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1 Introduction

> When the first encounter with some object *surprises* us, and we judge it as something new, or very different from what we have previously known or what we suppose that it has to be, it leads us to *admire* and be *astonished* by it; and so that this can arrive before we know in any way whether the object suits us or not, it seems to me that *admiration* is the first of all *passions*; and it does not have an opposite because if the object that presents itself has nothing in it that would *surprise* us, we are not *moved* by it, and we consider it without *passion*. (Descartes, 1824, p. 87, emphases added)

Mathematics educators are interested in affect (e.g., special issue of *Educational Studies in Mathematics, 63*[2]), a phenomenon that classical philosophers discussed under the *passions* because we are subject to (rather than the intentional causes of) them. We all know from personal experience that *astonishment* and surprise are thoroughly interlaced with passions in the form of emotions. Here, "*astonishment* [Verwunderung] is an affect [that occurs] when we present novelty that exceeds our expectation" (Kant, 1956, p. 363). That is, astonishment inherently makes thematic a form of emotion, whereas surprise characterizes the grasp that something unexpected has occurred. Moreover, whereas surprise and astonishment are of brief temporal extension, "*admiration* [Bewunderung] [is] an astonishment that does not cease once the novelty is gone" (p. 363). This ordered sequence of concepts might be a tremendous resource to mathematics educators interested in instilling in students long-term affective relations to the subject matter. I situate this investigation in the efforts that the Vygotsky began during the final years of his life—theorizing affect and intellect as inseparable phenomena (e.g., Vygotsky, 1987, 1999)—but that he could never complete.

Although researchers sometimes note that their student participants exhibited *astonishment* (e.g., Brousseau et al., 2009; de Bock et al., 2002) or that there existed *admiration* for a particular aspect of mathematics (e.g., Hemmi, 2010; Nosrati, 2015), neither phenomenon has received any notable attention. In fact, the observation that a research participant was surprised may be taken as evidence for a shortcoming or lack—e.g., that they were unaware of what they had done (e.g., Arsac et al., 1992). Yet Descartes considers admiration as the first and foremost passion, even preceding love, hate, desire, joy, and sadness. Admiration is associated with surprise and astonishment. In fact, it *enables understanding*; and next to it there is only ignorance. In admiration there "resides 'a great deal of force' of 'surprise' or of 'sudden arrival'" (Derrida, 1987, p. 438). Why should admiration in/of mathematics, as the astonishment from which it arises, not be a goal of mathematics education? As shown below, the admiration of mathematics generally and geometry specifically was an important feature already for Plato, who distinguished between those person who just accomplish routine operations and those who are reflectively aware of themselves as the source of admiration. What is it in geometry that led Plato to his admiration? It was Kant who realized that it is because of an overabundance, an excess of in the properties of representations, a fecundity that allows an infinite number of unexpected findings to be made: "The unexpected, the unforeseen, the unhoped for character of an overabundance of meaning is the proper

object of admiration" (Chrétien, 2002, p. 19). For Kant (1957), who, as seen below, uses many examples from geometry to substantiate his critical analysis of the phenomenon, admiration is "an always recurrent astonishment irrespective of the disappearance of doubt" (p. 475). Astonishment, therefore, that is, surprise tinged with positive affect, is the primary phenomenon underlying admiration. There have been suggestions that astonishment can be achieved voluntarily, and, therefore, can be cultured and become cultural practice. The mystery in teaching for astonishment lies in allowing the learner to disrupt the familiarity with themselves and the objects of their inquiries and to begin to think (Sherman, 2010). The purpose of this paper is to make an argument for a culture of astonishment; if not a practice, it could also be an attribute of discipline-specific inquiry practice. I begin with the description of a classroom episode and its analysis before articulating the phenomenon of interest.

2 A classroom episode

In this section, an episode is presented that previously had been recognized as exhibiting surprise, the intellectual dimension of astonishment (Roth, 2011a); but, failing to attend to its affective side, that study failed to develop an understanding of the phenomenon and its affordance for a holistic approach to learning and development.

2.1 Vignette

In a second grade classroom, the children are in their second week of a three-week curriculum on three-dimensional geometry. On this day, the children, working in pairs or groups of three, have been asked to use plasticine to make models of a mystery object hidden in a shoebox. They cannot see the object but, through a baffle, reach into the box allowing them to touch the object. In one group, the three girls (Jane, Melissa, and Sylvia) have made different plasticine shapes, two having the form of more-or-less flat slabs, Melissa having made a cube. The teacher eventually passes by to talk to the three girls insisting that there is only *one* mystery object so that there can be only *one* (correct) model. She also groups the two flat slabs and separates them from the cube. In the talk that follows, Jane and Sylvia attempt to convince Melissa that the mystery object is more like a slab, which Sylvia exemplifies twice: first in holding the two hands together as in Catholic praying, and second by configuring the thumb and index to form a caliper with a small opening. But Melissa repeatedly rejects what her peer is saying and provides reasons underpinning the claim that the mystery object has to be a cube: "it has three equal sides." Jane exemplifies how Melissa could test the correspondence by reaching into the shoebox with one hand and simultaneously touch the model with the other hand. But, having already reached into the shoebox seven times before, Melissa insists on the cubical nature: The mystery object feels like a cube. She then accepts another invitation to conduct a comparison, reaches into the shoebox for an eighth time apparently touching the object inside in the way she does with Jane's model on the outside. She feels out Jane's slab on the large surface, then rotates it 90 degrees; feels out the new side, and rotates the slab again, repeating the movement four times. She turns the slab over to touch the other side. Melissa then places the slab on one of the narrow sides, feels it, rotates it 90 degrees to feel it again. Altogether, she does so seven times, when the

