tool boxes have been opened in a number of papers dealing with climate models of the type just sketched. To explain stochastic resonance in a general potential landscape with finitely many minima, Freidlin [18] goes as far as large deviations' theory can take. In the absence of periodic external forcing, the exponential order of times at which successive transitions between metastable states happen corresponds to the work to be done against the potential gradient to leave a well divided by noise intensity (*Kramers' time*). The attractor basins are subdivided into a hierarchy of cycles with main states corresponding to the deepest among the cycle states. In the presence of periodic forcing with period time scale $e^{\frac{a}{\epsilon}}$, where ϵ denotes the noise intensity, in the limit $\epsilon \to 0$ transitions between (the main states of) cycles with critical hopping work close to a will be periodically observed. Transitions with smaller critical work may happen, but are negligible in the limit. Those with larger critical work are forbidden, this way creating the hierarchy of decoupling/coupling of states encountered in Fischer et al. [17] and Huisinga et al. [31]. In the simplest case of two minima of potential depth $\frac{V}{2}$ and $\frac{v}{2}$, $v < V$, the role of which switches periodically at time T, for T larger than $e^{\frac{v}{\epsilon}}$ the diffusion will be close to the deterministic periodic function jumping between the locations of the deepest wells. As T exceeds this exponential order, many short excursions to the wrong well during one period will occur. They will not count on the exponential scale, but trajectories will look less and less periodic. It therefore becomes clear that the physicists' quality measures of goodness for tuning cannot be explained by large deviations alone. The thesis by I. Pavljukevich [60], and the related papers ([33], [34]) deal with a mathematical foundation of the physical paradigm of stochastic resonance. As opposed to Berglund and Gentz [4] the main subject of their studies are - in the physical jargon - measures of quality of tuning of the *output* to the periodic *input*. The physicists' preferred measure, *spectral power amplification*, is extensively studied by means of spectral theory of the associated infinitesimal generator, and by comparison with the reduced dynamics of a Markov chain on the finite state space consisting only of the metastable states. Surprisingly, the reduced dynamics cannot capture the transition characteristics responsible for resonance phenomena of the diffusion, due to the importance of small fluctuations in the bottoms of the potential wells.

For parametrized deterministic dynamical systems passing through a pitchfork bifurcation point, the relaxation of solutions to stable equilibria are known to happen after well known delays. In [4], [5] and [6], Berglund and Gentz exploit this observation to derive pathwise estimates of the trajectories of noisy perturbations of these systems. These results are applied to situations in which the parameter moves the system in hysteresis loops back and forth through bifurcation points, for example in periodically changing double well potentials. The papers thus clarify questions concerning the trajectory-wise behaviour in the context of stochastic resonance, and shed new light on the stochastic perturbations of hysteresis cycles in one-dimensional stochastic differential equations, encountered for instance in Stommel models of the thermohaline circulation [52].

4 Overview of the Contributions to this Volume

The seven contributions to this special issue of Stochastics and Dynamics represent the range of topics that were discussed at the second meeting on Stochastic Climate Models in Chorin.

Berglund and Gentz go "beyond the Fokker-Planck equation" to investigate the properties of sample paths in multistable models of the kinds typically encountered in simple climate models, subject to slow external forcing. In the limit that the dynamical relaxation timescale τ_r, the timescale of the forcing τ_f, and the Kramers time τ_K are well separated:

$$\tau_r << \tau_f << \tau_K$$

asymptotic results are obtained concerning the deviations of sample paths of the systems away from the corresponding trajectories in the deterministic limit. Particular attention is paid to stochastic resonance, to the the structure of random hysteresis loops, and to the effect of noise on bifurcation delay.

The study of *Duan, Gao, and Schmalfuss* considers the stochastic dynamics of a zonally-averaged dynamic/thermodynamic ocean model coupled to a stochastic energy-balance atmosphere model. The existence of a finite-dimensional random attractor is rigorously proven, as is the ergodicity of the system. Bounds on the magnitude of the solution process are obtained, and are related to physical parameters such as the Prandtl and Rayleigh numbers.

Egger addresses the problem of determining the stochastic dynamics of a system from observational data. Using meteorological data obtained from a valley-wind field campaign in the Himalayas, he estimates the parameters of a Master equation for the joint evolution of the along-valley pressure gradient and wind. This analysis demonstrates the existence of a distinct diurnal cycle in the relationship between these variables which cannot be captured by a traditional simple regression model.

Farrell and Ioannou investigate the dynamics of linear systems in which the operator is uncertain, with uncertainty represented as stochastic fluctuations. Two relevant timescales are identified, associated with the mean operator and the decay timescale of the fluctuations, and three characteristic regimes are distinguished based on the relative sizes of these timescales.

Fraedrich considers the scaling behaviour of surface air and soil temperature time series, and finds long-term memory behaviour inconsistent with a system driven by white noise. A stochastic partial differential equation including Fickian diffusion as well as random fluxes is constructed and is demonstrated to reproduce the long-term memory characterising the observations.
Herrmann and Imkeller aim at a mathematically rigorous understanding of the interspike distributions in problems of stochastic resonance. A double well situation is considered, with the diffusion approximated by a two state Markov chain. The interspike distribution for the chain is given by a geometric law, the parameters of which can be identified from the system parameters.
Finally, *Monahan* extends an earlier study [52] in which variability in ocean mixing and surface fluxes are parameterised in a Stommel-type model of the THC by stochastic processes. The effects of correlations between, and of autocorrelations within, the stochastic processes on the number of THC regimes, and on the partitioning of probability mass between these regimes, are investigated using a combination of analytic and numerical approaches.

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