movement slows down and the feeling index finger moves a second time over the exposed surface. Within less than two seconds, her head has moved upward coming to face the others, her lips pucker up, and a hint of a smile (sheepish grin) emerges on her face. She grabs her cube and begins to work it, announcing "I making it else." Sylvia, holding up her own slab, invites Melissa to respond, "Are you making it like this now?," an invitation Melissa accepts by saying, "Yea."

2.2 A first analysis

In this fragment from a lesson, we observe astonishment in the making. Just prior to raising her head to face the others who are closely watching her, Melissa's movements express concerned involvement with testing Jane's model against the mystery object. Up until this eighth time of reaching into the shoebox apparently feeling out the mystery object, she has sensed a cube and has defended her sense by giving reasons not only stating that the three dimensions were the same but showing so with her cubical model. Then, in the course of two seconds, a change occurs that leads to the expression of surprise, together with a smile that goes with a positive affective tinge: there is astonishment. This astonishment is only a result of feeling over whatever is in the shoebox, which means that Melissa is being affected without knowing the causative *what*. In other words, she physically grasps something that she does not know what it is, and her intellectual grasp is the result of and coming after physical grasp has ended. The resulting surprise is associated with what we will come to know as the radical change in Melissa's perception: what has felt until now like a cube now feels like a slab, similar to the slabs that Sylvia and Jane have produced. That new perception surprises Melissa, it has been unexpected, unseen and therefore unforeseen. The realization that the mystery object feels like a slab suddenly arises, beginning somewhere near the end of the exploration by means of touch and facial expression. There is an intellectual consequence, which can be observed in the orientation to the cubical model that then comes to be reshaped, and in the acknowledgment that the new shape will indeed be like that of another model. The new feel comes as a surprise, because in the seven previous tactile explorations, which included instances where one hand has been in the shoebox and the other one was holding and rotating the cube, had led to the sense and have supported the assertion that there was a cube.

Reaching into the shoebox for another time, although under the auspices of a firm conviction that there is a cube also, and contradictorily, involves the willingness to experience a different feel. In Melissa's reaching into the shoebox, we observe the willingness to expose herself to be impressed in a new and different way—why else would she reach into it and feel the object over and over again if there were not the possibility that something else might be felt? There is the presence of doubt in the face of the search to affirm that the past impressions are repeatable and therefore correct. From the perspective of the observer, the conscious awareness for the different feel then is signaled by the facial expression of astonishment, manifesting her being stunned; but the hint of a smile also suggests a reflexive quality: A recognition that what she has claimed and has defended for such a long time has turned out not to be the case. It is as if she was surprised by the difference between what she had felt and what she now feels—much like

the experience of the suddenly sighted blind surprised to find that there is "'such a difference' between a tree and a human body" (Merleau-Ponty, 1945, p. 259).

From the first time the left hand has touched the slab to the instant of the final movement along one of its narrow sides, 22.44 seconds have passed. That is, Melissa has been feeling out and comparing the two objects for an extended amount of time. When she eventually knows the two to be similar, it is based on the sequentially accumulated ensemble of how the two objects feel. The end result, the sense that there is a slab hidden in the shoebox, has emerged as a cumulative sense experience.

Once the similarity between the two has emerged into conscious experience, a comparison (intellectual act) with her own model as representing something different is enabled. From the videotape, it is impossible to know how and when this grasping occurs that affectively has expressed itself in surprise. In the literature on learning, however, a phenomenological description of the process of becoming aware is available (Roth, 2015). Accordingly, awareness does not at all occur instantly. Instead, there is a *movement* of becoming aware, which begins with some dim and vague sense unaccompanied by a *what* that the sense might be about. This is so because the person is affected by something initially alien, that she does not know the source of—until, in the present case, it expresses itself in the fact that the object inside the box is indeed like the slab outside. But this experience is more than "an 'Aha' experience, as if emerging from the vague sea of the unconscious" (Otte & Zawadowski, 1985, p. 97), for the affect exceeds the "individual thinking process that ended in a happy 'Aha'" (Radford, 2009, p. 120) that marks the surprise. Without the person's active doing, the sense grows until the grasp of something becomes definite. In the present case, this might be the grasp of the object itself or a similarity between the slab and the mystery object. Once such a grasp exists, once the extended preceding experience of feeling the objects has gelled into the grasp of holding two slabs, associations with other aspects of a person's knowing may arise.

It is apparent that the entire process has nothing to do with "construction," as the new sense grows upon the individual. Melissa had expected to feel a cube and, therefore, a difference between the slab in her left hand and the mystery object in her right hand. If the concepts from the history of science about the relation between thematic frameworks or paradigms and what scientists perceive hold at all (Hanson, 1958; Kuhn, 1972), then Melissa should have "constructed" a cube again. Instead, to be able to *feel* anything differently, Melissa has had to allow herself to be affected (impressed) by whatever she has held in her right hand. However, she can know *what* affected her—the thing she later knows as cube or rectangular solid—only after she has become conscious of it. From a vague something then surges the sense of *a* (specific) something, and this specific something will have been unexpected. If she had indeed "constructed" the mystery object, she would not have been surprised, for the result of her doing would have been anticipated by what she already had aimed at constructing (Roth, 2011b). She is in the position of the poet, who "is typically unable to make clear exactly what it is that [s]he wants to do" (Rorty, 1989, p. 13). The construction metaphor is a "disanalogy" because "the craftsman typically knows what job he needs to do" (p. 12). The fact that she was stunned (astonished), overcome by what has given itself in her experience, gives evidence that she has been affected by something unknown that has given itself to her hand. She then grasps it as novel and, in this, that a previous conviction has been overturned.

Astonishment marks that she did not bring about the overturning; instead, the overturning occurred to her. We have here an instance of mathematics as revelation, where the unseen and therefore unforeseen reveals itself (Roth & Maheux, 2015).

2.3 Discussion

We might ask, "What is the advantage of theorizing such an event by means of the concept of astonishment?" Whereas *surprise* makes thematic the intellectual aspect of an experience, the novelty of the unsuspected and unforeseen, *astonishment* in addition makes thematic the inherent affective dimension. Melissa clearly was surprised by the sudden intellectual realization that what felt like a cube was a slab instead; but she also exhibits affect in the sheepish grin and the puckered lips. It is therefore one of those phenomena that clearly exhibit the connection between intellect and affect; and it might therefore serve as a paradigmatic case or model for all those pursuing the late and post-Vygotskian, Spinoza-inspired agenda based on the "*unity of affective and intellectual processes*" (Vygotsky, 1987, p. 50). The observable manifestation of astonishment is a sign of an affection that was unexpected: affection by something foreign, an unanticipated sensation or idea that *comes to* mind (rather than being constructed). Astonishment is that process of becoming aware of something heretofore alien, which, in having become part of conscious awareness, no longer is alien but has become one's own. "Astonished to find itself implicated in a world of object . . . consciousness will search in its memory the forgotten moment where, without knowing, it allied itself to the object or consented to perceive itself with it" (Levinas, 1978, p. 122).

Astonishment in that instance is quick but does not occur instantly: becoming astonished is a process stretching, in the case above, from the touching of an object to the realization that what *has been* felt is different from what was known before. Astonishment stretches back from the point of realization into the period of increasing involvement with the object, from which arises (grows) the new sense of the mystery object as a slab, itself tied to the sense of the similarity between the mystery object and Jane's slab on the outside of the box. Their similarity also is the basis of a becoming aware of the dissimilarity between the previously made cube and the mystery object. The sense of the similarity, was not suddenly there, but grew out of the sequentially accumulating sensations with the mystery object (e.g., Merleau-Ponty, 1945). Astonishment therefore is a dehiscent phenomenon, because the realization that manifests itself in the outside (affective) expression has grown from the exposure to be affected by whatever has given itself to the hand over the course of some temporal period. Astonishment is a temporal phenomenon rather than merely pertaining to the cognitive realization that often is said to occur in an instant. Astonishment can be thought of as a response, which begins with active listening and exposure to the saying of another, and reaches into the reply. A response not only unfolds in time but also stretches over it (Waldenfels, 2006).

There is a temporal delay between (a) the sensations and (b) the realization that what was sensed in the two hands touching different objects is the same or similar in some ways. The *what* of the grasp, the cause of the astonishment, is indeed tied to the delay with the *how* of the grasped, spread out over and with the extension of the feeling movement (Waldenfels, 2002). The effect has precedence over the cause. Only when the

feeling movement with the hand has ended, and, therefore, can appear as *one whole*, can it also be grasped cognitively. Astonishment as a fact is delayed with the experience of touch in which astonishment is grounded and from which it arises. Astonishment is the encounter with the alien, a something heretofore unfelt and unknown. Just as we do not understand what someone says without knowing to what the saying responds—the saying of another or the particulars of the situation (as in an unsolicited "Nice day today!" that addresses the neighbor)—we cannot understand astonishment unless we understand to *what* it is the reply. The phenomenon of astonishment, as the phenomenon of response, is grounded in the antecedence of the pathic experience (pathos) of touching and being touched and in the succession of the cause.

At the end of the exploration, there is an insight—there is no equivalent for the sense of touch, tact—seeing something that one has not seen before. A new idea has been born from the senses—and, unfortunately again, the etymology of idea lies in seeing, from Greek *ἰδεῖν* (idein), to see. It is unfortunate because common conceptions of seeing go with conceptions of instant grasp, neither being consistent with a scientific understanding of visual perception. Thus, like with the sense of touch, they eyes have to move in zigzags across, and from the beginning to the end of, a line to see this entity as such (Yarbus, 1967). The concept of insight refers to an intellectual event, seeing something for a first time, whereas the concept of astonishment includes affective qualities, both in the course of being affected and at the instant when the awareness "has sunk in" (or better, has finished sinking in). The notion of astonishment emphasizes that something is happening to us, over which we have no control, and because we have given ourselves to be affected (made ourselves vulnerable) in the first place. The phenomenon therefore also points out the limits of the constructivist approach (Nancy, 2008).

3 Astonishment: a philosophical historical review

> Plato is seized, when coming before the power attested by the deployment and extension of mathematical knowledge, by an "admiration" beyond what could ever be experienced by the "simple mathematician," who is absorbed in the object to the point of failing to ask about the power of the knowing subject. (Chrétien, 2002, p. 17–18)

Astonishment comes with both intellectual and affective characteristics. Astonishment is tinged by a positive affect that also characterizes wonder. The verb "*astonish* can imply a dazing or silencing or it may mean to surprise so greatly as to seem incredible . . . or sometimes merely unusual" (Merriam-Webster, 1984, p. 804). In contrast to "surprise"—the result of a sudden realization of something unknown or unexpected, experienced with positive or negative affect—dictionaries define astonishment as surprise with "(great) wonder," a positive affective tinge. Associated with affect and being-affected by the unseen and therefore unforeseen, astonishment requires us to re-theorize the subject—who neither has some stable identity nor is exclusively purely autopoeitic (agential). In the following subsections, I elaborate these two dimensions—intellect and affect—and their relation to the notion of the knowing and feeling subject.

3.1 Intellect

In the preceding lesson fragment, Melissa, as her two peers, has accepted the invitation to inquire about the nature of an unknown object and to represent it in the form of a plasticine model. In the face of the differences between her and the others' models, Melissa maintains the inquiry such that in the end she will have reached into the box eight times for a total of over three minutes. Admixed to the inquiry is a certain degree of curiosity to find out whether things are really the way they have been appearing to be. It is often assumed that children are curious and inquisitive by nature (Piaget)—which actually is not the case because curiosity and inquisitiveness, as all higher psychological function (Vygotsky, 1997), first were social relations with others (Meshcheryakov, 2009). Curiosity and inquisitiveness are concerned with the unknown, which, when the individual allows it, may be experienced as surprise and astonishment. The verb "to astonish" has its etymological roots in the old French *estoner*, which derives from the popular Latin *extonare* that itself has come from the classical *adtonare* (*ad-* + *tonare*, to thunder), hitting with thunder. It has "to stun" as a synonym, a verb that has the same etymological origin in *estonare*. The experience of astonishment signals the end of the birth to presence of something that was beyond the previous intellectual order, not just extra-ordinary in the sense of unusual, both beyond the horizon of what the preceding order admitted and that could have been thought within it.

In the lesson fragment, we observe how at a certain point, at what will have been the end of the eighth sensory exploration of the mystery object, there is a turning point. Something has happened so that in a few seconds, Melissa is moved to change the earlier representation she has made. That period of transition is marked in the manifestations of astonishment and the (intellectual) surprise it implies. As Kant (1957) describes, such astonishment arises from the incompatibility of a representation [*Vorstellung*] and the principles on which it is founded and that it presupposes. In astonishment, there is an excess of the new over the old intellectual order that cannot be anticipated or calculated. It "brings about a doubt whether I have seen well or judged well" (p. 475). Admiration accepts that something is as seen and experienced while acknowledging that this something is not or cannot be explained. Kant develops the concepts of astonishment and the admiration to which it gives rise by describing an example from mathematics to exemplify where astonishment and admiration may be experienced. He begins by considering the space created when a straight line is moved about an imagined fixed point. The construction is so simple that one does not anticipate behind it much of a manifold of insights. But he then proposes to have two lines intersect in a given point within the circle and intercepting its circumference. In that case, the lines turn out to intersect each other in proportion *in every case considered*. Fig. 1a states the problem. Kant himself does not provide the proof, but readers can find it in Fig. 1b, which indeed constitutes a complete proof account (cf. Livingston, 2008), even if a text is not provided. Whichever two cords are chosen, the theorem holds.

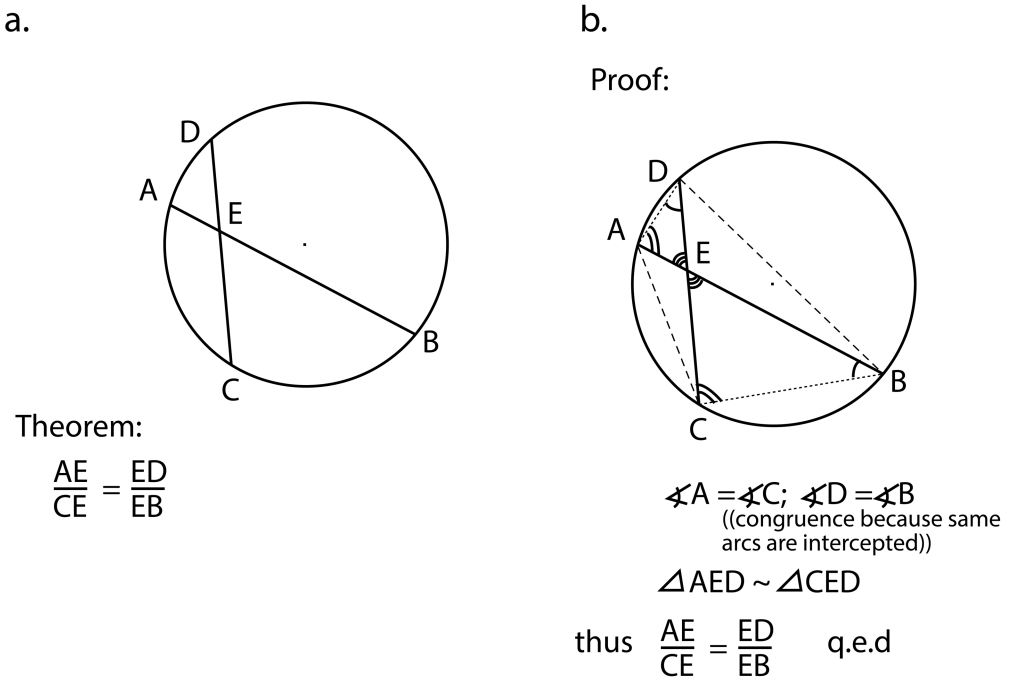

Fig. 1 **a** Kant's construction and theorem. **b** A proof account

Kant describes another theorem that appears to be from physics—the lengths of inclined planes that give rise to the same time of descent of a rolling object—but indeed can be solved by drawing on the geometry of the triangle. Fig. 2a provides enough material—is a sufficient proof account—for the reader to see it as a proof and to experience astonishment.[1]

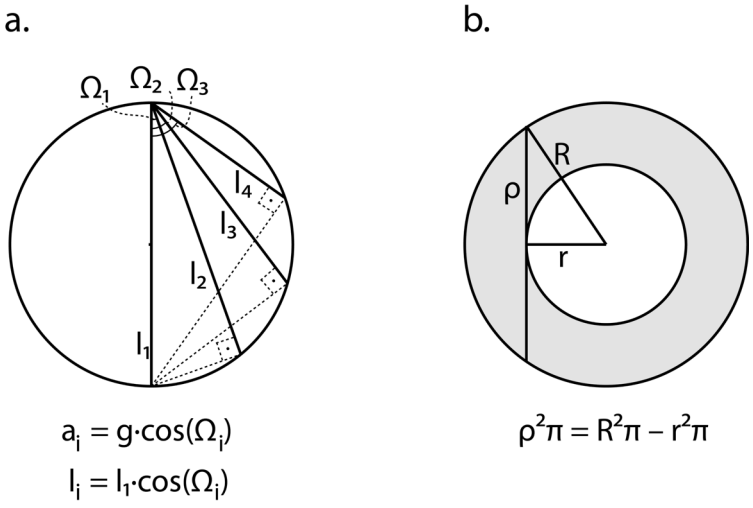

Fig. 2 The proof accounts for two further theorems Kant states without proving. **a** The lengths of the inclined planes that a rolling object takes equal time to descend. **b** The circle with the same area as the ring has a diameter the length of the chord in the larger circle that is tangent to the smaller circle

[1] The acceleration $a = g \cdot \cos(\Omega)$, where g is the gravitational constant and Ω the angle of incline; l_1 is the vertical diameter, corresponding to a free-falling object.

Kant adds another theorem, thereby demonstrating that the circle really is a fascinating object with the most astonishing properties that can throw us into continuous admiration: The diameter of the circle that has the same area as the ring of two equicentric circles is that chord of the larger circle that also is a tangent to the smaller one. Fig. 2b provides a visual proof account (as long as the reader sees the right triangle and uses the Pythagorean theorem). Kant concludes: "And indeed, one is surprised and justifiably put into *admiration* by such a curious combination of the manifold arising from such fruitful rules and such a poor and simple-minded thing, as is the circle" (Kant, 1960, p. 656, emphasis added). He notes that if there are already an infinite number of properties arising just from considering the circle, how much greater is the manifold of all harmonic relations were we to take into account other figures. It is so astonishing that we may continuously admire the beauty of nature. In part, admiration arises from the fact that the *cause* of what astonishes us is unavailable; admiration, Kant suggests, therefore is a daughter of ignorance. More positively framed, there is always excess in astonishment, something that escapes the intentionality of the agential subject. Admiration for an effect stops as soon as we can easily see the appropriateness of the cause that explains it. All admiration for the intricate working of the human body stops when it is taken to be the work of an almighty power, inherently able to achieve anything (Kant, 1960). The idea of the thing that causes astonishment, described or represented, "always has to be beautiful" (Kant, 1964, p. 569). If it were not, astonishment would become discouragement / deterrence [*Abschreckung*], "which is very different from admiration, as a judgment where one cannot stop being astonished" (p. 569).

Being unseen and unforeseen (unexpected), that which is new *gives itself* (Marion, 2001)—the mystery object has given itself in Melissa's sensations in a different way, which, as subsequently seen in her new model, will have been a flat rectangular solid. The donation—e.g., the unanticipated discovery that the mystery object feels like a flat rectangular solid rather than as a cube or that the chord is the diameter of the circle that has the same surface as the ring (Fig. 2b)—manifests itself in the new (gift) that astonished her (us), but as a process precedes this manifestation. The new uncontrollably advenes making me, the subject, subject and subjected to astonishment. The subject, therefore, also is what philosophers have called an *advenant,* the one to whom the new advenes (Romano, 1998) or the *gifted* [l'adonné], the one to whom something is given (Marion, 1998). The concept of advenant, suitable to theorize both teachers and students in the classroom (Roth, 2013), underscores that explorations—whether in the second-grade mathematics classroom or those we may engage in seeking to proof the theorems concerning the circle—also are *adventures* in all senses of this term.

In discovering something new heretofore unknown changes the subject in the sense that it can now do (e.g., manipulate or cognize) what it had not been able to do (manipulate or cognize) before. Melissa, following her astonishment, now is enabled to redo her model of the world and, in so doing, expressing a new understanding of it. Astonishment marks the instant in time when her possibilities as the subject of inquiry are reconfigured (e.g., Romano, 1998). As a subject, Melissa now also is afforded to understand herself in a new way and therefore as having been re/born; and, as in its natural counterpart, the subject neither did nor could foresee her own re/birth.

3.2 Affect

In the vignette, we watch Melissa raise her head, pucker her lips with a grin, and then smile saying that she is making "it something else." Apparent for everyone seeing her in the situation, and to viewers of the video, there is the manifestation of positively shaded surprise: astonishment. Having had to do geometrical proofs nearly half a century ago, I found myself in pleasant surprise when finding a proof for each of the three theorems Kant states while working on this text. Just as the philosopher, the repeated instances of being astonished became admixed with an admiration for the (pleasant) surprises that the mathematics of such a simple object as the circle had in stock for me. But affect is not part of Kant's (1956) understanding of astonishment, a phenomenon subordinate to admiration, which is the truly sublime because it goes without affect. As a result, in the Kantian system, "the true nobility of the sublime is therefore apathy, the absence of affect and *tone*" (Nancy, 2008, p. 100–101). This (constructivist) system of thought, therefore, exhibits "the most basic of defects of traditional approaches to the study of psychology, [which] has been the isolation of the intellectual from the volitional and affective aspects of consciousness" (Vygotsky, 1987, p. 50). A more complete understanding of astonishment, therefore, requires us to also attend to its affective dimensions without reducing these to the intellectual.

Philosophers have noted that astonishment involves a form of "pleasure [that] always has a surprise as its cause, thus an event, the coming of the new, the unanticipatable and non-repeatable" (Derrida, 1991, p. 185). Astonishment, admiration, and positive affect are allied. Thus, Descartes, attributes affective primacy to admiration: it is the most important affect. Admiration "translates emotion, astonishment, surprise, interrogation in the face of what oversteps the measure: in the face of the 'extraordinary,' says Descartes" (Derrida, 1987, p. 438). Astonishment is allied with wonder, one of the fundamental temperaments, a feeling that lies at the heart and constituting the beginning point of (philosophical) inquiry (Plato, 1881). We may initially be lost in wonder, which led Kant (1964) to note that astonishment is an excitation of feeling that may initially hinder the play of thoughts; but then, the feeling fosters, when experienced positively, the turning of thought towards the unexpected idea. "*This* affect *is called amazement* only, when we are uncertain whether the perception is happening awake or dreaming" (p. 593). But those deeply engaging in inquiry will be amazed about a truth [*Weisheit*] they were not aware of; and this is an affect that is due to reason. *Wonder*, however, "is not a problem to be solved. It is, moreover, not something to be learned, but at most, something to be practiced like dealing with death" (Waldenfels, 2008, p. 136). Wonder, astonishment, amazement, and admiration all are pathic experiences: these are phenomena that happen to us. In this way, they escape the constructivist theorizing, concerned as it is with the agential forms of relating to the world and with associated intellectualized versions of phenomena.

To be astonished indicates more than just to be surprised, which can be heard as referring to the unexpected nature of that which surprises. To be astonished also invokes the affect that goes with a surprise. Astonishment is the manifestation of *Being-affected* [*Getroffensein*], which has a perfective dimension in that what we are responding to already has happened and lies in the past (and therefore is articulated in the perfect tense). As we see in the experience of Melissa, the object in her hand *gives itself* in some way to

her sense of touch only to give itself in a different way. With the sense of touch, she exposes herself to the world to be touched by what was unintended and has given itself. She is affected even before the difference in the feeling becomes conscious of a difference; the consciousness of difference is preceded by a change in how the object feels. She could not interpret because the new feel of the object was only the end result of a process that led to intellectual surprise. The unexpected change in sensation is reflected intellectually after the fact, subsequently leading Melissa to change her plasticine model into a flat rectangular solid; and it is reflected affectively, as manifested in her facial expression that includes a smile. We are astonished because something has happened to us, has touched us, which we had not anticipated and could not have anticipated. We experience ourselves as *having-been-affected-by*, and that "by," the cause, is available only after the effect, the "being-affected." Only the reply part of the response reveals that which has affected us (Waldenfels, 2002). There is therefore an inversion of cause and effect as soon as the senses, affect, and the passions are involved. The fact that enters our awareness, the unseen and unforeseen, arises from an uncontrollable past (Marion, 2001), that is, it is in excess of the agency of the subject. We know the cause only after its effect, when we have been affected by something that we have not and could not have foreseen. This radically questions all attempts to theorize the subject as the result of an autopoietic (i.e., auto-constitutive) project—with respect to both knowledge and identity. This is so because the subject (to whom astonishment and admiration give themselves) actually is born; the subject thereby "finally admits <u>being unable</u>, or especially not being obliged, <u>to auto-constitute</u> by means of *cogitatio sui* [self-knowledge, self-consciousness] or *causa sui* [cause of itself], but <u>receives itself</u> from the given phenomenon and from it alone" (Marion, 1998, p. 361, original emphasis, underline added). That is, the subject is unable to constitute (i.e., construct) itself in that experience—in contrast to the many claims of identity and knowledge being the result of the individual's efforts.

In summary, therefore, the affective side of our phenomenon (astonishment) is not just important in its own right but it also requires us to reconsider our notions of the autopoeitic (agential) subject. It is common to read that students construct their knowledge, identity, and relations or negotiate different understandings. But in the experience of being-affected by what gives itself to their senses and its affective evaluation, we observe the pathic nature of the subject: subject and subjected to what is other. Moreover, there is a diastatic moment with respect to the person, because the person finding herself being astonished is different from the person to whom something has happened. The Melissa of the cube and the Melissa of the rectangular solid are different, they feel and experience different things: they live in different worlds. The *perežívanie* [experience]—i.e., the unity/identity of person and environment (Vygotsky, 1994)—also is different. A true cultural development has occurred. The difference and changeover comes to be marked and remarked in the experience of astonishment. Astonishment manifests that something has happened to ([thunder-] struck, stunned) *me*, which precedes the "I" that finds itself astonished. This "I" therefore is different from the "I" that had been exposed and has been affected. This questions the claims concerning individual identity that are often made in the constructivist literature of mathematics education, which orients itself to the agential aspects of the notion—e.g., in statements such as "[the individual] constructs her identity." Unsurprisingly, perhaps, there is therefore an "astonishing *diastasis of identity* and its rediscoveries" (Levinas, 1978, p. 52,

emphasis added). Thus, astonishment allows us to experience ourselves in flux and to become conscious thereof. Because of the unexpectedness of the new that astonishes us, what we will be and what we will know also lies outside of our reach, outside of our current horizon of possibilities to be.

4 Towards a culture of astonishment

Learning something really new is different from incrementally perfecting something that one already knows but that one gets better at it. The really new—as seen in the classroom episode or in our own proofing of the truths about the circle—that is, the unexpected, comes as a surprise that leads to our being astonished. This is why experienced mathematicians can continue to be astonished by and be in admiration of a certain beauty that arises from the plenitude of things one can learn from the simplest of mathematical phenomena. Why should we not organize school-based learning around astonishment and admiration? Astonishment requires exposure to and engagement with the unexpected and unknown, which thereby also comes with a challenge to the notions that the person has of herself: no longer in control but exposed to the vagaries, intemperies, and caprices of the adventure. In this context, we are required to think about passions both in terms of something that we undergo—as in the passions of Christ—and something that we have for the intellectual subject matter of mathematics.

Passions, in the philosophical tradition, are affects of our bodies (Spinoza, 2002); these happen to us and, therefore, exceed our agency, the relationship between will (what we want to do, mind) and our doing (affecting the world and others). Passions exceed the possibilities of any constructivist project (Roth, 2011b). It may come as no surprise that Vygotsky (1999), keen to overcome the false and problematic separation of intellect (will, mind) and affect (body) would near the end of his life turn to Spinoza, highlighting particularly the importance of astonishment. This is so because "the will . . . is determined to act not of itself, not by virtue of its absolute freedom, but according to indispensible laws of the spirit–body of man to which all passions, including astonishment, are subject" (p. 172). Astonishment is a model for how to think the relationship of the whole person in flesh and blood—a central concern for Vygotsky (e.g., 1987) near the end of his life—and an aspect of practical material life such as mathematics. But to have a passion for a cultural practice such as mathematics, to find oneself in admiration of mathematics, requires a *culture of astonishment*, a culture that supports exposure to the unknown and unexpected. There has to be the possibility that even having been wrong or having had an inappropriate conception may be experienced with positive affect, and, therefore as a positive surprise: astonishment.

Intellectual surprise only lies near the end of a movement of astonishment. In astonishment, a boundary (limit) is transgressed and a new intellectual horizon has opened that could not have been anticipated before. The new was not constructed but has happened to the person, who did not even know what was coming (i.e., Melissa did not know she would feel a rectangular prism). In school classrooms where astonishment is cultured (fostered), learners are led to curiosity, which then leads to further experiences of astonishment. For astonishment to occur we have to expose ourselves to the invisible other; the other so other than it cannot be conceived as the negative of what exists. With exposure comes vulnerability. Nurturing a culture of astonishment means abandoning

ways of teaching that lead children ask the teacher, "Am I (are we) right so far?" It means nurturing a culture where children can abandon themselves to forms of participation in which not only their current convictions and understandings are overturned but also where this overturning of prior conviction comes together with positive forms of affect: sense of wonder, gratitude, and excitement.

What might an effort to culture astonishment require? There are suggestions from the area of drama education that there needs to exist a particular form of "management": "management of the unforeseen, which relies on a scrupulous preparation, that makes teaching not a simple process of knowledge transmission but a space of astonishment and joint discovery" (Merlant, 2004, p. 68–69). This "management" will not be without reward, for "after the pleasure to be astonished, there is none greater than that of causing a surprise . . . that is to cause the pleasure to be astonished in the other" (Derrida, 1991, p. 185). That is, teachers, too, are afforded the pleasures of having brought about experiences of astonishment in their students. What such classrooms precisely look like remains to be explored—perhaps using the *design experiment* as method, where an initial curriculum comes to be continuously refined as a result of the research concurrently conducted. It may indeed turn out that we, researchers, may be astonished about what precisely is involved in making possible a culture where students experience mathematics in and through astonishment. At the outset, it would appear that the recognition of something as unforeseen could be made salient when the foreseen is articulated initially; and there needs to be an affective climate that makes possible and capitalizes on the opportunities arising from all those attempts even when the results are wrong. For example, admiration might be fostered: by allowing learners to wonder about the fact that in doubling something occurs once more and again and again. But the mere multiplication—1 x 2 = 2, 2 x 2 = 4, etc.—hinders becoming astonished (Waldenfels, 2002). That is, a generalization that "transform[s] an object of knowledge] into an object of consciousness" (Radford, 2010, p. 5) constitutes a "poetic moment of objectification" (p. 6). It is the basis of that aesthetic (i.e., affective) experience that transpires from Kant's and Waldenfels' examples.

The title of this text suggests that the rethinking of the curriculum according to the principle of astonishment is *post-constructivist* in nature. How might this be so? The transitive verb "to construct," which presupposes the object/motive that is constructed, undercuts, prevents, and occludes that we cannot ever anticipate the alien, the unknown, the unexpected, the unseen, and, therefore, the unforeseen. On the other hand, teaching for astonishment allows learners to appreciate *that* something has happened to them so that they now can do what they were unable to do before. In a classroom where astonishment is the norm, learners can appreciate that there is a purpose for school because of the exceptional and extraordinary that is designed to be happening to them. It has been noted that precisely because learning mathematics means coming to know something previously unknown, something inherently unseen and therefore *unforeseen*, it has an essentially pathic dimension (Roth, 2012). We come to know some aspect of mathematics unknown before. In our everyday lives, we often are not surprised, astonished, or amazed at the fact that there are things in our surroundings that we had never noticed before even though we may have passed them for decades. If we are not surprised, astonished, or amazed, we are not driven to wonder and investigate what is the cause of this thing or experience. The person who can no longer wonder is blasé. If we

can no longer be astonished—i.e., feel amazement when we become aware of something unexpected—then all inclination for learning something (really) new is gone. On the other hand, if classrooms are organized such that they culture astonishment, they also may lead to wonderment and admiration of the beauty of the field of inquiry, here mathematics. In creating a culture of astonishment we can maintain remain hopeful that students really become passionate about mathematics that leads so many students to experience in negative ways. Passions set us up to foster a particular affective relation of the learner to the subject matter that is supportive of continued inquiry into the invisible. We easily see here the possibility that an affective inversion may occur when students do not or cannot see the beauty in the manifold of properties that arise even from very simple mathematical objects. Instead of being led to admiration, students become discouraged to the point of developing aversion to mathematics.

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